Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/170-6.2.7-hyper^m-a+b-cosh^n-^p

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 8:19am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [85]. This is test number [170].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (85)	0.00 (0)
Mathematica	97.65 (83)	2.35 (2)
Maple	97.65 (83)	2.35 (2)
Fricas	89.41 (76)	10.59 (9)
Mupad	64.71 (55)	35.29 (30)
Giac	48.24 (41)	51.76 (44)
Maxima	40.00 (34)	60.00 (51)
Sympy	21.18 (18)	78.82 (67)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

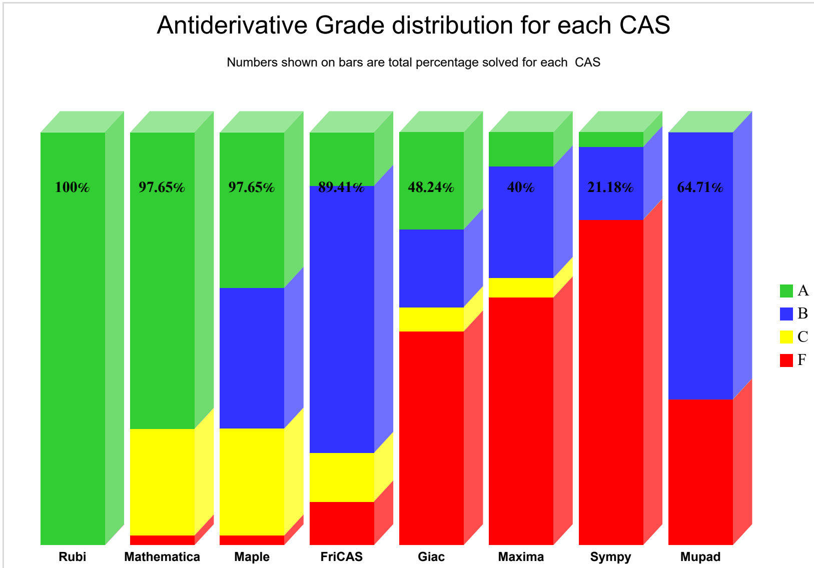
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

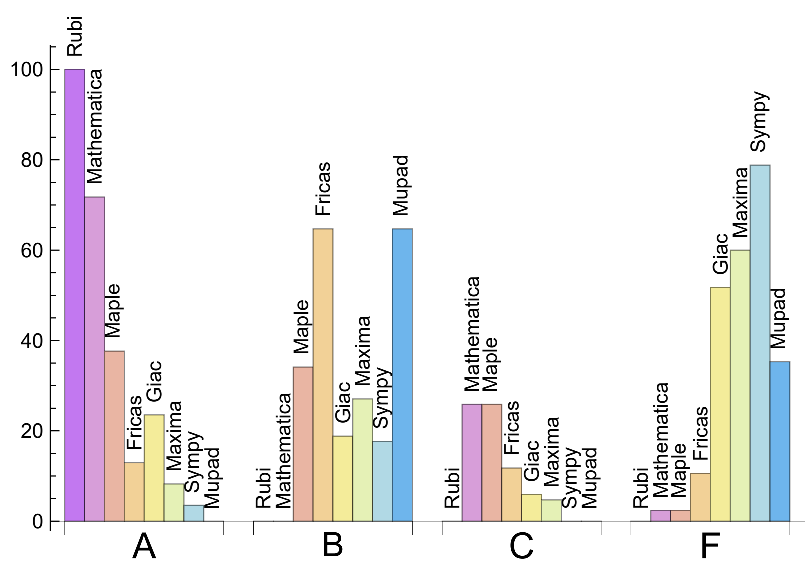
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.294	0.000	4.706	0.000
Mathematica	71.765	0.000	25.882	2.353
Maple	37.647	34.118	25.882	2.353
Giac	23.529	18.824	5.882	51.765
Fricas	12.941	64.706	11.765	10.588
Maxima	8.235	27.059	4.706	60.000
Sympy	3.529	17.647	0.000	78.824
Mupad	0.000	64.706	0.000	35.294

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	0.00	100.00	0.00
Maple	2	100.00	0.00	0.00
Fricas	9	66.67	0.00	33.33
Mupad	30	0.00	100.00	0.00
Giac	44	97.73	0.00	2.27
Maxima	51	100.00	0.00	0.00
Sympy	67	55.22	43.28	1.49

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.27
Rubi	0.36
Giac	0.39
Fricas	0.40
Mathematica	1.04
Maple	3.74
Mupad	5.67
Sympy	9.85

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	72.94	1.10	51.00	1.00
Maple	85.66	1.47	66.00	1.27
Rubi	87.88	1.07	49.00	1.00
Maxima	124.91	2.79	56.00	2.11
Giac	154.07	2.17	45.00	1.64
Mupad	443.51	5.79	243.00	3.33
Sympy	4434.50	151.69	119.00	3.81
Fricas	18607.54	89.10	353.00	8.70

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

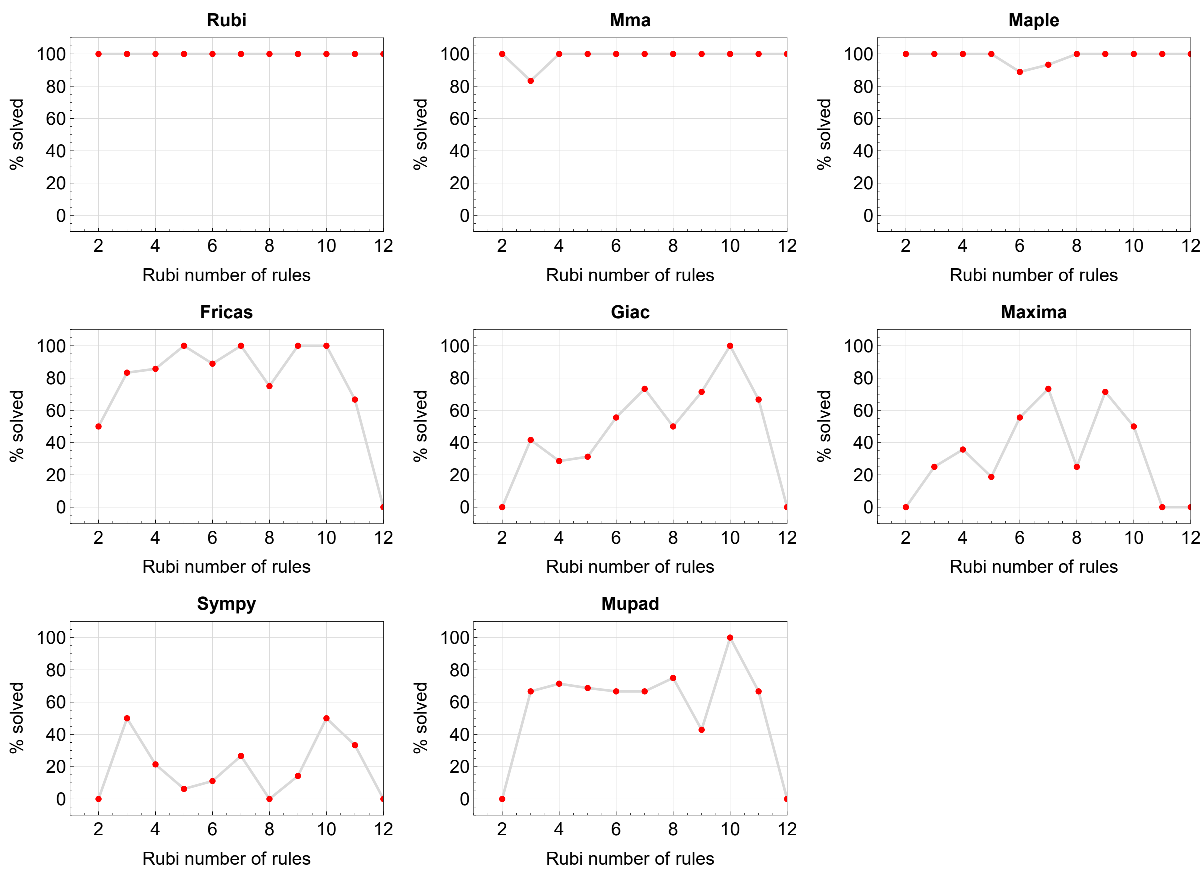


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

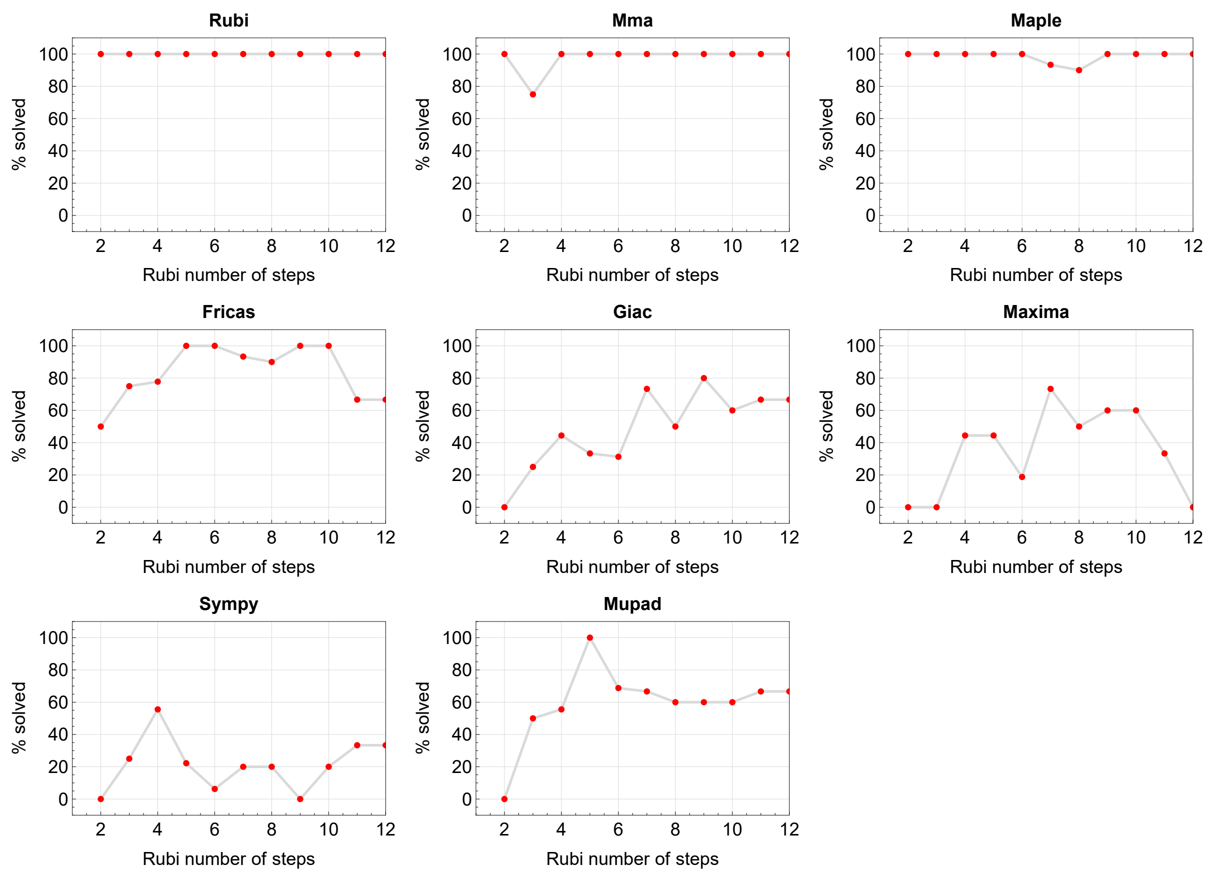


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

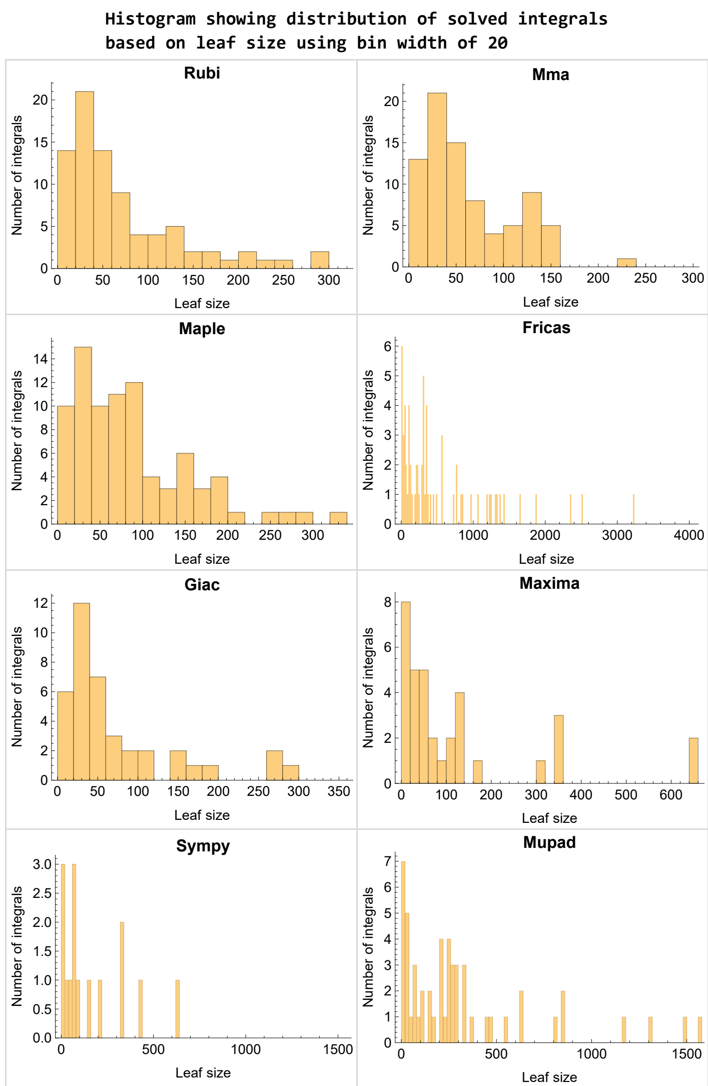


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

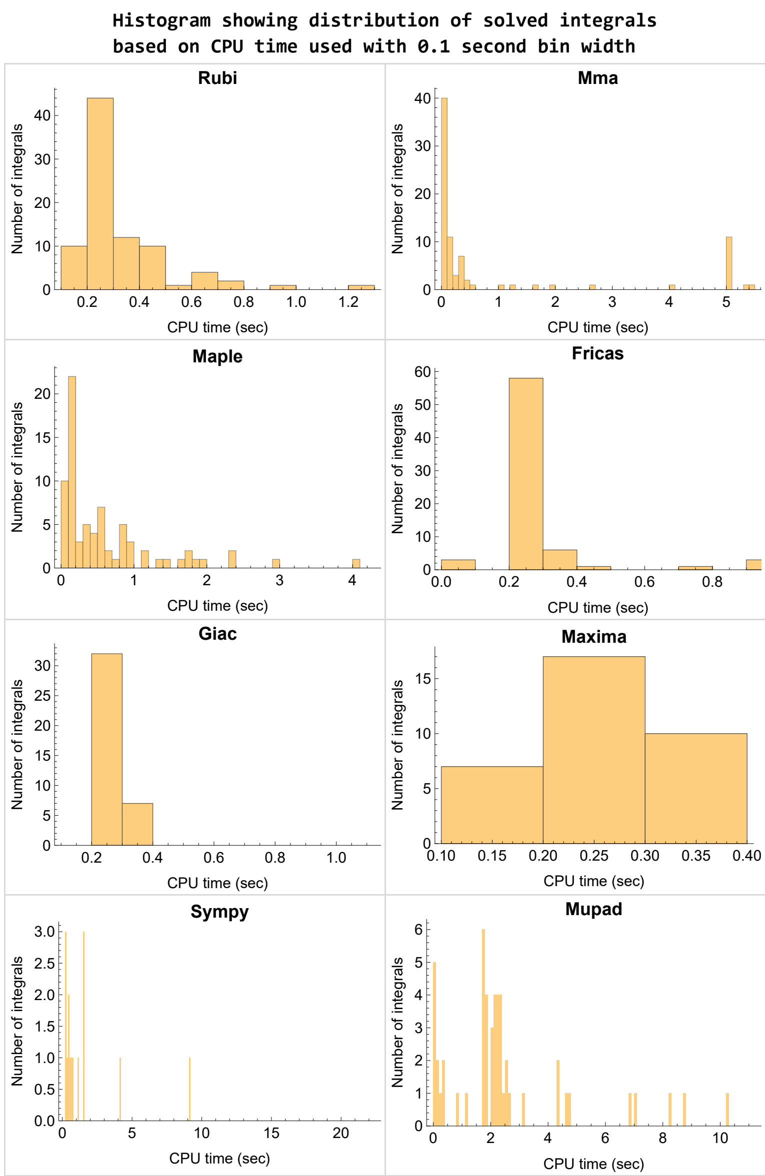


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

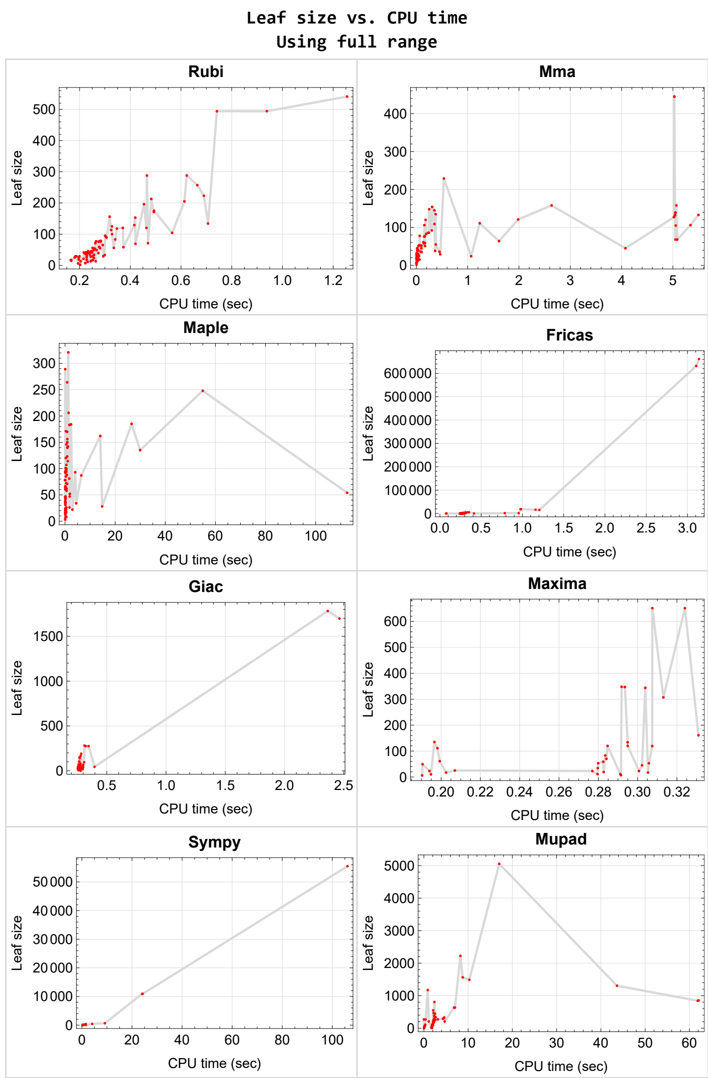


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

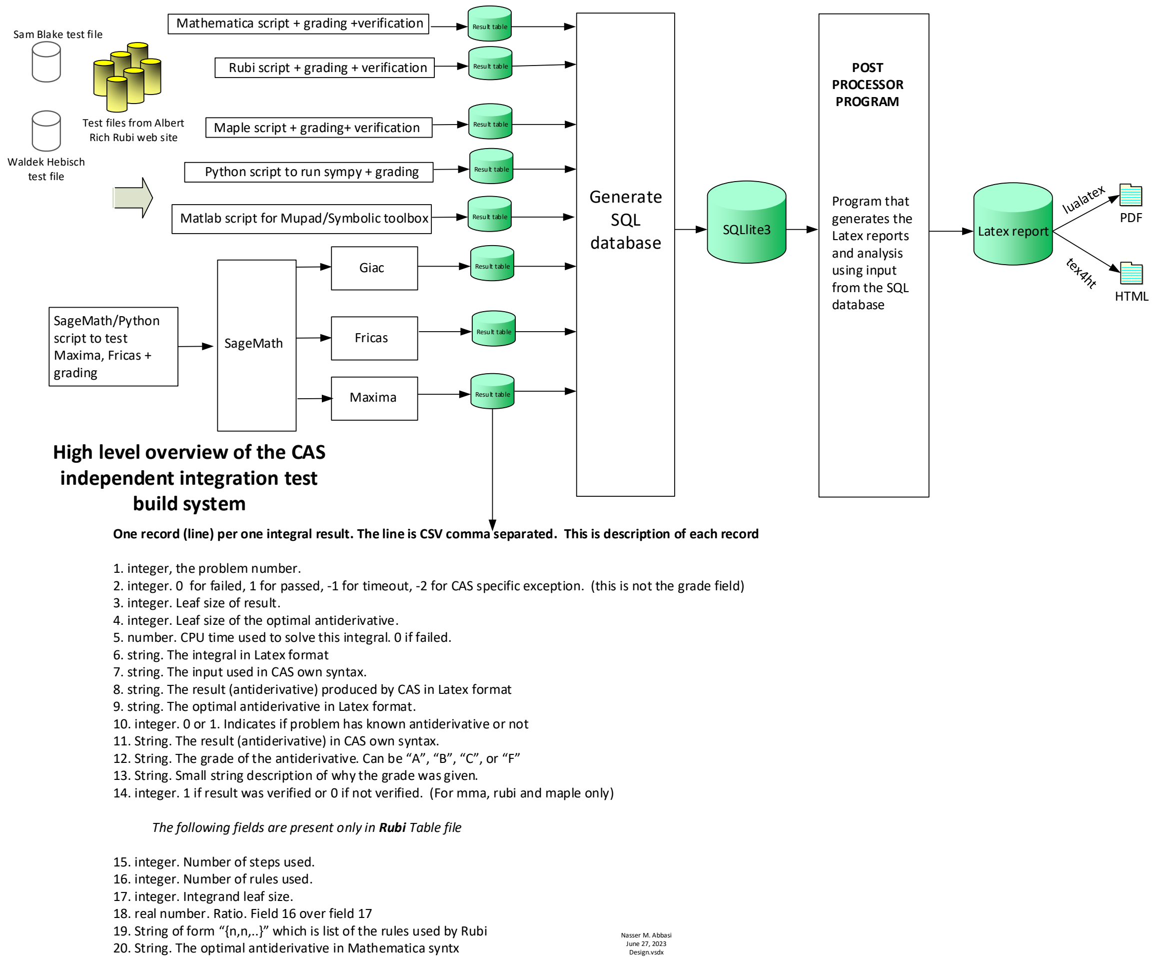
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85 }

B grade { }

C grade { 4, 5, 39, 40 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 61, 63, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85 }

B grade { }

C grade { 6, 7, 8, 10, 11, 12, 57, 58, 60, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 81 }

F normal fail { }

F(-1) timedout fail { 56, 59 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 37, 38, 39, 40, 43, 44, 45, 47, 48, 49, 50, 51, 54, 55, 76, 77, 78, 79, 84, 85 }

B grade { 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 41, 42, 46, 52, 53, 63 }

C grade { 19, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 81 }

F normal fail { 82, 83 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 15, 25, 44, 49, 54, 55, 84, 85 }

B grade { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 48, 51, 52, 53, 60, 61, 63, 66, 69, 70, 72, 73, 75, 76, 77, 78, 79, 80, 83 }

C grade { 56, 57, 58, 59, 62, 65, 68, 71, 74, 81 }

F normal fail { 41, 42, 45, 46, 47, 50 }

F(-1) timedout fail { }

F(-2) exception fail { 64, 67, 82 }

2.1.5 Maxima

A grade { 1, 3, 37, 44, 49, 54, 76 }

B grade { 2, 4, 5, 13, 14, 15, 16, 17, 18, 21, 23, 25, 27, 29, 31, 33, 34, 35, 36, 38, 39, 40, 63 }

C grade { 43, 48, 53, 80 }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 19, 20, 22, 24, 26, 28, 30, 32, 41, 42, 45, 46, 47, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 81, 82, 83, 84, 85 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 15, 16, 19, 25, 27, 29, 33, 37, 39, 40, 72, 74, 75, 76, 81 }

B grade { 13, 14, 21, 23, 35, 36, 38, 44, 49, 54, 58, 59, 60, 61, 63, 71 }

C grade { 43, 48, 53, 62, 80 }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 17, 18, 20, 22, 24, 26, 28, 30, 31, 32, 34, 41, 42, 45, 47, 50, 51, 52, 55, 56, 57, 64, 65, 66, 67, 68, 69, 70, 73, 77, 78, 79, 82, 83, 84, 85 }

F(-1) timedout fail { }

F(-2) exception fail { 46 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 56, 57, 58, 59, 60, 61, 62, 63, 65, 68, 71, 74, 75, 76, 81 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 64, 66, 67, 69, 70, 72, 73, 77, 78, 79, 80, 82, 83, 84, 85 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 2, 3, 19 }

B grade { 1, 9, 16, 26, 27, 35, 36, 37, 38, 39, 40, 58, 59, 63, 74 }

C grade { }

F normal fail { 4, 5, 10, 11, 17, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85 }

F(-1) timedout fail { 6, 7, 8, 12, 13, 14, 15, 18, 20, 21, 22, 23, 24, 25, 33, 34, 47, 48, 49, 50, 56, 57, 60, 61, 62, 71, 72, 73, 75 }

F(-2) exception fail { 70 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	19	26	25	14	153	26	25
N.S.	1	0.90	0.95	1.30	1.25	0.70	7.65	1.30	1.25
time (sec)	N/A	0.242	0.003	1.776	0.207	0.244	0.744	0.261	1.780

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	17	7	10	12	7
N.S.	1	1.00	1.00	1.14	2.43	1.00	1.43	1.71	1.00
time (sec)	N/A	0.237	0.002	0.523	0.203	0.252	0.444	0.263	1.706

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.194	0.000	0.167	0.190	0.255	0.273	0.260	0.030

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	24	22	22	61	100	0	21	21
N.S.	1	1.26	1.16	1.16	3.21	5.26	0.00	1.11	1.11
time (sec)	N/A	0.270	0.005	2.949	0.199	0.245	0.000	0.266	1.703

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	34	32	28	135	216	0	27	27
N.S.	1	1.17	1.10	0.97	4.66	7.45	0.00	0.93	0.93
time (sec)	N/A	0.279	0.004	14.812	0.197	0.253	0.000	0.261	1.709

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	148	94	0	2346	0	0	805
N.S.	1	1.00	1.90	1.21	0.00	30.08	0.00	0.00	10.32
time (sec)	N/A	0.304	0.251	0.057	0.000	0.313	0.000	0.000	2.382

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	120	54	0	1064	0	0	548
N.S.	1	1.00	2.22	1.00	0.00	19.70	0.00	0.00	10.15
time (sec)	N/A	0.274	0.181	112.590	0.000	0.287	0.000	0.000	2.115

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	83	34	0	416	0	0	257
N.S.	1	1.00	2.31	0.94	0.00	11.56	0.00	0.00	7.14
time (sec)	N/A	0.240	0.193	4.456	0.000	0.278	0.000	0.000	2.186

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	0	300	66	0	16
N.S.	1	1.00	1.00	0.68	0.00	12.00	2.64	0.00	0.64
time (sec)	N/A	0.206	0.037	0.434	0.000	0.274	0.347	0.000	1.782

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	106	52	0	349	0	0	462
N.S.	1	1.00	2.52	1.24	0.00	8.31	0.00	0.00	11.00
time (sec)	N/A	0.247	0.157	1.907	0.000	0.280	0.000	0.000	2.210

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	77	154	87	0	1332	0	0	2225
N.S.	1	1.26	2.52	1.43	0.00	21.84	0.00	0.00	36.48
time (sec)	N/A	0.291	0.308	6.491	0.000	0.296	0.000	0.000	8.262

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	125	229	135	0	5326	0	0	5056
N.S.	1	1.33	2.44	1.44	0.00	56.66	0.00	0.00	53.79
time (sec)	N/A	0.346	0.536	29.974	0.000	0.352	0.000	0.000	17.032

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	118	76	289	651	1308	0	166	248
N.S.	1	1.34	0.86	3.28	7.40	14.86	0.00	1.89	2.82
time (sec)	N/A	0.381	0.153	0.078	0.307	0.291	0.000	0.279	2.571

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	76	52	185	348	568	0	103	146
N.S.	1	1.29	0.88	3.14	5.90	9.63	0.00	1.75	2.47
time (sec)	N/A	0.302	0.098	26.566	0.292	0.281	0.000	0.266	2.115

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	36	88	120	300	0	52	79
N.S.	1	1.05	0.92	2.26	3.08	7.69	0.00	1.33	2.03
time (sec)	N/A	0.257	0.083	0.729	0.295	0.276	0.000	0.263	0.250

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	78	53	293	10924	39	267
N.S.	1	1.00	1.00	2.69	1.83	10.10	376.69	1.34	9.21
time (sec)	N/A	0.205	0.465	0.109	0.306	0.273	24.042	0.274	0.395

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	162	161	1875	0	0	245
N.S.	1	1.00	1.00	2.75	2.73	31.78	0.00	0.00	4.15
time (sec)	N/A	0.303	0.170	14.045	0.331	0.281	0.000	0.000	2.380

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	248	307	4977	0	0	333
N.S.	1	1.00	1.03	2.79	3.45	55.92	0.00	0.00	3.74
time (sec)	N/A	0.326	0.300	54.962	0.313	0.310	0.000	0.000	2.497

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	100	77	26	0	305	85	80	205
N.S.	1	1.02	0.79	0.27	0.00	3.11	0.87	0.82	2.09
time (sec)	N/A	0.349	0.164	0.532	0.000	0.256	0.482	0.277	4.709

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	206	0	2508	0	0	293
N.S.	1	1.00	1.10	2.64	0.00	32.15	0.00	0.00	3.76
time (sec)	N/A	0.296	0.233	1.423	0.000	0.283	0.000	0.000	2.345

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	120	76	170	651	1245	0	150	178
N.S.	1	1.36	0.86	1.93	7.40	14.15	0.00	1.70	2.02
time (sec)	N/A	0.400	0.163	0.843	0.324	0.283	0.000	0.269	2.260

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	146	0	1184	0	0	243
N.S.	1	1.00	1.09	2.61	0.00	21.14	0.00	0.00	4.34
time (sec)	N/A	0.274	0.137	0.518	0.000	0.280	0.000	0.000	2.180

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	76	52	120	347	573	0	95	142
N.S.	1	1.29	0.88	2.03	5.88	9.71	0.00	1.61	2.41
time (sec)	N/A	0.301	0.095	0.322	0.294	0.282	0.000	0.309	2.068

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	101	0	498	0	0	204
N.S.	1	1.00	1.00	2.66	0.00	13.11	0.00	0.00	5.37
time (sec)	N/A	0.234	0.028	0.214	0.000	0.278	0.000	0.000	2.057

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	92	120	317	0	50	376
N.S.	1	1.00	0.92	2.36	3.08	8.13	0.00	1.28	9.64
time (sec)	N/A	0.294	0.451	0.109	0.285	0.265	0.000	0.278	2.328

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	66	0	337	55498	0	87
N.S.	1	1.00	1.00	2.28	0.00	11.62	1913.72	0.00	3.00
time (sec)	N/A	0.217	0.012	0.104	0.000	0.263	106.055	0.000	2.072

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	78	53	293	10924	39	267
N.S.	1	1.00	1.00	2.69	1.83	10.10	376.69	1.34	9.21
time (sec)	N/A	0.197	0.019	0.000	0.280	0.265	24.281	0.261	0.003

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	45	85	0	360	0	0	208
N.S.	1	1.00	1.10	2.07	0.00	8.78	0.00	0.00	5.07
time (sec)	N/A	0.229	0.097	0.313	0.000	0.272	0.000	0.000	2.234

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	107	70	457	0	58	108
N.S.	1	1.00	1.00	2.82	1.84	12.03	0.00	1.53	2.84
time (sec)	N/A	0.250	0.366	0.534	0.284	0.261	0.000	0.276	0.311

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	71	58	123	0	963	0	0	447
N.S.	1	1.20	0.98	2.08	0.00	16.32	0.00	0.00	7.58
time (sec)	N/A	0.288	0.146	0.836	0.000	0.276	0.000	0.000	2.536

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	142	119	1377	0	0	239
N.S.	1	1.00	1.00	2.58	2.16	25.04	0.00	0.00	4.35
time (sec)	N/A	0.289	0.379	1.194	0.307	0.289	0.000	0.000	2.260

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	113	86	183	0	3239	0	0	1305
N.S.	1	1.26	0.96	2.03	0.00	35.99	0.00	0.00	14.50
time (sec)	N/A	0.348	0.245	1.742	0.000	0.303	0.000	0.000	43.632

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	171	134	1239	0	104	0
N.S.	1	1.00	1.05	2.63	2.06	19.06	0.00	1.60	0.00
time (sec)	N/A	0.321	5.084	0.356	0.295	0.271	0.000	0.269	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	120	106	264	344	5117	0	0	0
N.S.	1	1.12	0.99	2.47	3.21	47.82	0.00	0.00	0.00
time (sec)	N/A	0.497	5.343	0.852	0.304	0.335	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	34	66	60	34	50
N.S.	1	1.00	1.00	2.40	2.27	4.40	4.00	2.27	3.33
time (sec)	N/A	0.182	0.062	0.084	0.280	0.259	0.279	0.264	0.149

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	60	59	214	211	59	76
N.S.	1	1.00	1.00	1.71	1.69	6.11	6.03	1.69	2.17
time (sec)	N/A	0.254	0.102	0.210	0.283	0.256	1.162	0.272	1.886

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	56	51	72	83	575	428	71	112
N.S.	1	1.10	1.00	1.41	1.63	11.27	8.39	1.39	2.20
time (sec)	N/A	0.365	0.169	0.456	0.284	0.256	4.101	0.261	1.841

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	10	20	14	10	10
N.S.	1	1.00	1.00	1.50	5.00	10.00	7.00	5.00	5.00
time (sec)	N/A	0.223	0.003	0.046	0.195	0.248	0.212	0.267	0.067

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	21	17	16	49	84	34	18	18
N.S.	1	1.91	1.55	1.45	4.45	7.64	3.09	1.64	1.64
time (sec)	N/A	0.237	0.003	0.079	0.191	0.243	0.524	0.297	1.831

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	31	27	21	111	185	54	24	24
N.S.	1	1.63	1.42	1.11	5.84	9.74	2.84	1.26	1.26
time (sec)	N/A	0.248	0.003	0.092	0.198	0.240	1.572	0.275	0.072

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	114	0	0	0	0	0
N.S.	1	1.00	1.08	2.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.067	1.100	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	58	0	0	0	0	0
N.S.	1	1.00	1.06	3.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.021	0.623	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	11	33	0	31	13
N.S.	1	1.00	1.00	1.15	0.85	2.54	0.00	2.38	1.00
time (sec)	N/A	0.275	0.024	0.133	0.291	0.266	0.000	0.286	1.860

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	0	31	11
N.S.	1	1.00	1.00	1.27	1.00	0.18	0.00	2.82	1.00
time (sec)	N/A	0.265	0.023	0.109	0.280	0.256	0.000	0.280	1.779

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	62	0	0	0	0	0
N.S.	1	1.00	1.03	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.035	0.633	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	134	135	321	0	0	0	0	0
N.S.	1	1.01	1.02	2.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.752	0.377	1.327	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	58	51	99	0	0	0	0	0
N.S.	1	1.05	0.93	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	0.041	0.502	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	23	53	0	66	0
N.S.	1	1.00	0.76	0.64	0.70	1.61	0.00	2.00	0.00
time (sec)	N/A	0.334	0.050	0.124	0.277	0.257	0.000	0.300	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	23	19	0	66	0
N.S.	1	1.00	0.79	0.72	0.79	0.66	0.00	2.28	0.00
time (sec)	N/A	0.321	0.040	0.122	0.301	0.247	0.000	0.299	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	104	78	96	0	0	0	0	0
N.S.	1	1.03	0.77	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.612	0.066	0.425	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	66	0	132	0	0	0
N.S.	1	1.00	1.08	1.35	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	0.272	0.055	0.179	0.000	0.079	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	18	45	0	42	0	0	0
N.S.	1	1.00	1.06	2.65	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.172	0.030	0.143	0.000	0.078	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	30	34	19	49	0	40	0
N.S.	1	1.00	1.76	2.00	1.12	2.88	0.00	2.35	0.00
time (sec)	N/A	0.272	0.035	0.119	0.283	0.253	0.000	0.290	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	28	16	17	17	0	39	0
N.S.	1	1.00	1.87	1.07	1.13	1.13	0.00	2.60	0.00
time (sec)	N/A	0.266	0.037	0.121	0.305	0.257	0.000	0.300	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	61	0	39	0	0	0
N.S.	1	1.00	1.03	1.56	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.036	0.173	0.000	0.074	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	C	F(-1)	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	0	100	0	18612	0	0	633
N.S.	1	1.00	0.00	0.35	0.00	64.62	0.00	0.00	2.20
time (sec)	N/A	0.690	0.000	0.550	0.000	0.976	0.000	0.000	7.029

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	105	94	0	18612	0	0	633
N.S.	1	1.00	0.36	0.33	0.00	64.62	0.00	0.00	2.20
time (sec)	N/A	0.515	5.055	0.532	0.000	0.983	0.000	0.000	6.814

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	133	46	0	226	330	275	291
N.S.	1	1.00	1.46	0.51	0.00	2.48	3.63	3.02	3.20
time (sec)	N/A	0.327	5.498	0.070	0.000	0.275	1.513	0.346	4.324

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	C	B	B	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	0	46	0	234	320	275	295
N.S.	1	1.00	0.00	0.48	0.00	2.46	3.37	2.89	3.11
time (sec)	N/A	0.329	0.000	0.072	0.000	0.265	1.560	0.322	4.323

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	541	121	96	0	771	0	1781	1563
N.S.	1	1.50	0.34	0.27	0.00	2.14	0.00	4.93	4.33
time (sec)	N/A	1.279	1.984	0.334	0.000	0.291	0.000	2.365	8.766

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	156	109	96	0	779	0	1697	1487
N.S.	1	1.54	1.08	0.95	0.00	7.71	0.00	16.80	14.72
time (sec)	N/A	0.329	0.353	0.338	0.000	0.296	0.000	2.464	10.275

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	196	45	36	0	133	0	281	205
N.S.	1	1.11	0.26	0.20	0.00	0.76	0.00	1.60	1.16
time (sec)	N/A	0.494	4.075	0.223	0.000	0.263	0.000	0.312	1.106

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	44	45	115	75	43	61
N.S.	1	1.00	0.96	1.76	1.80	4.60	3.00	1.72	2.44
time (sec)	N/A	0.189	1.068	0.187	0.302	0.255	0.634	0.256	0.133

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	139	156	0	0	0	0	0
N.S.	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.043	5.052	0.914	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	132	140	0	15201	0	0	844
N.S.	1	1.00	0.77	0.82	0.00	88.89	0.00	0.00	4.94
time (sec)	N/A	0.540	5.040	0.940	0.000	1.206	0.000	0.000	61.873

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	257	158	184	0	661324	0	0	0
N.S.	1	1.05	0.64	0.75	0.00	2699.28	0.00	0.00	0.00
time (sec)	N/A	0.737	5.068	2.375	0.000	3.140	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	139	150	0	0	0	0	0
N.S.	1	1.00	0.28	0.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.833	5.045	0.889	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	132	140	0	16379	0	0	855
N.S.	1	1.00	0.75	0.80	0.00	93.59	0.00	0.00	4.89
time (sec)	N/A	0.531	5.041	0.937	0.000	1.159	0.000	0.000	62.083

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	158	184	0	631813	0	0	0
N.S.	1	1.00	0.74	0.86	0.00	2966.26	0.00	0.00	0.00
time (sec)	N/A	0.520	2.635	2.384	0.000	3.107	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	445	62	0	836	0	0	0
N.S.	1	1.00	2.00	0.28	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.724	5.028	0.134	0.000	0.298	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	68	71	0	315	0	140	337
N.S.	1	1.00	0.82	0.86	0.00	3.80	0.00	1.69	4.06
time (sec)	N/A	0.352	5.049	0.861	0.000	0.266	0.000	0.272	2.667

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	127	47	0	737	0	1	0
N.S.	1	1.00	0.98	0.36	0.00	5.71	0.00	0.01	0.00
time (sec)	N/A	0.453	5.020	1.829	0.000	0.282	0.000	0.280	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	445	64	0	852	0	0	0
N.S.	1	1.00	2.17	0.31	0.00	4.16	0.00	0.00	0.00
time (sec)	N/A	0.667	5.028	0.130	0.000	0.297	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	111	47	0	358	632	10	329
N.S.	1	1.00	1.56	0.66	0.00	5.04	8.90	0.14	4.63
time (sec)	N/A	0.502	1.235	0.179	0.000	0.277	9.151	0.266	4.603

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	81	0	358	0	45	271
N.S.	1	1.00	0.93	1.17	0.00	5.19	0.00	0.65	3.93
time (sec)	N/A	0.448	1.612	1.665	0.000	0.262	0.000	0.397	3.162

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	15	22	23	47	0	23	27
N.S.	1	1.27	1.00	1.47	1.53	3.13	0.00	1.53	1.80
time (sec)	N/A	0.210	0.007	0.425	0.194	0.253	0.000	0.255	0.096

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	39	42	0	357	0	0	0
N.S.	1	1.15	1.00	1.08	0.00	9.15	0.00	0.00	0.00
time (sec)	N/A	0.244	0.019	0.163	0.000	0.413	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	0	248	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	9.54	0.00	0.00	0.00
time (sec)	N/A	0.249	0.012	0.140	0.000	0.304	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	63	0	0	0
N.S.	1	1.00	1.00	0.92	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.210	0.008	0.113	0.000	0.274	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	16	19	7	112	0	38	0
N.S.	1	1.00	1.23	1.46	0.54	8.62	0.00	2.92	0.00
time (sec)	N/A	0.287	0.010	0.164	0.292	0.268	0.000	0.276	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	145	93	0	1435	0	191	1173
N.S.	1	1.00	0.95	0.61	0.00	9.38	0.00	1.25	7.67
time (sec)	N/A	0.447	0.352	4.073	0.000	0.955	0.000	0.284	0.879

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.013	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	1648	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	36.62	0.00	0.00	0.00
time (sec)	N/A	0.254	0.016	0.000	0.000	0.786	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	113	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	3.90	0.00	0.00	0.00
time (sec)	N/A	0.270	0.016	0.118	0.000	0.257	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	45	45	38	0	156	0	0	0
N.S.	1	0.96	0.96	0.81	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.265	0.016	0.067	0.000	0.263	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [37] had the largest ratio of [1.12500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	0.90	16	0.438
2	A	7	7	1.00	16	0.438
3	A	4	4	1.00	16	0.250
4	C	7	6	1.26	16	0.375
5	C	8	7	1.17	16	0.438
6	A	6	5	1.00	15	0.333
7	A	6	5	1.00	15	0.333
8	A	6	5	1.00	15	0.333
9	A	5	4	1.00	13	0.308
10	A	7	6	1.00	13	0.462
11	A	8	7	1.26	15	0.467
12	A	9	8	1.33	15	0.533
13	A	11	10	1.34	15	0.667
14	A	8	7	1.29	15	0.467
15	A	7	6	1.05	15	0.400
16	A	4	3	1.00	10	0.300
17	A	5	4	1.00	15	0.267
18	A	6	5	1.00	15	0.333
19	A	11	10	1.02	13	0.769
20	A	5	4	1.00	15	0.267
21	A	9	8	1.36	15	0.533
22	A	5	4	1.00	15	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	1.29	15	0.400
24	A	5	4	1.00	15	0.267
25	A	6	5	1.00	15	0.333
26	A	4	3	1.00	13	0.231
27	A	4	3	1.00	10	0.300
28	A	6	5	1.00	13	0.385
29	A	5	4	1.00	15	0.267
30	A	8	7	1.20	15	0.467
31	A	5	4	1.00	15	0.267
32	A	10	9	1.26	15	0.600
33	A	8	7	1.00	10	0.700
34	A	10	9	1.12	10	0.900
35	A	4	3	1.00	8	0.375
36	A	7	6	1.00	8	0.750
37	A	10	9	1.10	8	1.125
38	A	8	7	1.00	10	0.700
39	C	6	5	1.91	10	0.500
40	C	8	7	1.63	10	0.700
41	A	4	4	1.00	12	0.333
42	A	2	2	1.00	10	0.200
43	A	7	7	1.00	12	0.583
44	A	7	7	1.00	10	0.700
45	A	4	4	1.00	12	0.333
46	A	11	11	1.01	12	0.917
47	A	8	8	1.05	10	0.800
48	A	9	9	1.00	12	0.750
49	A	9	9	1.00	10	0.900
50	A	12	12	1.03	12	1.000
51	A	4	4	1.00	12	0.333
52	A	2	2	1.00	10	0.200
53	A	7	7	1.00	12	0.583
54	A	7	7	1.00	10	0.700
55	A	4	4	1.00	12	0.333
56	A	3	3	1.00	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	1.00	11	0.273
58	A	3	3	1.00	8	0.375
59	A	3	3	1.00	10	0.300
60	A	10	9	1.50	10	0.900
61	A	5	4	1.54	11	0.364
62	A	9	8	1.11	8	1.000
63	A	5	4	1.00	10	0.400
64	A	3	3	1.00	10	0.300
65	A	6	5	1.00	10	0.500
66	A	6	5	1.05	10	0.500
67	A	3	3	1.00	11	0.273
68	A	6	5	1.00	11	0.455
69	A	6	5	1.00	11	0.455
70	A	3	3	1.00	8	0.375
71	A	6	5	1.00	8	0.625
72	A	6	5	1.00	8	0.625
73	A	3	3	1.00	10	0.300
74	A	12	11	1.00	10	1.100
75	A	12	11	1.00	10	1.100
76	A	7	6	1.27	11	0.545
77	A	7	6	1.15	15	0.400
78	A	6	5	1.00	15	0.333
79	A	6	5	1.00	13	0.385
80	A	10	9	1.00	15	0.600
81	A	6	5	1.00	15	0.333
82	A	7	6	1.00	15	0.400
83	A	8	7	1.00	15	0.467
84	A	7	6	1.00	15	0.400
85	A	8	7	0.96	15	0.467

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sinh^4(x)}{a-a\cosh^2(x)} dx$

3.1.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{\sinh^4(x)}{a-a\cosh^2(x)} dx = \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a}$$

output
```
1/2*x/a-1/2*cosh(x)*sinh(x)/a
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^4(x)}{a-a\cosh^2(x)} dx = -\frac{-\frac{x}{2}+\frac{1}{4}\sinh(2x)}{a}$$

input
```
Integrate[Sinh[x]^4/(a - a*Cosh[x]^2),x]
```

output
```
-((-1/2*x + Sinh[2*x]/4)/a)
```

3.1.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3654, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a-a\cosh^2(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(\frac{\pi}{2}+ix\right)^4}{a-a\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \; 3654$$

$$\frac{\int -\sinh^2(x)dx}{a}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \sinh^2(x)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int -\sin(ix)^2dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \sin(ix)^2dx}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)}{a}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)}{a}$$

input
```
Int[Sinh[x]^4/(a - a*Cosh[x]^2),x]
```

output
```
(x/2 - (Cosh[x]*Sinh[x])/2)/a
```

3.1.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.1.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{x}{2a}-\frac{{\mathrm e}^{2 x}}{8 a}+\frac{{\mathrm e}^{-2 x}}{8 a}$	26
default	$\frac{\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+1\right)}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{2}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)-1\right)}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{2}}{a}$	65

input
```
int(sinh(x)^4/(a-a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a-1/8/a*exp(2*x)+1/8/a*exp(-2*x)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{a - a\cosh^2(x)}\,dx = -\frac{\cosh(x)\sinh(x) - x}{2\,a}$$

input
```
integrate(sinh(x)^4/(a-a*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(cosh(x)*sinh(x) - x)/a
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(14) = 28.

Time = 0.74 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\begin{aligned}\int \frac{\sinh^4(x)}{a - a\cosh^2(x)}\,dx &= \frac{x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} \\ &\quad - \frac{2x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} \\ &\quad + \frac{x}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} \\ &\quad - \frac{2\tanh^3\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} \\ &\quad - \frac{2\tanh\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}\end{aligned}$$

input
```
integrate(sinh(x)**4/(a-a*cosh(x)**2),x)
```

output
```
x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)
**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*
a*tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**
2 + 2*a) - 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^4(x)}{a - a\cosh^2(x)}\,dx = \frac{x}{2\,a} - \frac{e^{(2\,x)}}{8\,a} + \frac{e^{(-2\,x)}}{8\,a}$$

input
```
integrate(sinh(x)^4/(a-a*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/2*x/a - 1/8*e^(2*x)/a + 1/8*e^(-2*x)/a
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^4(x)}{a - a\cosh^2(x)}\,dx = -\frac{\left(2\,e^{(2\,x)} - 1\right)e^{(-2\,x)} - 4\,x + e^{(2\,x)}}{8\,a}$$

input
```
integrate(sinh(x)^4/(a-a*cosh(x)^2),x, algorithm="giac")
```

output
```
-1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))/a
```

3.1.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^4(x)}{a - a\cosh^2(x)}\,dx = \frac{\mathrm{e}^{-2\,x}}{8\,a} - \frac{\mathrm{e}^{2\,x}}{8\,a} + \frac{x}{2\,a}$$

input
```
int(sinh(x)^4/(a - a*cosh(x)^2),x)
```

output
```
exp(-2*x)/(8*a) - exp(2*x)/(8*a) + x/(2*a)
```

3.2 $\int \frac{\sinh^3(x)}{a-a\cosh^2(x)}\,dx$

3.2.1 Optimal result

Integrand size = 16, antiderivative size = 7

$$\int \frac{\sinh^3(x)}{a-a\cosh^2(x)}\,dx=-\frac{\cosh(x)}{a}$$

output
```
-cosh(x)/a
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a-a\cosh^2(x)}\,dx=-\frac{\cosh(x)}{a}$$

input
```
Integrate[Sinh[x]^3/(a - a*Cosh[x]^2),x]
```

output
```
-(Cosh[x]/a)
```

3.2.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 26, 3654, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a-a\cosh^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\cos\left(\frac{\pi}{2}+ix\right)^3}{a-a\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^3}{a-a\sin\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 3654$$

$$-\frac{i\int -i\sinh(x)dx}{a}$$

$$\Big\downarrow \ 26$$

$$-\frac{\int \sinh(x)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int -i\sin(ix)dx}{a}$$

$$\Big\downarrow \ 26$$

$$\frac{i\int \sin(ix)dx}{a}$$

$$\Big\downarrow \ 3118$$

$$-\frac{\cosh(x)}{a}$$

input
```
Int[Sinh[x]^3/(a - a*Cosh[x]^2),x]
```

output
```
-(Cosh[x]/a)
```

3.2.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.2.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\frac{\cosh(x)}{a}$	8
default	$-\frac{\cosh(x)}{a}$	8
risch	$-\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}$	18

input
```
int(sinh(x)^3/(a-a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)/a
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a - a\cosh^2(x)}\,dx = -\frac{\cosh(x)}{a}$$

input
```
integrate(sinh(x)^3/(a-a*cosh(x)^2),x, algorithm="fricas")
```

output
```
-cosh(x)/a
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int \frac{\sinh^3(x)}{a - a\cosh^2(x)}\,dx = \frac{2}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

input
```
integrate(sinh(x)**3/(a-a*cosh(x)**2),x)
```

output
```
2/(a*tanh(x/2)**2 - a)
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \frac{\sinh^3(x)}{a - a\cosh^2(x)}\,dx = -\frac{e^{(-x)}}{2\,a} - \frac{e^x}{2\,a}$$

input
```
integrate(sinh(x)^3/(a-a*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x)/a - 1/2*e^x/a
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{\sinh^3(x)}{a - a\cosh^2(x)}\, dx = -\frac{e^{(-x)} + e^x}{2\,a}$$

input
```
integrate(sinh(x)^3/(a-a*cosh(x)^2),x, algorithm="giac")
```

output
```
-1/2*(e^(-x) + e^x)/a
```

3.2.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a - a\cosh^2(x)}\, dx = -\frac{\cosh(x)}{a}$$

input
```
int(sinh(x)^3/(a - a*cosh(x)^2),x)
```

output
```
-cosh(x)/a
```

3.3 $\int \frac{\sinh^2(x)}{a-a\cosh^2(x)}\,dx$

3.3.1 Optimal result

Integrand size = 16, antiderivative size = 6

$$\int \frac{\sinh^2(x)}{a-a\cosh^2(x)}\,dx = -\frac{x}{a}$$

output
```
-x/a
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{a-a\cosh^2(x)}\,dx = -\frac{x}{a}$$

input
```
Integrate[Sinh[x]^2/(a - a*Cosh[x]^2),x]
```

output
```
-(x/a)
```

3.3.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 25, 3654, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a-a\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(\frac{\pi}{2}+ix\right)^2}{a-a\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2}{a-a\sin\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3654}$$

$$-\frac{\int 1dx}{a}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{x}{a}$$

input
```
Int[Sinh[x]^2/(a - a*Cosh[x]^2),x]
```

output
```
-(x/a)
```

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.3.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
risch	$-\frac{x}{a}$	7
default	$-\frac{2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	11

input
```
int(sinh(x)^2/(a-a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x/a
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{a - a\cosh^2(x)}\, dx = -\frac{x}{a}$$

input
```
integrate(sinh(x)^2/(a-a*cosh(x)^2),x, algorithm="fricas")
```

output
```
-x/a
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{a - a\cosh^2(x)}\,dx = -\frac{x}{a}$$

input
```
integrate(sinh(x)**2/(a-a*cosh(x)**2),x)
```

output
```
-x/a
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{a - a\cosh^2(x)}\,dx = -\frac{x}{a}$$

input
```
integrate(sinh(x)^2/(a-a*cosh(x)^2),x, algorithm="maxima")
```

output
```
-x/a
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{a - a\cosh^2(x)}\,dx = -\frac{x}{a}$$

input
```
integrate(sinh(x)^2/(a-a*cosh(x)^2),x, algorithm="giac")
```

output
```
-x/a
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{a - a\cosh^2(x)}\,dx = -\frac{x}{a}$$

input
```
int(sinh(x)^2/(a - a*cosh(x)^2),x)
```

output
```
-x/a
```

3.4 $\int \frac{\operatorname{csch}^2(x)}{a-a\cosh^2(x)}\,dx$

3.4.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{\operatorname{csch}^2(x)}{a-a\cosh^2(x)}\,dx = -\frac{\coth(x)}{a}+\frac{\coth^3(x)}{3a}$$

output
```
-coth(x)/a+1/3*coth(x)^3/a
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{csch}^2(x)}{a-a\cosh^2(x)}\,dx = -\frac{\frac{2\coth(x)}{3}-\frac{1}{3}\coth(x)\operatorname{csch}^2(x)}{a}$$

input
```
Integrate[Csch[x]^2/(a - a*Cosh[x]^2),x]
```

output
```
-(((2*Coth[x])/3 - (Coth[x]*Csch[x]^2)/3)/a)
```

3.4.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 25, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{a-a\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(\frac{\pi}{2}+ix\right)^2\left(a-a\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix+\frac{\pi}{2}\right)^2\left(a-a\sin\left(ix+\frac{\pi}{2}\right)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$-\frac{\int \operatorname{csch}^4(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc(ix)^4dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{i\int \left(1-\coth^2(x)\right)d(-i\coth(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)}{a}$$

input
```
Int[Csch[x]^2/(a - a*Cosh[x]^2),x]
```

output
```
((-I)*((-I)*Coth[x] + (I/3)*Coth[x]^3))/a
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.4.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
risch	$\frac{4\,{\mathrm e}^{2 x}-\frac{4}{3}}{\left({\mathrm e}^{2 x}-1\right)^{3} a}$	22
default	$\frac{\frac{\tanh \left(\frac{x}{2}\right)^{3}}{3}-3 \tanh \left(\frac{x}{2}\right)+\frac{1}{3 \tanh \left(\frac{x}{2}\right)^{3}}-\frac{3}{\tanh \left(\frac{x}{2}\right)}}{8 a}$	37

input
```
int(csch(x)^2/(a-a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
4/3*(3*exp(2*x)-1)/(exp(2*x)-1)^3/a
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.26

$$\int \frac{\operatorname{csch}^2(x)}{a - a\cosh^2(x)}\, dx$$

$$= \frac{8\left(\cosh\left(x\right) + 2\,\mathrm{s}\right.}{3\left(a\cosh\left(x\right)^{5} + 5\,a\cosh\left(x\right)\sinh\left(x\right)^{4} + a\sinh\left(x\right)^{5} - 3\,a\cosh\left(x\right)^{3} + \left(10\,a\cosh\left(x\right)^{2} - 3\,a\right)\sinh\left(x\right)^{3} + \left(10\right.\right.}$$

input
```
integrate(csch(x)^2/(a-a*cosh(x)^2),x, algorithm="fricas")
```

output
```
8/3*(cosh(x) + 2*sinh(x))/(a*cosh(x)^5 + 5*a*cosh(x)*sinh(x)^4 + a*sinh(x)
^5 - 3*a*cosh(x)^3 + (10*a*cosh(x)^2 - 3*a)*sinh(x)^3 + (10*a*cosh(x)^3 -
9*a*cosh(x))*sinh(x)^2 + 2*a*cosh(x) + (5*a*cosh(x)^4 - 9*a*cosh(x)^2 + 4*
a)*sinh(x))
```

3.4.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a - a\cosh^2(x)}\, dx = -\frac{\int \frac{\operatorname{csch}^2(x)}{\cosh^2(x)-1}\, dx}{a}$$

input
```
integrate(csch(x)**2/(a-a*cosh(x)**2),x)
```

output
```
-Integral(csch(x)**2/(cosh(x)**2 - 1), x)/a
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{\operatorname{csch}^2(x)}{a - a\cosh^2(x)}\, dx = -\frac{4\,e^{(-2\,x)}}{3\,ae^{(-2\,x)} - 3\,ae^{(-4\,x)} + ae^{(-6\,x)} - a} + \frac{4}{3\,(3\,ae^{(-2\,x)} - 3\,ae^{(-4\,x)} + ae^{(-6\,x)} - a)}$$

input
```
integrate(csch(x)^2/(a-a*cosh(x)^2),x, algorithm="maxima")
```

output
```
-4*e^(-2*x)/(3*a*e^(-2*x) - 3*a*e^(-4*x) + a*e^(-6*x) - a) + 4/3/(3*a*e^(-
2*x) - 3*a*e^(-4*x) + a*e^(-6*x) - a)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(x)}{a - a\cosh^2(x)}\,dx = \frac{4\left(3\,e^{(2\,x)} - 1\right)}{3\,a\left(e^{(2\,x)} - 1\right)^3}$$

input
```
integrate(csch(x)^2/(a-a*cosh(x)^2),x, algorithm="giac")
```

output
```
4/3*(3*e^(2*x) - 1)/(a*(e^(2*x) - 1)^3)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(x)}{a - a\cosh^2(x)}\,dx = \frac{4\left(3\,e^{2\,x} - 1\right)}{3\,a\left(e^{2\,x} - 1\right)^3}$$

input
```
int(1/(sinh(x)^2*(a - a*cosh(x)^2)),x)
```

output
```
(4*(3*exp(2*x) - 1))/(3*a*(exp(2*x) - 1)^3)
```

3.5 $\int \frac{\operatorname{csch}^4(x)}{a-a\cosh^2(x)}\,dx$

3.5.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\operatorname{csch}^4(x)}{a-a\cosh^2(x)}\,dx=\frac{\coth(x)}{a}-\frac{2\coth^3(x)}{3a}+\frac{\coth^5(x)}{5a}$$

output
```
coth(x)/a-2/3*coth(x)^3/a+1/5*coth(x)^5/a
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^4(x)}{a-a\cosh^2(x)}\,dx=-\frac{-\frac{8\coth(x)}{15}+\frac{4}{15}\coth(x)\operatorname{csch}^2(x)-\frac{1}{5}\coth(x)\operatorname{csch}^4(x)}{a}$$

input
```
Integrate[Csch[x]^4/(a - a*Cosh[x]^2),x]
```

output
```
-(((-8*Coth[x])/15 + (4*Coth[x]*Csch[x]^2)/15 - (Coth[x]*Csch[x]^4)/5)/a)
```

3.5.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3654, 25, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{a-a\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(\frac{\pi}{2}+ix\right)^4\left(a-a\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int -\operatorname{csch}^6(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \operatorname{csch}^6(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\csc(ix)^6dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \csc(ix)^6dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\int \left(\coth^4(x)-2\coth^2(x)+1\right)d(-i\coth(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{5}i\coth^5(x)+\frac{2}{3}i\coth^3(x)-i\coth(x)\right)}{a}$$

input
```
Int[Csch[x]^4/(a - a*Cosh[x]^2),x]
```

output
```
(I*((-I)*Coth[x] + ((2*I)/3)*Coth[x]^3 - (I/5)*Coth[x]^5))/a
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.5.4 Maple [A] (verified)

Time = 14.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{\frac{32\,{\mathrm e}^{4 x}}{3}-\frac{16\,{\mathrm e}^{2 x}}{3}+\frac{16}{15}}{\left({\mathrm e}^{2 x}-1\right)^{5} a}$	28
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}-\frac{5 \tanh\left(\frac{x}{2}\right)^{3}}{3}+10 \tanh\left(\frac{x}{2}\right)+\frac{1}{5 \tanh\left(\frac{x}{2}\right)^{5}}+\frac{10}{\tanh\left(\frac{x}{2}\right)}-\frac{5}{3 \tanh\left(\frac{x}{2}\right)^{3}}}{32 a}$	53

input
```
int(csch(x)^4/(a-a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
16/15*(10*exp(4*x)-5*exp(2*x)+1)/(exp(2*x)-1)^5/a
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 7.45

$$\int \frac{\operatorname{csch}^4(x)}{a - a\cosh^2(x)}\,dx$$

$$= \frac{}{15\left(a\cosh(x)^8 + 8\,a\cosh(x)\sinh(x)^7 + a\sinh(x)^8 - 5\,a\cosh(x)^6 + \left(28\,a\cosh(x)^2 - 5\,a\right)\sinh(x)^6 + 2\,(\right.}$$

input
```
integrate(csch(x)^4/(a-a*cosh(x)^2),x, algorithm="fricas")
```

output
```
16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 - 5)/(a*cosh(x)^8
+ 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 - 5*a*cosh(x)^6 + (28*a*cosh(x)^2 -
5*a)*sinh(x)^6 + 2*(28*a*cosh(x)^3 - 15*a*cosh(x))*sinh(x)^5 + 10*a*cosh(x
)^4 + 5*(14*a*cosh(x)^4 - 15*a*cosh(x)^2 + 2*a)*sinh(x)^4 + 4*(14*a*cosh(x
)^5 - 25*a*cosh(x)^3 + 10*a*cosh(x))*sinh(x)^3 - 11*a*cosh(x)^2 + (28*a*co
sh(x)^6 - 75*a*cosh(x)^4 + 60*a*cosh(x)^2 - 11*a)*sinh(x)^2 + 2*(4*a*cosh(
x)^7 - 15*a*cosh(x)^5 + 20*a*cosh(x)^3 - 9*a*cosh(x))*sinh(x) + 5*a)
```

3.5.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a - a\cosh^2(x)}\,dx = -\frac{\int \frac{\operatorname{csch}^4(x)}{\cosh^2(x)-1}\,dx}{a}$$

input
```
integrate(csch(x)**4/(a-a*cosh(x)**2),x)
```

output
```
-Integral(csch(x)**4/(cosh(x)**2 - 1), x)/a
```

3.5.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.66

$$\int \frac{\operatorname{csch}^4(x)}{a - a\cosh^2(x)}\,dx = \frac{16\,e^{(-2\,x)}}{3\,(5\,ae^{(-2\,x)} - 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} - 5\,ae^{(-8\,x)} + ae^{(-10\,x)} - a)} - \frac{32\,e^{(-4\,x)}}{3\,(5\,ae^{(-2\,x)} - 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} - 5\,ae^{(-8\,x)} + ae^{(-10\,x)} - a)} - \frac{16}{15\,(5\,ae^{(-2\,x)} - 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} - 5\,ae^{(-8\,x)} + ae^{(-10\,x)} - a)}$$

input
```
integrate(csch(x)^4/(a-a*cosh(x)^2),x, algorithm="maxima")
```

output
```
16/3*e^(-2*x)/(5*a*e^(-2*x) - 10*a*e^(-4*x) + 10*a*e^(-6*x) - 5*a*e^(-8*x)
 + a*e^(-10*x) - a) - 32/3*e^(-4*x)/(5*a*e^(-2*x) - 10*a*e^(-4*x) + 10*a*e
^(-6*x) - 5*a*e^(-8*x) + a*e^(-10*x) - a) - 16/15/(5*a*e^(-2*x) - 10*a*e^(
-4*x) + 10*a*e^(-6*x) - 5*a*e^(-8*x) + a*e^(-10*x) - a)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{csch}^4(x)}{a - a\cosh^2(x)}\,dx = \frac{16\,(10\,e^{(4\,x)} - 5\,e^{(2\,x)} + 1)}{15\,a(e^{(2\,x)} - 1)^5}$$

input
```
integrate(csch(x)^4/(a-a*cosh(x)^2),x, algorithm="giac")
```

output
```
16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(a*(e^(2*x) - 1)^5)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{csch}^4(x)}{a - a\cosh^2(x)}\,dx = \frac{16\,(10\,e^{4\,x} - 5\,e^{2\,x} + 1)}{15\,a\,(e^{2\,x} - 1)^5}$$

input
```
int(1/(sinh(x)^4*(a - a*cosh(x)^2)),x)
```

output
```
(16*(10*exp(4*x) - 5*exp(2*x) + 1))/(15*a*(exp(2*x) - 1)^5)
```

3.6 $\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx$

3.6.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = -\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{(a^2+3ab+3b^2)\cosh(x)}{b^3} - \frac{(a+3b)\cosh^3(x)}{3b^2}+\frac{\cosh^5(x)}{5b}$$

output
```
(a^2+3*a*b+3*b^2)*cosh(x)/b^3-1/3*(a+3*b)*cosh(x)^3/b^2+1/5*cosh(x)^5/b-(a
+b)^3*arctan(cosh(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.6.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = -\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}} - \frac{(a+b)^3\arctan\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}} + \frac{(8a^2+22ab+19b^2)\cosh(x)}{8b^3}-\frac{(4a+9b)\cosh(3x)}{48b^2}+\frac{\cosh(5x)}{80b}$$

input
```
Integrate[Sinh[x]^7/(a + b*Cosh[x]^2),x]
```

output
```
-(((a + b)^3*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/(Sqrt[a]
*b^(7/2))) - ((a + b)^3*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]
])/(Sqrt[a]*b^(7/2)) + ((8*a^2 + 22*a*b + 19*b^2)*Cosh[x])/(8*b^3) - ((4*a
 + 9*b)*Cosh[3*x])/(48*b^2) + Cosh[5*x]/(80*b)
```

3.6.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(\frac{\pi}{2}+ix\right)^7}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^7}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \frac{\left(1-\cosh^2(x)\right)^3}{b\cosh^2(x)+a}d\cosh(x)$$

$$\Big\downarrow \text{300}$$

$$-\int \left(-\frac{\cosh^4(x)}{b}+\frac{(a+3b)\cosh^2(x)}{b^2}-\frac{a^2+3ba+3b^2}{b^3}+\frac{a^3+3ba^2+3b^2a+b^3}{b^3\left(b\cosh^2(x)+a\right)}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2+3ab+3b^2\right)\cosh(x)}{b^3}-\frac{(a+b)^3\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a+3b)\cosh^3(x)}{3b^2}+\frac{\cosh^5(x)}{5b}$$

input
```
Int[Sinh[x]^7/(a + b*Cosh[x]^2),x]
```

output
```
-(((a + b)^3*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2))) + ((a^2
 + 3*a*b + 3*b^2)*Cosh[x])/b^3 - ((a + 3*b)*Cosh[x]^3)/(3*b^2) + Cosh[x]^5
/(5*b)
```

3.6.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.6.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\frac{\frac{\cosh(x)^5 b^2}{5} - \frac{ab\cosh(x)^3}{3} - b^2\cosh(x)^3 + a^2\cosh(x) + 3ab\cosh(x) + 3b^2\cosh(x)}{b^3} + \frac{(-a^3 - 3a^2b - 3a\,b^2 - b^3)\,\mathrm{a}}{b^3\sqrt{ab}}$$

input
```
int(sinh(x)^7/(a+b*cosh(x)^2),x)
```

output
```
1/b^3*(1/5*cosh(x)^5*b^2-1/3*a*b*cosh(x)^3-b^2*cosh(x)^3+a^2*cosh(x)+3*a*b
*cosh(x)+3*b^2*cosh(x))+(-a^3-3*a^2*b-3*a*b^2-b^3)/b^3/(a*b)^(1/2)*arctan(
b*cosh(x)/(a*b)^(1/2))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(66) = 132$.

Time = 0.31 (sec) , antiderivative size = 2346, normalized size of antiderivative = 30.08

$$\int \frac{\sinh^7(x)}{a + b\cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/480*(3*a*b^3*cosh(x)^10 + 30*a*b^3*cosh(x)*sinh(x)^9 + 3*a*b^3*sinh(x)^
10 - 5*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^8 + 5*(27*a*b^3*cosh(x)^2 - 4*a^2*b^2
 - 9*a*b^3)*sinh(x)^8 + 40*(9*a*b^3*cosh(x)^3 - (4*a^2*b^2 + 9*a*b^3)*cosh
(x))*sinh(x)^7 + 30*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^6 + 10*(63*a
*b^3*cosh(x)^4 + 24*a^3*b + 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 + 9*a*b^
3)*cosh(x)^2)*sinh(x)^6 + 4*(189*a*b^3*cosh(x)^5 - 70*(4*a^2*b^2 + 9*a*b^3
)*cosh(x)^3 + 45*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x))*sinh(x)^5 + 30
*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^4 + 10*(63*a*b^3*cosh(x)^6 - 35
*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^4 + 24*a^3*b + 66*a^2*b^2 + 57*a*b^3 + 45*(
8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)^2)*sinh(x)^4 + 3*a*b^3 + 40*(9*a*
b^3*cosh(x)^7 - 7*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^5 + 15*(8*a^3*b + 22*a^2*b
^2 + 19*a*b^3)*cosh(x)^3 + 3*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x))*si
nh(x)^3 - 5*(4*a^2*b^2 + 9*a*b^3)*cosh(x)^2 + 5*(27*a*b^3*cosh(x)^8 - 28*(
4*a^2*b^2 + 9*a*b^3)*cosh(x)^6 + 90*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh
(x)^4 - 4*a^2*b^2 - 9*a*b^3 + 36*(8*a^3*b + 22*a^2*b^2 + 19*a*b^3)*cosh(x)
^2)*sinh(x)^2 - 240*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^5 + 5*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4*sinh(x) + 10*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cosh(x)^3*sinh(x)^2 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2*s
inh(x)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^4 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*sinh(x)^5)*sqrt(-a*b)*log((b*cosh(x)^4 + 4*b*cos...
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**7/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.6.7 Maxima [F]

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\sinh(x)^7}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/480*(3*b^2*e^(10*x) + 3*b^2 - 5*(4*a*b + 9*b^2)*e^(8*x) + 30*(8*a^2 + 22
*a*b + 19*b^2)*e^(6*x) + 30*(8*a^2 + 22*a*b + 19*b^2)*e^(4*x) - 5*(4*a*b +
 9*b^2)*e^(2*x))*e^(-5*x)/b^3 - 1/128*integrate(256*((a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*e^(3*x) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^x)/(b^4*e^(4*x) + b
^4 + 2*(2*a*b^3 + b^4)*e^(2*x)), x)
```

3.6.8 Giac [F]

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\sinh(x)^7}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.6.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 805, normalized size of antiderivative = 10.32

$$\int \frac{\sinh^7(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{-5x}}{160\,b} + \frac{e^{5x}}{160\,b} + \frac{e^{-x}\,(8\,a^2+22\,a\,b+19\,b^2)}{16\,b^3}$$

$$- \frac{\left(2\,\mathrm{atan}\left(\frac{e^{x}\,(a+b)^3\,\sqrt{a\,b^7}}{2\,a\,b^3\,\sqrt{(a+b)^6}}\right) - 2\,\mathrm{atan}\left(\frac{2\,e^{3\,x}\,\left(a^7\,\sqrt{a\,b^7}+b^7\,\sqrt{a\,b^7}+7\,a\,b^6\,\sqrt{a\,b^7}+7\,a^6\,b\,\sqrt{a\,b^7}+21\,a^2\,b^5\,\sqrt{a\,b^7}+35\,a^3\,b^4\,\sqrt{a\,b^7}+35\,a^4\,b^3\cdots\right.}{a\,b^3\,\sqrt{(a+b)^6}\,(4\,a^4+16\,a^3\,b+24\,a^2\,b^2+16\,a\,b^3+4\,b^4)}\cdots\right.\right.}{\cdots}$$

$$- \frac{e^{-3\,x}\,(4\,a+9\,b)}{96\,b^2} - \frac{e^{3\,x}\,(4\,a+9\,b)}{96\,b^2} + \frac{e^{x}\,(8\,a^2+22\,a\,b+19\,b^2)}{16\,b^3}$$

input
```
int(sinh(x)^7/(a + b*cosh(x)^2),x)
```

output
```
exp(-5*x)/(160*b) + exp(5*x)/(160*b) + (exp(-x)*(22*a*b + 8*a^2 + 19*b^2))
/(16*b^3) - ((2*atan((exp(x)*(a + b)^3*(a*b^7)^(1/2))/(2*a*b^3*((a + b)^6)
^(1/2))) - 2*atan((2*exp(3*x)*(a^7*(a*b^7)^(1/2) + b^7*(a*b^7)^(1/2) + 7*a
*b^6*(a*b^7)^(1/2) + 7*a^6*b*(a*b^7)^(1/2) + 21*a^2*b^5*(a*b^7)^(1/2) + 35
*a^3*b^4*(a*b^7)^(1/2) + 35*a^4*b^3*(a*b^7)^(1/2) + 21*a^5*b^2*(a*b^7)^(1/
2)))/(a*b^3*((a + b)^6)^(1/2)*(16*a*b^3 + 16*a^3*b + 4*a^4 + 4*b^4 + 24*a^
2*b^2)) + (a*b^8*exp(x)*(a*b^7)^(1/2)*((4*(2*a*b^7*(6*a*b^5 + 6*a^5*b + a^
6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2) + 8*a^2*b^6*(6*a*b^5
 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2) + 12*
a^3*b^5*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*
b^2)^(1/2) + 8*a^4*b^4*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^
3*b^3 + 15*a^4*b^2)^(1/2) + 2*a^5*b^3*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*
a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))/(a^2*b^15*(a + b)^3) + (2*(a^7*
(a*b^7)^(1/2) + b^7*(a*b^7)^(1/2) + 7*a*b^6*(a*b^7)^(1/2) + 7*a^6*b*(a*b^7
)^(1/2) + 21*a^2*b^5*(a*b^7)^(1/2) + 35*a^3*b^4*(a*b^7)^(1/2) + 35*a^4*b^3
*(a*b^7)^(1/2) + 21*a^5*b^2*(a*b^7)^(1/2)))/(a^2*b^11*(a*b^7)^(1/2)*((a +
b)^6)^(1/2))))/(16*a*b^3 + 16*a^3*b + 4*a^4 + 4*b^4 + 24*a^2*b^2)))*(6*a*b
^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2)^(1/2))/(2
*(a*b^7)^(1/2)) - (exp(-3*x)*(4*a + 9*b))/(96*b^2) - (exp(3*x)*(4*a + 9*b)
)/(96*b^2) + (exp(x)*(22*a*b + 8*a^2 + 19*b^2))/(16*b^3)
```

3.7 $\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}\,dx$

3.7.1 Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}\,dx=\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\cosh(x)}{b^2}+\frac{\cosh^3(x)}{3b}$$

output
```
-(a+2*b)*cosh(x)/b^2+1/3*cosh(x)^3/b+(a+b)^2*arctan(cosh(x)*b^(1/2)/a^(1/2
))/b^(5/2)/a^(1/2)
```

3.7.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.22

$$\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}\,dx$$
$$=\frac{\frac{12(a+b)^2\arctan\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{12(a+b)^2\arctan\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}-3\sqrt{b}(4a+7b)\cosh(x)+b^{3/2}\cosh(3x)}{12b^{5/2}}$$

input
```
Integrate[Sinh[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
((12*(a + b)^2*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/Sqrt[a
] + (12*(a + b)^2*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/Sqr
t[a] - 3*Sqrt[b]*(4*a + 7*b)*Cosh[x] + b^(3/2)*Cosh[3*x])/(12*b^(5/2))
```

3.7.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3669, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(\frac{\pi}{2}+ix\right)^5}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^5}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{\left(1-\cosh^2(x)\right)^2}{a+b\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{a^2+2ab+b^2}{b^2\left(a+b\cosh^2(x)\right)}-\frac{a+2b}{b^2}+\frac{\cosh^2(x)}{b}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\cosh(x)}{b^2}+\frac{\cosh^3(x)}{3b}$$

input
```
Int[Sinh[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
((a + b)^2*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*
b)*Cosh[x])/b^2 + Cosh[x]^3/(3*b)
```

3.7.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.7.4 Maple [A] (verified)

Time = 112.59 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{-\frac{b\cosh(x)^3}{3}+a\cosh(x)+2b\cosh(x)}{b^2}+\frac{(a^2+2ab+b^2)\arctan\left(\frac{b\cosh(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
default	$-\frac{-\frac{b\cosh(x)^3}{3}+a\cosh(x)+2b\cosh(x)}{b^2}+\frac{(a^2+2ab+b^2)\arctan\left(\frac{b\cosh(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{e^{3x}}{24b}-\frac{a\,e^{x}}{2b^2}-\frac{7\,e^{x}}{8b}-\frac{a\,e^{-x}}{2b^2}-\frac{7\,e^{-x}}{8b}+\frac{e^{-3x}}{24b}-\frac{\ln\left(e^{2x}-\frac{2a\,e^{x}}{\sqrt{-ab}}+1\right)a^2}{2\sqrt{-ab}\,b^2}-\frac{\ln\left(e^{2x}-\frac{2a\,e^{x}}{\sqrt{-ab}}+1\right)a}{\sqrt{-ab}\,b}-\frac{\ln\left(e^{2x}-\frac{2a\,e}{\sqrt{-}}\right.}{2\sqrt{-a}}$

input
```
int(sinh(x)^5/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/3*b*cosh(x)^3+a*cosh(x)+2*b*cosh(x))+(a^2+2*a*b+b^2)/b^2/(a*b)^
(1/2)*arctan(b*cosh(x)/(a*b)^(1/2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(44) = 88$.

Time = 0.29 (sec) , antiderivative size = 1064, normalized size of antiderivative = 19.70

$$\int \frac{\sinh^5(x)}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/24*(a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sinh(x)^5 + a*b^2*sinh(x)^6 - 3*(
4*a^2*b + 7*a*b^2)*cosh(x)^4 + 3*(5*a*b^2*cosh(x)^2 - 4*a^2*b - 7*a*b^2)*s
inh(x)^4 + 4*(5*a*b^2*cosh(x)^3 - 3*(4*a^2*b + 7*a*b^2)*cosh(x))*sinh(x)^3
 + a*b^2 - 3*(4*a^2*b + 7*a*b^2)*cosh(x)^2 + 3*(5*a*b^2*cosh(x)^4 - 4*a^2*
b - 7*a*b^2 - 6*(4*a^2*b + 7*a*b^2)*cosh(x)^2)*sinh(x)^2 - 12*((a^2 + 2*a*
b + b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^2*sinh(x) + 3*(a^2 + 2*
a*b + b^2)*cosh(x)*sinh(x)^2 + (a^2 + 2*a*b + b^2)*sinh(x)^3)*sqrt(-a*b)*l
og((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a - b)*cosh(x
)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a - b)*c
osh(x))*sinh(x) - 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh
(x)^2 + 1)*sinh(x) + cosh(x))*sqrt(-a*b) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*s
inh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a +
b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 6*(a*b^
2*cosh(x)^5 - 2*(4*a^2*b + 7*a*b^2)*cosh(x)^3 - (4*a^2*b + 7*a*b^2)*cosh(x
))*sinh(x))/(a*b^3*cosh(x)^3 + 3*a*b^3*cosh(x)^2*sinh(x) + 3*a*b^3*cosh(x)
*sinh(x)^2 + a*b^3*sinh(x)^3), 1/24*(a*b^2*cosh(x)^6 + 6*a*b^2*cosh(x)*sin
h(x)^5 + a*b^2*sinh(x)^6 - 3*(4*a^2*b + 7*a*b^2)*cosh(x)^4 + 3*(5*a*b^2*co
sh(x)^2 - 4*a^2*b - 7*a*b^2)*sinh(x)^4 + 4*(5*a*b^2*cosh(x)^3 - 3*(4*a^2*b
 + 7*a*b^2)*cosh(x))*sinh(x)^3 + a*b^2 - 3*(4*a^2*b + 7*a*b^2)*cosh(x)^2 +
 3*(5*a*b^2*cosh(x)^4 - 4*a^2*b - 7*a*b^2 - 6*(4*a^2*b + 7*a*b^2)*cosh(...
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(x)}{a + b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**5/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.7.7 Maxima [F]

$$\int \frac{\sinh^5(x)}{a + b\cosh^2(x)}\,dx = \int \frac{\sinh(x)^5}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/24*(b*e^(6*x) - 3*(4*a + 7*b)*e^(4*x) - 3*(4*a + 7*b)*e^(2*x) + b)*e^(-3
*x)/b^2 + 1/32*integrate(64*((a^2 + 2*a*b + b^2)*e^(3*x) - (a^2 + 2*a*b +
b^2)*e^x)/(b^3*e^(4*x) + b^3 + 2*(2*a*b^2 + b^3)*e^(2*x)), x)
```

3.7.8 Giac [F]

$$\int \frac{\sinh^5(x)}{a + b\cosh^2(x)}\,dx = \int \frac{\sinh(x)^5}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 548, normalized size of antiderivative = 10.15

$$\int \frac{\sinh^5(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{-3x}}{24\,b} + \frac{e^{3x}}{24\,b} - \frac{e^{-x}\,(4\,a+7\,b)}{8\,b^2} + \frac{2\,\mathrm{atan}\left(\frac{e^x\,(a+b)^2\,\sqrt{a\,b^5}}{2\,a\,b^2\,\sqrt{(a+b)^4}}\right) - 2\,\mathrm{atan}\left(\frac{a\,b^6\,e^x\,\left(\frac{4\,(6\,a^2\,b^4\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}+6\,a^3\,b^3\,\sqrt{a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4}+2\,a^4\,b^2\,\sqrt{\cdots}\cdots}{a^2\,b^{11}\,(a+b)^2}\cdots\right)}{\cdots}\right)\cdots}{\cdots} - \frac{e^x\,(4\,a+7\,b)}{8\,b^2}$$

input
```
int(sinh(x)^5/(a + b*cosh(x)^2),x)
```

output
```
exp(-3*x)/(24*b) + exp(3*x)/(24*b) - (exp(-x)*(4*a + 7*b))/(8*b^2) + ((2*a
tan((exp(x)*(a + b)^2*(a*b^5)^(1/2))/(2*a*b^2*((a + b)^4)^(1/2))) - 2*atan
((a*b^6*exp(x)*((4*(6*a^2*b^4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^
(1/2) + 6*a^3*b^3*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2) + 2*a^
4*b^2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2) + 2*a*b^5*(4*a*b^3
 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2)))/(a^2*b^11*(a + b)^2) + (2*(a^5
*(a*b^5)^(1/2) + b^5*(a*b^5)^(1/2) + 5*a*b^4*(a*b^5)^(1/2) + 5*a^4*b*(a*b^
5)^(1/2) + 10*a^2*b^3*(a*b^5)^(1/2) + 10*a^3*b^2*(a*b^5)^(1/2)))/(a^2*b^8*
(a*b^5)^(1/2)*((a + b)^4)^(1/2)))*(a*b^5)^(1/2))/(12*a*b^2 + 12*a^2*b + 4*
a^3 + 4*b^3) + (2*exp(3*x)*(a^5*(a*b^5)^(1/2) + b^5*(a*b^5)^(1/2) + 5*a*b^
4*(a*b^5)^(1/2) + 5*a^4*b*(a*b^5)^(1/2) + 10*a^2*b^3*(a*b^5)^(1/2) + 10*a^
3*b^2*(a*b^5)^(1/2)))/(a*b^2*((a + b)^4)^(1/2)*(12*a*b^2 + 12*a^2*b + 4*a^
3 + 4*b^3))))*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)^(1/2))/(2*(a*b^5
)^(1/2)) - (exp(x)*(4*a + 7*b))/(8*b^2)
```

3.8 $\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx$

3.8.1 Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx = -\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{\cosh(x)}{b}$$

output
```
cosh(x)/b-(a+b)*arctan(cosh(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.8.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.31

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx = -\frac{(a+b)\left(\arctan\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+\arctan\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)\right)}{\sqrt{a}b^{3/2}} + \frac{\cosh(x)}{b}$$

input
```
Integrate[Sinh[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
-(((a + b)*(ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + ArcTan[(
Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]]))/(Sqrt[a]*b^(3/2))) + Cosh[x]
/b
```

3.8.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 3669, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(\frac{\pi}{2}+ix\right)^3}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^3}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \frac{1-\cosh^2(x)}{b\cosh^2(x)+a}d\cosh(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{\cosh(x)}{b}-\frac{(a+b)\int \frac{1}{b\cosh^2(x)+a}d\cosh(x)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\cosh(x)}{b}-\frac{(a+b)\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Int[Sinh[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
-(((a + b)*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + Cosh[x]
/b
```

3.8.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.8.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\cosh(x)}{b}+\frac{(-a-b)\arctan\left(\frac{b\cosh(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
default	$\frac{\cosh(x)}{b}+\frac{(-a-b)\arctan\left(\frac{b\cosh(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{e^{x}}{2b}+\frac{e^{-x}}{2b}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{\sqrt{-ab}}+1\right)a}{2\sqrt{-ab}\,b}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{\sqrt{-ab}}+1\right)}{2\sqrt{-ab}}+\frac{\ln\left(e^{2x}-\frac{2a\,e^{x}}{\sqrt{-ab}}+1\right)a}{2\sqrt{-ab}\,b}+\frac{\ln\left(e^{2x}-\frac{2a\,e^{x}}{\sqrt{-ab}}+1\right)}{2\sqrt{-ab}}$	130

input
```
int(sinh(x)^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)/b+(-a-b)/b/(a*b)^(1/2)*arctan(b*cosh(x)/(a*b)^(1/2))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 416, normalized size of antiderivative = 11.56

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx$$
$$= \left[\frac{ab\cosh(x)^2 + 2\,ab\cosh(x)\sinh(x) + ab\sinh(x)^2 - \sqrt{-ab}((a+b)\cosh(x) + (a+b)\sinh(x))\log\left(\frac{b\cosh\ldots}{\ldots}\right)}{\ldots}\right.$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - sqrt(-a*b)*(
(a + b)*cosh(x) + (a + b)*sinh(x))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^
3 + b*sinh(x)^4 - 2*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh
(x)^2 + 4*(b*cosh(x)^3 - (2*a - b)*cosh(x))*sinh(x) + 4*(cosh(x)^3 + 3*cos
h(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))*sqrt(-a*
b) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*c
osh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a +
 b)*cosh(x))*sinh(x) + b)) + a*b)/(a*b^2*cosh(x) + a*b^2*sinh(x)), 1/2*(a*
b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - 2*sqrt(a*b)*((a + b)
*cosh(x) + (a + b)*sinh(x))*arctan(1/2*sqrt(a*b)*(cosh(x) + sinh(x))/a) +
2*sqrt(a*b)*((a + b)*cosh(x) + (a + b)*sinh(x))*arctan(1/2*(b*cosh(x)^3 +
3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + b)*cosh(x) + (3*b*cosh(x)^2 +
 4*a + b)*sinh(x))*sqrt(a*b)/(a*b)) + a*b)/(a*b^2*cosh(x) + a*b^2*sinh(x))
]
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.8.7 Maxima [F]

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\sinh(x)^3}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) + 1)*e^(-x)/b - 1/8*integrate(16*((a + b)*e^(3*x) - (a + b)*e
^x)/(b^2*e^(4*x) + b^2 + 2*(2*a*b + b^2)*e^(2*x)), x)
```

3.8.8 Giac [F]

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\sinh(x)^3}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.8.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 7.14

$$\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{-x}}{2\,b} + \frac{e^{x}}{2\,b} + \frac{\left(2\,\mathrm{atan}\left(\frac{e^{3\,x}\,\left(a^3\,\sqrt{a\,b^3}+b^3\,\sqrt{a\,b^3}+3\,a\,b^2\,\sqrt{a\,b^3}+3\,a^2\,b\,\sqrt{a\,b^3}\right)}{2\,a\,b\,\left({\left(a+b\right)}^2\right)^{3/2}} + \frac{a\,b^4\,e^{x}\,\sqrt{a\,b^3}\,\left(\frac{8\,\left(a^2+2\,a\,b+b^2\right)^{3/2}}{a\,b^6\,{\left(a+b\right)}^3} + \frac{2\,\left(a^3\,\sqrt{a\,b^3}+b^3\,\sqrt{a\,b^3}+3\,a\,b^2\,\sqrt{a\,b^3}+\cdots\right.}{a^2\,b^5\,\sqrt{a\,b^3}\,\left({\left(a+b\right)}^2\right)^{3/2}\cdots}\right.}{4}\cdots\right.\right.}{2\,\sqrt{a\,b^3}}$$

input
```
int(sinh(x)^3/(a + b*cosh(x)^2),x)
```

output
```
exp(-x)/(2*b) + exp(x)/(2*b) + ((2*atan((exp(3*x)*(a^3*(a*b^3)^(1/2) + b^3
*(a*b^3)^(1/2) + 3*a*b^2*(a*b^3)^(1/2) + 3*a^2*b*(a*b^3)^(1/2)))/(2*a*b*((
a + b)^2)^(3/2)) + (a*b^4*exp(x)*(a*b^3)^(1/2)*((8*(2*a*b + a^2 + b^2)^(3/
2))/(a*b^6*(a + b)^3) + (2*(a^3*(a*b^3)^(1/2) + b^3*(a*b^3)^(1/2) + 3*a*b^
2*(a*b^3)^(1/2) + 3*a^2*b*(a*b^3)^(1/2)))/(a^2*b^5*(a*b^3)^(1/2)*((a + b)^
2)^(3/2))))/4) - 2*atan((exp(x)*(a + b)^3*(a*b^3)^(1/2))/(2*a*b*((a + b)^2
)^(3/2))))*(2*a*b + a^2 + b^2)^(1/2))/(2*(a*b^3)^(1/2))
```

3.8. $\int \frac{\sinh^3(x)}{a+b\cosh^2(x)}\,dx$

3.9 $\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx$

3.9.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(cosh(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[Sinh[x]/(a + b*Cosh[x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.9.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3669, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{a+b\cosh^2(x)}d\cosh(x)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[Sinh[x]/(a + b*Cosh[x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

3.9.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.9.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\cosh(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
default	$\frac{\arctan\left(\frac{b\cosh(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
risch	$-\frac{\ln\left(\mathrm{e}^{2x}-\frac{2a\,\mathrm{e}^{x}}{\sqrt{-ab}}+1\right)}{2\sqrt{-ab}}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{\sqrt{-ab}}+1\right)}{2\sqrt{-ab}}$	54

input
```
int(sinh(x)/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*cosh(x)/(a*b)^(1/2))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 12.00

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2\,(2\,a-b)\cosh(x)^2+2\left(3\,b\cosh(x)^2-2\,a+b\right)\sinh(x)^2+4\left(b\cosh(x)^3-(2\,a-b)\,c\cdots\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2\cdots\right.}\right.}{2\,ab}\cdots\right.$$

input
```
integrate(sinh(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 -
2*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(
x)^3 - (2*a - b)*cosh(x))*sinh(x) - 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + cosh(x))*sqrt(-a*b) + b)/(b*cosh(x)
^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*
cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(
x) + b))/(a*b), (sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(x) + sinh(x))/a) - s
qrt(a*b)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (
4*a + b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(a*b)/(a*b)))/(a
*b)]
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\cosh(x)} & \text{for } a=0 \wedge b=0 \\ \frac{\cosh(x)}{a} & \text{for } b=0 \\ -\frac{1}{b\cosh(x)} & \text{for } a=0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\cosh(x)\right)}{2b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\cosh(x)\right)}{2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)**2),x)
```

output
```
Piecewise((zoo/cosh(x), Eq(a, 0) & Eq(b, 0)), (cosh(x)/a, Eq(b, 0)), (-1/(
b*cosh(x)), Eq(a, 0)), (log(-sqrt(-a/b) + cosh(x))/(2*b*sqrt(-a/b)) - log(
sqrt(-a/b) + cosh(x))/(2*b*sqrt(-a/b)), True))
```

3.9.7 Maxima [F]

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\sinh(x)}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sinh(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(sinh(x)/(b*cosh(x)^2 + a), x)
```

3.9.8 Giac [F]

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\sinh(x)}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sinh(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{\sinh(x)}{a+b\cosh^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{b\cosh(x)}{\sqrt{a\,b}}\right)}{\sqrt{a\,b}}$$

input
```
int(sinh(x)/(a + b*cosh(x)^2),x)
```

output
```
atan((b*cosh(x))/(a*b)^(1/2))/(a*b)^(1/2)
```

3.10 $\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx$

3.10.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}-\frac{\operatorname{arctanh}(\cosh(x))}{a+b}$$

output
```
-arctanh(cosh(x))/(a+b)-arctan(cosh(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1
/2)
```

3.10.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.52

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}$$

input
```
Integrate[Csch[x]/(a + b*Cosh[x]^2),x]
```

output
```
-(((Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/Sqrt[a] +
 (Sqrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]])/Sqrt[a] + L
og[Cosh[x/2]] - Log[Sinh[x/2]])/(a + b))
```

3.10.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 26, 3669, 303, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cos\left(\frac{\pi}{2}+ix\right)\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos\left(ix+\frac{\pi}{2}\right)\left(b\sin\left(ix+\frac{\pi}{2}\right)^2+a\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$-\int \frac{1}{(1-\cosh^2(x))\left(b\cosh^2(x)+a\right)}d\cosh(x)$$

$$\Big\downarrow \text{303}$$

$$-\frac{\int \frac{1}{1-\cosh^2(x)}d\cosh(x)}{a+b}-\frac{b\int \frac{1}{b\cosh^2(x)+a}d\cosh(x)}{a+b}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\int \frac{1}{1-\cosh^2(x)}d\cosh(x)}{a+b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}-\frac{\operatorname{arctanh}(\cosh(x))}{a+b}$$

input
```
Int[Csch[x]/(a + b*Cosh[x]^2),x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*(a + b))) - ArcTanh
[Cosh[x]]/(a + b)
```

3.10.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.10.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}-\frac{b\arctan\left(\frac{2(a+b)\tanh\left(\frac{x}{2}\right)^{2}-2a+2b}{4\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}$	52
risch	$\frac{\ln(e^{x}-1)}{a+b}-\frac{\ln(e^{x}+1)}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2x}-\frac{2\sqrt{-ab}\,e^{x}}{b}+1\right)}{2a(a+b)}-\frac{\sqrt{-ab}\,\ln\left(e^{2x}+\frac{2\sqrt{-ab}\,e^{x}}{b}+1\right)}{2a(a+b)}$	97

input
```
int(csch(x)/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*ln(tanh(1/2*x))-b/(a+b)/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x
)^2-2*a+2*b)/(a*b)^(1/2))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 349, normalized size of antiderivative = 8.31

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2\,(2\,a-b)\cosh(x)^2+2\left(3\,b\cosh(x)^2-2\,a+b\right)\sinh(x)^2+4\left(b\cosh(x)^3-(2\,a-b)\cosh\cdots\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^{2}\cdots\right.}\right.}{\cdots}\right. \\ -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{1}{2}\sqrt{\frac{b}{a}}(\cosh(x)+\sinh(x))\right)-\sqrt{\frac{b}{a}}\arctan\left(\frac{\left(b\cosh(x)^3+3\,b\cosh(x)\sinh(x)^2+b\sinh(x)^3+(4\,a+b)\cosh(x)+\cdots\right.}{2\,b}\cdots\right.}{a+b}$$

input
```
integrate(csch(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 -
2*(2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(
x)^3 - (2*a - b)*cosh(x))*sinh(x) - 4*(a*cosh(x)^3 + 3*a*cosh(x)*sinh(x)^2
 + a*sinh(x)^3 + a*cosh(x) + (3*a*cosh(x)^2 + a)*sinh(x))*sqrt(-b/a) + b)/
(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2
 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh
(x))*sinh(x) + b)) - 2*log(cosh(x) + sinh(x) + 1) + 2*log(cosh(x) + sinh(x
) - 1))/(a + b), -(sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(x) + sinh(x))) - s
qrt(b/a)*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (
4*a + b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + b)*sinh(x))*sqrt(b/a)/b) + log(c
osh(x) + sinh(x) + 1) - log(cosh(x) + sinh(x) - 1))/(a + b)]
```

3.10.6 Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(csch(x)/(a+b*cosh(x)**2),x)
```

output
```
Integral(csch(x)/(a + b*cosh(x)**2), x)
```

3.10.7 Maxima [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\operatorname{csch}(x)}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-log(e^x + 1)/(a + b) + log(e^x - 1)/(a + b) - 2*integrate((b*e^(3*x) - b*
e^x)/(a*b + b^2 + (a*b + b^2)*e^(4*x) + 2*(2*a^2 + 3*a*b + b^2)*e^(2*x)),
x)
```

3.10.8 Giac [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\operatorname{csch}\left(x\right)}{b\cosh\left(x\right)^2+a}\,dx$$

input
```
integrate(csch(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 462, normalized size of antiderivative = 11.00

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh^2(x)}\,dx=-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\left(16\,a^2\,\sqrt{-a^2-2\,a\,b-b^2}+b^2\,\sqrt{-a^2-2\,a\,b-b^2}+8\,a\,b\,\sqrt{-a^2-2\,a\,b-b^2}\right)}{16\,a^3+24\,a^2\,b+9\,a\,b^2+b^3}\right)}{\sqrt{-a^2-2\,a\,b-b^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\mathrm{e}^x\,\sqrt{a\,{\left(a+b\right)}^2}}{2\,a\,\left(a+b\right)}\right)-2\,\mathrm{atan}\left(\frac{\left(a^3\,b^{5/2}\,\sqrt{a^3+2\,a^2\,b+a\,b^2}+a^2\,b^{7/2}\,\sqrt{a^3+2\,a^2\,b+a\,b^2}\right)\,\left(\mathrm{e}^x\,\left(\frac{64\,\left(8\,a^3+10\,a^2\,b+2\cdots\right.}{a\,b^3\,\sqrt{a\,{\left(a+b\right)}^2}\,\left(a^2+b\,a\right)\,\sqrt{a\cdots}}\right.\right.}{\cdots}\right.\right.}{2\,\sqrt{a^3+2\,a^2\,b\cdots}}$$

input
```
int(1/(sinh(x)*(a + b*cosh(x)^2)),x)
```

output
```
- (2*atan((exp(x)*(16*a^2*(- 2*a*b - a^2 - b^2)^(1/2) + b^2*(- 2*a*b - a^2
 - b^2)^(1/2) + 8*a*b*(- 2*a*b - a^2 - b^2)^(1/2)))/(9*a*b^2 + 24*a^2*b +
16*a^3 + b^3)))/(- 2*a*b - a^2 - b^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*ex
p(x)*(a*(a + b)^2)^(1/2))/(2*a*(a + b))) - 2*atan(((a^3*b^(5/2)*(a*b^2 + 2
*a^2*b + a^3)^(1/2) + a^2*b^(7/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))*(exp(x)*(
(64*(2*a*b^2 + 10*a^2*b + 8*a^3))/(a*b^3*(a*(a + b)^2)^(1/2)*(a*b + a^2)*(
a*b^2 + 2*a^2*b + a^3)^(1/2)) + (32*(b^(3/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2)
 + 4*a*b^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(a^2*b^(5/2)*(a + b)*(a*b +
 a^2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))) + (32*exp(3*x)*(b^(3/2)*(a*b^2 + 2*a
^2*b + a^3)^(1/2) + 4*a*b^(1/2)*(a*b^2 + 2*a^2*b + a^3)^(1/2)))/(a^2*b^(5/
2)*(a + b)*(a*b + a^2)*(a*b^2 + 2*a^2*b + a^3)^(1/2))))/(256*a + 64*b))))/
(2*(a*b^2 + 2*a^2*b + a^3)^(1/2))
```

3.11 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx$

3.11.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx=\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2}+\frac{(a+3b)\operatorname{arctanh}(\cosh(x))}{2(a+b)^2}-\frac{\coth(x)\operatorname{csch}(x)}{2(a+b)}$$

output
```
1/2*(a+3*b)*arctanh(cosh(x))/(a+b)^2-1/2*coth(x)*csch(x)/(a+b)+b^(3/2)*arc
tan(cosh(x)*b^(1/2)/a^(1/2))/(a+b)^2/a^(1/2)
```

3.11.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.52

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx$$
$$=\frac{8b^{3/2}\arctan\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+8b^{3/2}\arctan\left(\frac{\sqrt{b}+i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right)-\sqrt{a}(a+b)\operatorname{csch}^2\left(\frac{x}{2}\right)+4\sqrt{a}(a+3b)\,(\mathrm{lo}}{8\sqrt{a}(a+b)^2}$$

input
```
Integrate[Csch[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
(8*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + 8*b^(3/2)
*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] - Sqrt[a]*(a + b)*Csc
h[x/2]^2 + 4*Sqrt[a]*(a + 3*b)*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) - Sqrt[a]
*(a + b)*Sech[x/2]^2)/(8*Sqrt[a]*(a + b)^2)
```

3.11.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3669, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cos\left(\frac{\pi}{2}+ix\right)^3\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos\left(ix+\frac{\pi}{2}\right)^3\left(b\sin\left(ix+\frac{\pi}{2}\right)^2+a\right)}dx$$

$$\Big\downarrow \text{3669}$$

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^2\left(a+b\cosh^2(x)\right)}d\cosh(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{b\cosh^2(x)+a+2b}{\left(1-\cosh^2(x)\right)\left(b\cosh^2(x)+a\right)}d\cosh(x)}{2(a+b)}+\frac{\cosh(x)}{2(a+b)\left(1-\cosh^2(x)\right)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2b^2\int \frac{1}{b\cosh^2(x)+a}d\cosh(x)}{a+b}+\frac{(a+3b)\int \frac{1}{1-\cosh^2(x)}d\cosh(x)}{a+b}}{2(a+b)}+\frac{\cosh(x)}{2(a+b)\left(1-\cosh^2(x)\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+3b)\int \frac{1}{1-\cosh^2(x)}d\cosh(x)}{a+b}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{\cosh(x)}{2(a+b)\left(1-\cosh^2(x)\right)}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{(a+3b)\text{arctanh}(\cosh(x))}{a+b}}{2(a+b)}+\frac{\cosh(x)}{2(a+b)\left(1-\cosh^2(x)\right)}$$

input
```
Int[Csch[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
((2*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((a + 3
*b)*ArcTanh[Cosh[x]])/(a + b))/(2*(a + b)) + Cosh[x]/(2*(a + b)*(1 - Cosh[
x]^2))
```

3.11.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.11.4 Maple [A] (verified)

Time = 6.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)^2}{8a+8b}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2}+\frac{(-2a-6b)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^2}+\frac{b^2\arctan\left(\frac{2(a+b)\tanh\left(\frac{x}{2}\right)^2-2a+2b}{4\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}$
risch	$-\frac{e^x\left(1+e^{2x}\right)}{\left(e^{2x}-1\right)^2(a+b)}+\frac{a\ln(e^x+1)}{2a^2+4ab+2b^2}+\frac{3\ln(e^x+1)b}{2(a^2+2ab+b^2)}-\frac{\ln(e^x-1)a}{2(a^2+2ab+b^2)}-\frac{3\ln(e^x-1)b}{2(a^2+2ab+b^2)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2x}+\frac{2\sqrt{-ab}\,e^x}{b}+1\right)}{2a(a+b)^2}-$

input
```
int(csch(x)^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*tanh(1/2*x)^2/(a+b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(-2*a-6*b)*ln(
tanh(1/2*x))+b^2/(a+b)^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/2*x)^2-2*a
+2*b)/(a*b)^(1/2))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 1332, normalized size of antiderivative = 21.84

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(a + b)*cosh(x)^3 + 6*(a + b)*cosh(x)*sinh(x)^2 + 2*(a + b)*sinh(
x)^3 - (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cosh(x)^2
+ 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*cosh(x))*sinh(x) +
b)*sqrt(-b/a)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(
2*a - b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^
3 - (2*a - b)*cosh(x))*sinh(x) + 4*(a*cosh(x)^3 + 3*a*cosh(x)*sinh(x)^2 +
a*sinh(x)^3 + a*cosh(x) + (3*a*cosh(x)^2 + a)*sinh(x))*sqrt(-b/a) + b)/(b*
cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 +
2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x)
)*sinh(x) + b)) + 2*(a + b)*cosh(x) - ((a + 3*b)*cosh(x)^4 + 4*(a + 3*b)*c
osh(x)*sinh(x)^3 + (a + 3*b)*sinh(x)^4 - 2*(a + 3*b)*cosh(x)^2 + 2*(3*(a +
 3*b)*cosh(x)^2 - a - 3*b)*sinh(x)^2 + 4*((a + 3*b)*cosh(x)^3 - (a + 3*b)*
cosh(x))*sinh(x) + a + 3*b)*log(cosh(x) + sinh(x) + 1) + ((a + 3*b)*cosh(x
)^4 + 4*(a + 3*b)*cosh(x)*sinh(x)^3 + (a + 3*b)*sinh(x)^4 - 2*(a + 3*b)*co
sh(x)^2 + 2*(3*(a + 3*b)*cosh(x)^2 - a - 3*b)*sinh(x)^2 + 4*((a + 3*b)*cos
h(x)^3 - (a + 3*b)*cosh(x))*sinh(x) + a + 3*b)*log(cosh(x) + sinh(x) - 1)
+ 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x))/((a^2 + 2*a*b + b^2)*cosh(x)^4
+ 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4
- 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(x)^2 - a
^2 - 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2...
```

3.11.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(csch(x)**3/(a+b*cosh(x)**2),x)
```

output
```
Integral(csch(x)**3/(a + b*cosh(x)**2), x)
```

3.11.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}(x)^3}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/2*(a + 3*b)*log(e^x + 1)/(a^2 + 2*a*b + b^2) - 1/2*(a + 3*b)*log(e^x - 1
)/(a^2 + 2*a*b + b^2) - (e^(3*x) + e^x)/((a + b)*e^(4*x) - 2*(a + b)*e^(2*
x) + a + b) + 8*integrate(1/4*(b^2*e^(3*x) - b^2*e^x)/(a^2*b + 2*a*b^2 + b
^3 + (a^2*b + 2*a*b^2 + b^3)*e^(4*x) + 2*(2*a^3 + 5*a^2*b + 4*a*b^2 + b^3)
*e^(2*x)), x)
```

3.11.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}(x)^3}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.11.9 Mupad [B] (verification not implemented)

Time = 8.26 (sec) , antiderivative size = 2225, normalized size of antiderivative = 36.48

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)^3*(a + b*cosh(x)^2)),x)
```

output
```
((2*atan((b^2*exp(x)*(a*(a + b)^4)^(1/2))/(2*a*(a + b)^2*(b^3)^(1/2))) - 2
*atan((exp(x)*((32*(b^8*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1
/2) + 36*a^2*b^6*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 4
7*a^3*b^5*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 30*a^4*b
^4*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 9*a^5*b^3*(a*b^
4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + a^6*b^2*(a*b^4 + 4*a^4*
b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 12*a*b^7*(a*b^4 + 4*a^4*b + a^5 +
 4*a^2*b^3 + 6*a^3*b^2)^(1/2)))/(a^2*b^2*(a + b)^7*(a*b + a^2)*(b^3)^(1/2)
*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(9*a*b^2 + 6*a^2*b +
a^3 + b^3)*(a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2)) + (64*(2
0*a^3*(b^3)^(5/2) + 232*a^6*(b^3)^(3/2) + 2*a^9*(b^3)^(1/2) + 10*a^2*b^4*(
b^3)^(3/2) + 20*a^4*b^2*(b^3)^(3/2) + 18*a^2*b^7*(b^3)^(1/2) + 102*a^3*b^6
*(b^3)^(1/2) + 242*a^4*b^5*(b^3)^(1/2) + 310*a^5*b^4*(b^3)^(1/2) + 98*a^7*
b^2*(b^3)^(1/2) + 2*a*b^5*(b^3)^(3/2) + 10*a^5*b*(b^3)^(3/2) + 22*a^8*b*(b
^3)^(1/2)))/(a*b^4*(a + b)^5*(a*b + a^2)*(a*(a + b)^4)^(1/2)*(2*a*b + a^2
+ b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(9*a*b^2 + 6*a^2*b + a^3 + b^3)*(a*
b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2))) + (32*exp(3*x)*(b^8*(
a*b^4 + 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 36*a^2*b^6*(a*b^4 +
 4*a^4*b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 47*a^3*b^5*(a*b^4 + 4*a^4*
b + a^5 + 4*a^2*b^3 + 6*a^3*b^2)^(1/2) + 30*a^4*b^4*(a*b^4 + 4*a^4*b + ...
```

3.11. $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh^2(x)}\,dx$

3.12 $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx$

3.12.1 Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3} - \frac{(3a^2+10ab+15b^2)\operatorname{arctanh}(\cosh(x))}{8(a+b)^3} + \frac{(3a+7b)\coth(x)\operatorname{csch}(x)}{8(a+b)^2} - \frac{\coth(x)\operatorname{csch}^3(x)}{4(a+b)}$$

output
```
-1/8*(3*a^2+10*a*b+15*b^2)*arctanh(cosh(x))/(a+b)^3+1/8*(3*a+7*b)*coth(x)*
csch(x)/(a+b)^2-1/4*coth(x)*csch(x)^3/(a+b)-b^(5/2)*arctan(cosh(x)*b^(1/2)
/a^(1/2))/(a+b)^3/a^(1/2)
```

3.12.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.44

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = \frac{2\sqrt{a}(3a^2+10ab+7b^2)\operatorname{csch}^2\left(\frac{x}{2}\right) - \sqrt{a}(a+b)^2\operatorname{csch}^4\left(\frac{x}{2}\right) - 8\left(8b^{5/2}\arctan\left(\frac{\sqrt{b}-i\sqrt{a+b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a}}\right) + 8b^{5/2}\operatorname{arcta}\right.}{}$$

input
```
Integrate[Csch[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
(2*Sqrt[a]*(3*a^2 + 10*a*b + 7*b^2)*Csch[x/2]^2 - Sqrt[a]*(a + b)^2*Csch[x
/2]^4 - 8*(8*b^(5/2)*ArcTan[(Sqrt[b] - I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] +
 8*b^(5/2)*ArcTan[(Sqrt[b] + I*Sqrt[a + b]*Tanh[x/2])/Sqrt[a]] + Sqrt[a]*(
3*a^2 + 10*a*b + 15*b^2)*(Log[Cosh[x/2]] - Log[Sinh[x/2]])) + 2*Sqrt[a]*(3
*a^2 + 10*a*b + 7*b^2)*Sech[x/2]^2 + Sqrt[a]*(a + b)^2*Sech[x/2]^4)/(64*Sq
rt[a]*(a + b)^3)
```

3.12.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 26, 3669, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i}{\cos\left(\frac{\pi}{2}+ix\right)^5\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{1}{\cos\left(ix+\frac{\pi}{2}\right)^5\left(b\sin\left(ix+\frac{\pi}{2}\right)^2+a\right)}dx$$

$$\Big\downarrow \text{ 3669}$$

$$-\int \frac{1}{\left(1-\cosh^2(x)\right)^3\left(b\cosh^2(x)+a\right)}d\cosh(x)$$

$$\Big\downarrow \text{ 316}$$

$$-\frac{\int \frac{3b\cosh^2(x)+3a+4b}{\left(1-\cosh^2(x)\right)^2\left(b\cosh^2(x)+a\right)}d\cosh(x)}{4(a+b)}-\frac{\cosh(x)}{4(a+b)\left(1-\cosh^2(x)\right)^2}$$

$$\Big\downarrow \text{ 402}$$

$$-\frac{\frac{\int \frac{3a^2+7ba+8b^2+b(3a+7b)\cosh^2(x)}{(1-\cosh^2(x))(b\cosh^2(x)+a)}d\cosh(x)}{2(a+b)}+\frac{(3a+7b)\cosh(x)}{2(a+b)(1-\cosh^2(x))}}{4(a+b)}-\frac{\cosh(x)}{4(a+b)\left(1-\cosh^2(x)\right)^2}$$

$\big\downarrow$ 397

$$-\frac{\frac{\frac{(3a^2+10ab+15b^2)\int \frac{1}{1-\cosh^2(x)}d\cosh(x)}{a+b}+\frac{8b^3\int \frac{1}{b\cosh^2(x)+a}d\cosh(x)}{a+b}}{2(a+b)}+\frac{(3a+7b)\cosh(x)}{2(a+b)(1-\cosh^2(x))}}{4(a+b)}-\frac{\cosh(x)}{4(a+b)\left(1-\cosh^2(x)\right)^2}$$

$\big\downarrow$ 218

$$-\frac{\frac{\frac{(3a^2+10ab+15b^2)\int \frac{1}{1-\cosh^2(x)}d\cosh(x)}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\cosh(x)}{2(a+b)(1-\cosh^2(x))}}{4(a+b)}-\frac{\cosh(x)}{4(a+b)\left(1-\cosh^2(x)\right)^2}$$

$\big\downarrow$ 219

$$-\frac{\frac{\frac{(3a^2+10ab+15b^2)\text{arctanh}(\cosh(x))}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\cosh(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\cosh(x)}{2(a+b)(1-\cosh^2(x))}}{4(a+b)}-\frac{\cosh(x)}{4(a+b)\left(1-\cosh^2(x)\right)^2}$$

input
```
Int[Csch[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
-1/4*Cosh[x]/((a + b)*(1 - Cosh[x]^2)^2) - (((8*b^(5/2)*ArcTan[(Sqrt[b]*Co
sh[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[Co
sh[x]])/(a + b))/(2*(a + b)) + ((3*a + 7*b)*Cosh[x])/(2*(a + b)*(1 - Cosh[
x]^2)))/(4*(a + b))
```

3.12.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.12. $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx$

3.12.4 Maple [A] (verified)

Time = 29.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.44

method	result
default	$\frac{\left(\tanh\left(\frac{x}{2}\right)^{2} a+\tanh\left(\frac{x}{2}\right)^{2} b-4a-8b\right)^{2}}{64(a+b)^{3}}-\frac{1}{64(a+b)\tanh\left(\frac{x}{2}\right)^{4}}-\frac{-4a-8b}{32(a+b)^{2}\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(6a^{2}+20ab+30b^{2})\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{16(a+b)^{3}}-\frac{b^{3}\,\mathrm{arcta}}{}$
risch	$\frac{\mathrm{e}^{x}\left(3a\,\mathrm{e}^{6x}+7b\,\mathrm{e}^{6x}-11a\,\mathrm{e}^{4x}-15b\,\mathrm{e}^{4x}-11a\,\mathrm{e}^{2x}-15b\,\mathrm{e}^{2x}+3a+7b\right)}{4(\mathrm{e}^{2x}-1)^{4}(a+b)^{2}}+\frac{3\ln(\mathrm{e}^{x}-1)a^{2}}{8(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})}+\frac{5\ln(\mathrm{e}^{x}-1)ab}{4(a^{3}+3a^{2}b+3a\,b^{2}+b^{3})}+\frac{15\ln(}{8(a^{3}+3a^{2}}$

input
```
int(csch(x)^5/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(tanh(1/2*x)^2*a+tanh(1/2*x)^2*b-4*a-8*b)^2/(a+b)^3-1/64/(a+b)/tanh(1
/2*x)^4-1/32*(-4*a-8*b)/(a+b)^2/tanh(1/2*x)^2+1/16/(a+b)^3*(6*a^2+20*a*b+3
0*b^2)*ln(tanh(1/2*x))-b^3/(a+b)^3/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*tanh(1/
2*x)^2-2*a+2*b)/(a*b)^(1/2))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2724 vs. $2(80) = 160$.

Time = 0.35 (sec) , antiderivative size = 5326, normalized size of antiderivative = 56.66

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(csch(x)**5/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.12.7 Maxima [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}(x)^5}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/8*(3*a^2 + 10*a*b + 15*b^2)*log(e^x + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3
) + 1/8*(3*a^2 + 10*a*b + 15*b^2)*log(e^x - 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3) + 1/4*((3*a + 7*b)*e^(7*x) - (11*a + 15*b)*e^(5*x) - (11*a + 15*b)*e^
(3*x) + (3*a + 7*b)*e^x)/(a^2 + 2*a*b + b^2 + (a^2 + 2*a*b + b^2)*e^(8*x)
- 4*(a^2 + 2*a*b + b^2)*e^(6*x) + 6*(a^2 + 2*a*b + b^2)*e^(4*x) - 4*(a^2 +
 2*a*b + b^2)*e^(2*x)) - 32*integrate(1/16*(b^3*e^(3*x) - b^3*e^x)/(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4 + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*e^(4*x)
+ 2*(2*a^4 + 7*a^3*b + 9*a^2*b^2 + 5*a*b^3 + b^4)*e^(2*x)), x)
```

3.12.8 Giac [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}(x)^5}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.12.9 Mupad [B] (verification not implemented)

Time = 17.03 (sec) , antiderivative size = 5056, normalized size of antiderivative = 53.79

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)^5*(a + b*cosh(x)^2)),x)
```

output
```
(atan((exp(x)*(243*a^12*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20
*a^3*b^3 - 15*a^4*b^2)^(3/2) + 3840*b^12*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6
- 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 110560*a*b^11*(- 6*a*b^5 -
 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 4050*
a^11*b*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4
*b^2)^(3/2) + 976143*a^2*b^10*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^
4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 2740050*a^3*b^9*(- 6*a*b^5 - 6*a^5*b
- a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 4252775*a^4*b^
8*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)
^(3/2) + 4316760*a^5*b^7*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 2
0*a^3*b^3 - 15*a^4*b^2)^(3/2) + 3087390*a^6*b^6*(- 6*a*b^5 - 6*a^5*b - a^6
 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 1608364*a^7*b^5*(-
6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2
) + 615750*a^8*b^4*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*
b^3 - 15*a^4*b^2)^(3/2) + 171000*a^9*b^3*(- 6*a*b^5 - 6*a^5*b - a^6 - b^6
- 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2) + 33075*a^10*b^2*(- 6*a*b^5
- 6*a^5*b - a^6 - b^6 - 15*a^2*b^4 - 20*a^3*b^3 - 15*a^4*b^2)^(3/2)))/(81*
a^19*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 256*b^
19*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 9504*a*b
^18*(300*a*b^3 + 60*a^3*b + 9*a^4 + 225*b^4 + 190*a^2*b^2)^(1/2) + 1809...
```

3.13 $\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx$

3.13.1 Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{(8a^2+20ab+15b^2)\,x}{8b^3} - \frac{(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}b^3} - \frac{(4a+7b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh(x)\sinh^3(x)}{4b}$$

output
```
1/8*(8*a^2+20*a*b+15*b^2)*x/b^3-1/8*(4*a+7*b)*cosh(x)*sinh(x)/b^2+1/4*cosh
(x)*sinh(x)^3/b-(a+b)^(5/2)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/b^3/a^(1/
2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{4(8a^2+20ab+15b^2)\,x - \frac{32(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}} - 8b(a+2b)\sinh(2x) + b^2\sinh(4x)}{32b^3}$$

input
```
Integrate[Sinh[x]^6/(a + b*Cosh[x]^2),x]
```

output
```
(4*(8*a^2 + 20*a*b + 15*b^2)*x - (32*(a + b)^(5/2)*ArcTanh[(Sqrt[a]*Tanh[x
])/Sqrt[a + b]])/Sqrt[a] - 8*b*(a + 2*b)*Sinh[2*x] + b^2*Sinh[4*x])/(32*b^
3)
```

3.13.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 25, 3670, 316, 25, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(\frac{\pi}{2}+ix\right)^6}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^6}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx$$

$$\Big\downarrow \text{3670}$$

$$-\int \frac{1}{\left(1-\coth^2(x)\right)^3\left(a-(a+b)\coth^2(x)\right)}d\coth(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\int -\frac{3(a+b)\coth^2(x)+a+4b}{\left(1-\coth^2(x)\right)^2\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{4b}+\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\int \frac{3(a+b)\coth^2(x)+a+4b}{\left(1-\coth^2(x)\right)^2\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{4b}$$

$$\Big\downarrow \text{402}$$

$$\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{-\frac{\int -\frac{4a^2+9ba+8b^2+(a+b)(4a+7b)\coth^2(x)}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{(4a+7b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \; 25$$

$$\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\int \frac{4a^2+9ba+8b^2+(a+b)(4a+7b)\coth^2(x)}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{(4a+7b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \; 397$$

$$\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\frac{8(a+b)^3\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{\left(8a^2+20ab+15b^2\right)\int \frac{1}{1-\coth^2(x)}d\coth(x)}{b}}{2b}-\frac{(4a+7b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \; 219$$

$$\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\frac{8(a+b)^3\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{\left(8a^2+20ab+15b^2\right)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{(4a+7b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \; 221$$

$$\frac{\coth(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\frac{8(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\left(8a^2+20ab+15b^2\right)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{(4a+7b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

input
```
Int[Sinh[x]^6/(a + b*Cosh[x]^2),x]
```

output
```
Coth[x]/(4*b*(1 - Coth[x]^2)^2) - ((-(((8*a^2 + 20*a*b + 15*b^2)*ArcTanh[C
oth[x]])/b) + (8*(a + b)^(5/2)*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(Sq
rt[a]*b))/(2*b) - ((4*a + 7*b)*Coth[x])/(2*b*(1 - Coth[x]^2)))/(4*b)
```

3.13. $\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx$

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.13.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(74) = 148$.

Time = 0.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.28

$$\frac{2(a^3+3a^2b+3a\,b^2+b^3)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^2+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^2-2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}\right)}{b^3}+\frac{}{4b\,(\mathrm{ta}}$$

input
```
int(sinh(x)^6/(a+b*cosh(x)^2),x)
```

output
```
2/b^3*(a^3+3*a^2*b+3*a*b^2+b^3)*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^(1/2)*ln
((a+b)^(1/2)*tanh(1/2*x)^2-2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2)))+1/4/b/(tanh
(1/2*x)-1)^4+1/2/b/(tanh(1/2*x)-1)^3-1/8*(4*a+7*b)/b^2/(tanh(1/2*x)-1)-1/8
*(5*b+4*a)/b^2/(tanh(1/2*x)-1)^2+1/8/b^3*(-8*a^2-20*a*b-15*b^2)*ln(tanh(1/
2*x)-1)-1/4/b/(tanh(1/2*x)+1)^4+1/2/b/(tanh(1/2*x)+1)^3-1/8*(4*a+7*b)/b^2/
(tanh(1/2*x)+1)-1/8*(-5*b-4*a)/b^2/(tanh(1/2*x)+1)^2+1/8*(8*a^2+20*a*b+15*
b^2)/b^3*ln(tanh(1/2*x)+1)
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 1308, normalized size of antiderivative = 14.86

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 8*(a*b +
2*b^2)*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - 2*a*b - 4*b^2)*sinh(x)^6 + 8*(8*a^
2 + 20*a*b + 15*b^2)*x*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 6*(a*b + 2*b^2)*co
sh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 - 60*(a*b + 2*b^2)*cosh(x)^2 + 4*(8
*a^2 + 20*a*b + 15*b^2)*x)*sinh(x)^4 + 8*(7*b^2*cosh(x)^5 - 20*(a*b + 2*b^
2)*cosh(x)^3 + 4*(8*a^2 + 20*a*b + 15*b^2)*x*cosh(x))*sinh(x)^3 + 8*(a*b +
 2*b^2)*cosh(x)^2 + 4*(7*b^2*cosh(x)^6 - 30*(a*b + 2*b^2)*cosh(x)^4 + 12*(
8*a^2 + 20*a*b + 15*b^2)*x*cosh(x)^2 + 2*a*b + 4*b^2)*sinh(x)^2 + 32*((a^2
 + 2*a*b + b^2)*cosh(x)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(x)^3*sinh(x) + 6*(a
^2 + 2*a*b + b^2)*cosh(x)^2*sinh(x)^2 + 4*(a^2 + 2*a*b + b^2)*cosh(x)*sinh
(x)^3 + (a^2 + 2*a*b + b^2)*sinh(x)^4)*sqrt((a + b)/a)*log((b^2*cosh(x)^4
+ 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*
(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*c
osh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) + 4*(a*b*cosh(x)^2 + 2*a*b*cosh(
x)*sinh(x) + a*b*sinh(x)^2 + 2*a^2 + a*b)*sqrt((a + b)/a))/(b*cosh(x)^4 +
4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(
x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) +
b)) - b^2 + 8*(b^2*cosh(x)^7 - 6*(a*b + 2*b^2)*cosh(x)^5 + 4*(8*a^2 + 20*a
*b + 15*b^2)*x*cosh(x)^3 + 2*(a*b + 2*b^2)*cosh(x))*sinh(x))/(b^3*cosh(x)^
4 + 4*b^3*cosh(x)^3*sinh(x) + 6*b^3*cosh(x)^2*sinh(x)^2 + 4*b^3*cosh(x)...
```

3.13.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(x)}{a + b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**6/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(74) = 148$.

Time = 0.31 (sec) , antiderivative size = 651, normalized size of antiderivative = 7.40

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = -\frac{15\,(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}b} + \frac{5\,\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{32\,\sqrt{(a+b)a}} + \frac{3\,(2\,a+b)x}{2\,b^2} + \frac{15\,x}{16\,b} - \frac{\left(4\,(2\,a+b)e^{(-2\,x)}-b\right)e^{(4\,x)}}{64\,b^2} - \frac{3\,e^{(2\,x)}}{16\,b} + \frac{3\,e^{(-2\,x)}}{16\,b} + \frac{\left(4\,(2\,a+b)e^{(2\,x)}-b\right)e^{(-4\,x)}}{64\,b^2} - \frac{3\,(2\,a+b)\log\left(be^{(4\,x)}+2\,(2\,a+b)e^{(2\,x)}+b\right)}{16\,b^2} + \frac{3\,(2\,a+b)\log\left(2\,(2\,a+b)e^{(-2\,x)}+be^{(-4\,x)}+b\right)}{16\,b^2} - \frac{3\,(8\,a^2+8\,ab+b^2)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}b^2} + \frac{3\,(8\,a^2+8\,ab+b^2)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}b^2} + \frac{(16\,a^2+16\,ab+3\,b^2)x}{8\,b^3} - \frac{(16\,a^2+16\,ab+3\,b^2)\log\left(be^{(4\,x)}+2\,(2\,a+b)e^{(2\,x)}+b\right)}{64\,b^3} + \frac{(16\,a^2+16\,ab+3\,b^2)\log\left(2\,(2\,a+b)e^{(-2\,x)}+be^{(-4\,x)}+b\right)}{64\,b^3} - \frac{(32\,a^3+48\,a^2b+18\,ab^2+b^3)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{128\,\sqrt{(a+b)a}b^3} + \frac{(32\,a^3+48\,a^2b+18\,ab^2+b^3)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{128\,\sqrt{(a+b)a}b^3}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-15/64*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x)
+ 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) + 5/32*log((b*e^(-2*x)
 + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a))
)/sqrt((a + b)*a) + 3/2*(2*a + b)*x/b^2 + 15/16*x/b - 1/64*(4*(2*a + b)*e^
(-2*x) - b)*e^(4*x)/b^2 - 3/16*e^(2*x)/b + 3/16*e^(-2*x)/b + 1/64*(4*(2*a
+ b)*e^(2*x) - b)*e^(-4*x)/b^2 - 3/16*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b
)*e^(2*x) + b)/b^2 + 3/16*(2*a + b)*log(2*(2*a + b)*e^(-2*x) + b*e^(-4*x)
+ b)/b^2 - 3/64*(8*a^2 + 8*a*b + b^2)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a
 + b)*a))/(b*e^(2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2)
 + 3/64*(8*a^2 + 8*a*b + b^2)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a
))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) + 1/8
*(16*a^2 + 16*a*b + 3*b^2)*x/b^3 - 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(b*e^
(4*x) + 2*(2*a + b)*e^(2*x) + b)/b^3 + 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(
2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^3 - 1/128*(32*a^3 + 48*a^2*b + 18
*a*b^2 + b^3)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2
*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^3) + 1/128*(32*a^3 + 48*a^
2*b + 18*a*b^2 + b^3)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^
(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^3)
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{be^{(4x)} - 8\,ae^{(2x)} - 16\,be^{(2x)}}{64\,b^2} + \frac{(8\,a^2 + 20\,ab + 15\,b^2)x}{8\,b^3} - \frac{\left(48\,a^2e^{(4x)} + 120\,abe^{(4x)} + 90\,b^2e^{(4x)} - 8\,abe^{(2x)} - 16\,b^2e^{(2x)} + b^2\right)e^{(-4x)}}{64\,b^3} - \frac{(a^3 + 3\,a^2b + 3\,ab^2 + b^3)\arctan\left(\frac{be^{(2x)}+2\,a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^3}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
1/64*(b*e^(4*x) - 8*a*e^(2*x) - 16*b*e^(2*x))/b^2 + 1/8*(8*a^2 + 20*a*b +
15*b^2)*x/b^3 - 1/64*(48*a^2*e^(4*x) + 120*a*b*e^(4*x) + 90*b^2*e^(4*x) -
8*a*b*e^(2*x) - 16*b^2*e^(2*x) + b^2)*e^(-4*x)/b^3 - (a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 -
 a*b)*b^3)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.82

$$\int \frac{\sinh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{4x}}{64\,b}-\frac{e^{-4x}}{64\,b}+\frac{x\,(8\,a^2+20\,a\,b+15\,b^2)}{8\,b^3}+\frac{e^{-2x}\,(a+2\,b)}{8\,b^2}-\frac{e^{2x}\,(a+2\,b)}{8\,b^2}+\frac{\ln\left(\frac{4\,(a+b)^5\,(2\,a\,b+8\,a^2\,e^{2x}+b^2\,e^{2x}+b^2+8\,a\,b\,e^{2x})}{a\,b^8}-\frac{8\,(a+b)^{11/2}\,(b+4\,a\,e^{2x}+2\,b\,e^{2x})}{\sqrt{a}\,b^8}\right)\,(a+b)^{5/2}}{2\,\sqrt{a}\,b^3}-\frac{\ln\left(\frac{8\,(a+b)^{11/2}\,(b+4\,a\,e^{2x}+2\,b\,e^{2x})}{\sqrt{a}\,b^8}+\frac{4\,(a+b)^5\,(2\,a\,b+8\,a^2\,e^{2x}+b^2\,e^{2x}+b^2+8\,a\,b\,e^{2x})}{a\,b^8}\right)\,(a+b)^{5/2}}{2\,\sqrt{a}\,b^3}$$

input
```
int(sinh(x)^6/(a + b*cosh(x)^2),x)
```

output
```
exp(4*x)/(64*b) - exp(-4*x)/(64*b) + (x*(20*a*b + 8*a^2 + 15*b^2))/(8*b^3)
 + (exp(-2*x)*(a + 2*b))/(8*b^2) - (exp(2*x)*(a + 2*b))/(8*b^2) + (log((4*
(a + b)^5*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/
(a*b^8) - (8*(a + b)^(11/2)*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(1/2)*b^
8))*(a + b)^(5/2))/(2*a^(1/2)*b^3) - (log((8*(a + b)^(11/2)*(b + 4*a*exp(2
*x) + 2*b*exp(2*x)))/(a^(1/2)*b^8) + (4*(a + b)^5*(2*a*b + 8*a^2*exp(2*x)
+ b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*b^8))*(a + b)^(5/2))/(2*a^(1/2)
*b^3)
```

3.14 $\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx$

3.14.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx=-\frac{(2a+3b)x}{2b^2}+\frac{(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}b^2}+\frac{\cosh(x)\sinh(x)}{2b}$$

output
```
-1/2*(2*a+3*b)*x/b^2+1/2*cosh(x)*sinh(x)/b+(a+b)^(3/2)*arctanh(a^(1/2)*tan
h(x)/(a+b)^(1/2))/b^2/a^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx=\frac{-4ax-6bx+\frac{4(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}+b\sinh(2x)}{4b^2}$$

input
```
Integrate[Sinh[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(-4*a*x - 6*b*x + (4*(a + b)^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])
/Sqrt[a] + b*Sinh[2*x])/(4*b^2)
```

3.14.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3670, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(\frac{\pi}{2}+ix\right)^4}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{1}{\left(1-\coth^2(x)\right)^2\left(a-(a+b)\coth^2(x)\right)}d\coth(x)$$

$$\Big\downarrow \text{316}$$

$$-\frac{\int -\frac{(a+b)\coth^2(x)+a+2b}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(a+b)\coth^2(x)+a+2b}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2(a+b)^2\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{(2a+3b)\int \frac{1}{1-\coth^2(x)}d\coth(x)}{b}}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2(a+b)^2\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{(2a+3b)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{(2a+3b)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

input
```
Int[Sinh[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(-(((2*a + 3*b)*ArcTanh[Coth[x]])/b) + (2*(a + b)^(3/2)*ArcTanh[(Sqrt[a +
b]*Coth[x])/Sqrt[a]])/(Sqrt[a]*b))/(2*b) - Coth[x]/(2*b*(1 - Coth[x]^2))
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(47) = 94$.

Time = 26.57 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.14

method	result
default	$\frac{2(a^2+2ab+b^2)\left(\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^2+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^2-2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}\right)}{b^2}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{}{2b}$
risch	$-\frac{ax}{b^2}-\frac{3x}{2b}+\frac{e^{2x}}{8b}-\frac{e^{-2x}}{8b}+\frac{\sqrt{a(a+b)}\,\ln\left(e^{2x}-\frac{2\sqrt{a(a+b)}-2a-b}{b}\right)}{2b^2}+\frac{\sqrt{a(a+b)}\,\ln\left(e^{2x}-\frac{2\sqrt{a(a+b)}-2a-b}{b}\right)}{2ab}-\frac{\sqrt{a(a+b)}\,\ln\left(\right.}{}$

input
```
int(sinh(x)^4/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(a^2+2*a*b+b^2)*(1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^
2+2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2))-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2
)*tanh(1/2*x)^2-2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2)))-1/2/b/(tanh(1/2*x)+1)^
2+1/2/b/(tanh(1/2*x)+1)+1/2/b^2*(-2*a-3*b)*ln(tanh(1/2*x)+1)+1/2/b/(tanh(1
/2*x)-1)^2+1/2/b/(tanh(1/2*x)-1)+1/2*(2*a+3*b)/b^2*ln(tanh(1/2*x)-1)
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 568, normalized size of antiderivative = 9.63

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx$$
$$= \left[\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-4\,(2\,a+3\,b)x\cosh(x)^2+2\,\left(3\,b\cosh(x)^2-2\,(2\,a+3\,b\right.}{}\right.$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/8*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a + 3*b)*x*
cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*(2*a + 3*b)*x)*sinh(x)^2 + 4*((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2)*sqrt((a + b)/a)*lo
g((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^
2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b
 + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(a*b*cosh(x
)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + 2*a^2 + a*b)*sqrt((a + b)/a)
)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)
^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*co
sh(x))*sinh(x) + b)) + 4*(b*cosh(x)^3 - 2*(2*a + 3*b)*x*cosh(x))*sinh(x) -
 b)/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2), 1/8*(b*cosh(x
)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a + 3*b)*x*cosh(x)^2 + 2*
(3*b*cosh(x)^2 - 2*(2*a + 3*b)*x)*sinh(x)^2 + 8*((a + b)*cosh(x)^2 + 2*(a
+ b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2)*sqrt(-(a + b)/a)*arctan(1/2*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-(a + b)/a)/(
a + b)) + 4*(b*cosh(x)^3 - 2*(2*a + 3*b)*x*cosh(x))*sinh(x) - b)/(b^2*cosh
(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)]
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{a + b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**4/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.90

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx = \frac{(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}b} - \frac{3\,\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{16\,\sqrt{(a+b)a}} - \frac{(2\,a+b)x}{b^2} - \frac{x}{b} + \frac{e^{(2\,x)}}{8\,b} - \frac{e^{(-2\,x)}}{8\,b} + \frac{(2\,a+b)\log\left(be^{(4\,x)}+2\,(2\,a+b)e^{(2\,x)}+b\right)}{8\,b^2} - \frac{(2\,a+b)\log\left(2\,(2\,a+b)e^{(-2\,x)}+be^{(-4\,x)}+b\right)}{8\,b^2} + \frac{(8\,a^2+8\,ab+b^2)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{32\,\sqrt{(a+b)a}b^2} - \frac{(8\,a^2+8\,ab+b^2)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{32\,\sqrt{(a+b)a}b^2}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2
*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 3/16*log((b*e^(-2*x) +
2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/s
qrt((a + b)*a) - (2*a + b)*x/b^2 - x/b + 1/8*e^(2*x)/b - 1/8*e^(-2*x)/b +
1/8*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b)*e^(2*x) + b)/b^2 - 1/8*(2*a + b)
*log(2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^2 + 1/32*(8*a^2 + 8*a*b + b^
2)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*a + b + 2*
sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) - 1/32*(8*a^2 + 8*a*b + b^2)*log((
b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((
a + b)*a)))/(sqrt((a + b)*a)*b^2)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.75

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx = -\frac{(2a+3b)x}{2b^2} + \frac{e^{(2x)}}{8b} + \frac{\left(4ae^{(2x)}+6be^{(2x)}-b\right)e^{(-2x)}}{8b^2} + \frac{(a^2+2ab+b^2)\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}\,b^2}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
-1/2*(2*a + 3*b)*x/b^2 + 1/8*e^(2*x)/b + 1/8*(4*a*e^(2*x) + 6*b*e^(2*x) -
b)*e^(-2*x)/b^2 + (a^2 + 2*a*b + b^2)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqr
t(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.47

$$\int \frac{\sinh^4(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{2x}}{8b} - \frac{e^{-2x}}{8b} - \frac{x(2a+3b)}{2b^2} + \frac{\ln\left(-\frac{4e^{2x}(a+b)^2}{b^3} - \frac{2(a+b)^{3/2}(b+2ae^{2x}+be^{2x})}{\sqrt{a}\,b^3}\right)(a+b)^{3/2}}{2\sqrt{a}\,b^2} - \frac{\ln\left(\frac{2(a+b)^{3/2}(b+2ae^{2x}+be^{2x})}{\sqrt{a}\,b^3} - \frac{4e^{2x}(a+b)^2}{b^3}\right)(a+b)^{3/2}}{2\sqrt{a}\,b^2}$$

input
```
int(sinh(x)^4/(a + b*cosh(x)^2),x)
```

output
```
exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (x*(2*a + 3*b))/(2*b^2) + (log(- (4*exp
(2*x)*(a + b)^2)/b^3 - (2*(a + b)^(3/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(
a^(1/2)*b^3))*(a + b)^(3/2))/(2*a^(1/2)*b^2) - (log((2*(a + b)^(3/2)*(b +
2*a*exp(2*x) + b*exp(2*x)))/(a^(1/2)*b^3) - (4*exp(2*x)*(a + b)^2)/b^3)*(a
 + b)^(3/2))/(2*a^(1/2)*b^2)
```

3.15 $\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx$

3.15.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx=\frac{x}{b}-\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}b}$$

output
```
x/b-arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx=\frac{x-\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}}{b}$$

input
```
Integrate[Sinh[x]^2/(a + b*Cosh[x]^2),x]
```

output
```
(x - (Sqrt[a + b]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/Sqrt[a])/b
```

3.15.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3670, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\cos\left(\frac{\pi}{2}+ix\right)^2}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx$$
$$\Big\downarrow \text{3670}$$
$$-\int \frac{1}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)$$
$$\Big\downarrow \text{303}$$
$$\frac{\int \frac{1}{1-\coth^2(x)}d\coth(x)}{b}-\frac{(a+b)\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}(\coth(x))}{b}-\frac{(a+b)\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}$$
$$\Big\downarrow \text{221}$$
$$\frac{\text{arctanh}(\coth(x))}{b}-\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}b}$$

input
```
Int[Sinh[x]^2/(a + b*Cosh[x]^2),x]
```

output
```
ArcTanh[Coth[x]]/b - (Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/
(Sqrt[a]*b)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

method	result
risch	$\frac{x}{b}+\frac{\sqrt{a(a+b)}\,\ln\left(\mathrm{e}^{2x}+\frac{2\sqrt{a(a+b)}+2a+b}{b}\right)}{2ab}-\frac{\sqrt{a(a+b)}\,\ln\left(\mathrm{e}^{2x}-\frac{2\sqrt{a(a+b)}-2a-b}{b}\right)}{2ab}$
default	$\frac{2(a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}-2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln(\tanh(}{b}$

input
```
int(sinh(x)^2/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b+1/2/a*(a*(a+b))^(1/2)/b*ln(exp(2*x)+(2*(a*(a+b))^(1/2)+2*a+b)/b)-1/2/a
*(a*(a+b))^(1/2)/b*ln(exp(2*x)-(2*(a*(a+b))^(1/2)-2*a-b)/b)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 7.69

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx$$
$$= \left[\frac{\sqrt{\frac{a+b}{a}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,(2\,ab+b^2)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+2\,ab+b^2\right)\sinh(x)^2+8\,a^2+8\,ab+b^2+4}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2\right.}\right.}{2\,b}\right.$$
$$\left.-\frac{\sqrt{-\frac{a+b}{a}}\arctan\left(\frac{\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2+2\,a+b\right)\sqrt{-\frac{a+b}{a}}}{2\,(a+b)}\right)-x}{b}\right]$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt((a + b)/a)*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*s
inh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))
*sinh(x) + 4*(a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 + 2*a^
2 + a*b)*sqrt((a + b)/a))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)
^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*
cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 2*x)/b, -(sqrt(-(a + b)/a)*
arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqr
t(-(a + b)/a)/(a + b)) - x)/b]
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**2/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(31) = 62.

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.08

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}b} + \frac{\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}} + \frac{x}{b}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) +
2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) + 1/4*log((b*e^(-2*x) +
2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/s
qrt((a + b)*a) + x/b
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{(a+b)\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b} + \frac{x}{b}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
-(a + b)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a
*b)*b) + x/b
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.03

$$\int \frac{\sinh^2(x)}{a+b\cosh^2(x)}\,dx = \frac{x}{b}+\frac{\operatorname{atan}\left(\frac{\sqrt{-a\,b^2}}{2\,a\,\sqrt{a+b}}+\frac{\sqrt{-a\,b^2}}{b\,\sqrt{a+b}}+\frac{e^{2\,x}\,\sqrt{-a\,b^2}}{2\,a\,\sqrt{a+b}}\right)\,\sqrt{a+b}}{\sqrt{-a\,b^2}}$$

input
```
int(sinh(x)^2/(a + b*cosh(x)^2),x)
```

output
```
x/b + (atan((-a*b^2)^(1/2)/(2*a*(a + b)^(1/2)) + (-a*b^2)^(1/2)/(b*(a + b)
^(1/2)) + (exp(2*x)*(-a*b^2)^(1/2))/(2*a*(a + b)^(1/2)))*(a + b)^(1/2))/(-
a*b^2)^(1/2)
```

3.16 $\int \frac{1}{a+b\cosh^2(x)}\,dx$

3.16.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{1}{a+b\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(1/2)/(a+b)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

3.16.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^2(x)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$
$$\Big\downarrow \; 3660$$
$$\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)$$
$$\Big\downarrow \; 221$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

3.16.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.16.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(21) = 42$.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

method	result	size
default	$\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{2\sqrt{a}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}-2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{2\sqrt{a}\,\sqrt{a+b}}$	78
risch	$\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}+2a^{2}+2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}}$	128

input
```
int(1/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)
+(a+b)^(1/2))-1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2-2*tanh(
1/2*x)*a^(1/2)+(a+b)^(1/2))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 10.10

$$\int \frac{1}{a+b\cosh^2(x)}\,dx$$

$$= \left[\frac{\log\left(\frac{b^{2}\cosh(x)^{4}+4\,b^{2}\cosh(x)\sinh(x)^{3}+b^{2}\sinh(x)^{4}+2\,(2\,ab+b^{2})\cosh(x)^{2}+2\,\left(3\,b^{2}\cosh(x)^{2}+2\,ab+b^{2}\right)\sinh(x)^{2}+8\,a^{2}+8\,ab+b^{2}+4\,\left(b^{2}\cosh(\right.}{b\cosh(x)^{4}+4\,b\cosh(x)\sinh(x)^{3}+b\sinh(x)^{4}+2\,(2\,a+b)\cosh(x)^{2}+2\,\left(3\,b\cosh(x)^{2}+2\,a+b\right)\mathrm{s}}\right.}{2\sqrt{a^{2}+ab}}\right.$$

input
```
integrate(1/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a
*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2
+ 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(
b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2
+ 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(
x))*sinh(x) + b))/sqrt(a^2 + a*b), sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a
*b))/(a^2 + a*b)]
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(27) = 54$.

Time = 24.04 (sec) , antiderivative size = 10924, normalized size of antiderivative = 376.69

$$\int \frac{1}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)**2),x)
```

output
```
Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(
x/2)/(2*b) - 1/(2*b*tanh(x/2)), Eq(a, -b)), (2*tanh(x/2)/(b*(tanh(x/2)**2
+ 1)), Eq(a, 0)), (-a**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b)
)*log(-sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tanh(x/2))/(2*
a**4*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b
/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(a/(a + b) - b/(a + b) -
2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) +
 8*a**3*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt
(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(a/(a +
b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt
(-a*b)/(a + b)) + 2*a*b**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a*b**2*sqrt(-a*
b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(
a + b) + 2*sqrt(-a*b)/(a + b))) + a**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt
(-a*b)/(a + b))*log(sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) + t
anh(x/2))/(2*a**4*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(
a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(a/(a + b) -
 b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*
b)/(a + b)) + 8*a**3*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/
(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b...
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{1}{a+b\cosh^2(x)}\,dx = -\frac{\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b
+ 2*sqrt((a + b)*a)))/sqrt((a + b)*a)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{a+b\cosh^2(x)}\,dx = \frac{\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}}$$

input
```
integrate(1/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/sqrt(-a^2 - a*b)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.21

$$\int \frac{1}{a+b\cosh^2(x)}\,dx =$$

$$-\frac{\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^{2\,x}\,{\left(-a^2-b\,a\right)}^{3/2}\,\left(\frac{4\,\left(4\,a+2\,b\right)\,\left(8\,a^3+12\,a^2\,b+4\,a\,b^2\right)}{b^5\,{\left(-a^2-b\,a\right)}^{3/2}\,\sqrt{-a\,\left(a+b\right)}}+\frac{2\,\left(8\,a^2+8\,a\,b+b^2\right)\,\left(8\,a^2\,\sqrt{-a^2-b\,a}+b^2\,\sqrt{-a^2-b\,a}+8\,a\,b\,\sqrt{-a^2-b\,a}\right)}{a\,b^5\,\left(a+b\right)\,{\left(-a^2-b\,a\right)}^{3/2}}\right)}{4}+\frac{(2\,a^2\,b+}{b^3}\right.}{\sqrt{-a^2-b\,a}}$$

input
```
int(1/(a + b*cosh(x)^2),x)
```

output
```
-atan((b^2*exp(2*x)*(- a*b - a^2)^(3/2)*((4*(4*a + 2*b)*(4*a*b^2 + 12*a^2*
b + 8*a^3))/(b^5*(- a*b - a^2)^(3/2)*(-a*(a + b))^(1/2)) + (2*(8*a*b + 8*a
^2 + b^2)*(8*a^2*(- a*b - a^2)^(1/2) + b^2*(- a*b - a^2)^(1/2) + 8*a*b*(-
a*b - a^2)^(1/2)))/(a*b^5*(a + b)*(- a*b - a^2)^(3/2))))/4 + ((2*a*b^2 + 2
*a^2*b)*(4*a + 2*b))/(b^3*(-a*(a + b))^(1/2)) + ((b^2*(- a*b - a^2)^(1/2)
+ 2*a*b*(- a*b - a^2)^(1/2))*(8*a*b + 8*a^2 + b^2))/(2*a*b^3*(a + b)))/(-
a*b - a^2)^(1/2)
```

3.16. $\int \frac{1}{a+b\cosh^2(x)}\,dx$

3.17 $\int \frac{\text{csch}^4(x)}{a+b\cosh^2(x)}\,dx$

3.17.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\text{csch}^4(x)}{a+b\cosh^2(x)}\,dx=\frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{(a+2b)\coth(x)}{(a+b)^2}-\frac{\coth^3(x)}{3(a+b)}$$

output
```
(a+2*b)*coth(x)/(a+b)^2-1/3*coth(x)^3/(a+b)+b^2*arctanh(a^(1/2)*tanh(x)/(a
+b)^(1/2))/(a+b)^(5/2)/a^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}^4(x)}{a+b\cosh^2(x)}\,dx=\frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{\coth(x)\left(-2a-5b+(a+b)\text{csch}^2(x)\right)}{3(a+b)^2}$$

input
```
Integrate[Csch[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(b^2*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) - (Co
th[x]*(-2*a - 5*b + (a + b)*Csch[x]^2))/(3*(a + b)^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(\frac{\pi}{2}+ix\right)^4\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{3670}$$

$$\int \frac{\left(1-\coth^2(x)\right)^2}{a-(a+b)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{b^2}{(a+b)^2\left(a-(a+b)\coth^2(x)\right)}-\frac{\coth^2(x)}{a+b}+\frac{a+2b}{(a+b)^2}\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}-\frac{\coth^3(x)}{3(a+b)}+\frac{(a+2b)\coth(x)}{(a+b)^2}$$

input
```
Int[Csch[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(b^2*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((a
 + 2*b)*Coth[x])/(a + b)^2 - Coth[x]^3/(3*(a + b))
```

3.17.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(49) = 98$.

Time = 14.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.75

method	result
default	$-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^{3}}{3}+\frac{b\tanh\left(\frac{x}{2}\right)^{3}}{3}-3a\tanh\left(\frac{x}{2}\right)-7b\tanh\left(\frac{x}{2}\right)}{8(a+b)^{2}}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-3a-7b}{8(a+b)^{2}\tanh\left(\frac{x}{2}\right)}-\frac{2b^{2}\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\cdots\right.}{4\sqrt{a}\,\sqrt{\cdots}}\cdots\right.}{\cdots}$
risch	$-\frac{2\left(-3b\,{e}^{4x}+6a\,{e}^{2x}+12b\,{e}^{2x}-2a-5b\right)}{3\left({e}^{2x}-1\right)^{3}(a+b)^{2}}+\frac{b^{2}\ln\left({e}^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}\,(a+b)^{2}}-\frac{b^{2}\ln\left({e}^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}+2a^{2}+2a\cdots}{b\sqrt{a^{2}+ab}}\right.}{2\sqrt{a^{2}+ab}\,(a+b)^{2}}$

input
```
int(csch(x)^4/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a+b)^2*(1/3*a*tanh(1/2*x)^3+1/3*b*tanh(1/2*x)^3-3*a*tanh(1/2*x)-7*b*
tanh(1/2*x))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-7*b)/tanh(1/2*x)-2
*b^2/(a+b)^2*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*tanh
(1/2*x)*a^(1/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*x)^2-2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2)))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 1875, normalized size of antiderivative = 31.78

$$\int \frac{\operatorname{csch}^4(x)}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b + a*b^2)*cosh(x)^4 + 48*(a^2*b + a*b^2)*cosh(x)*sinh(x)^3
+ 12*(a^2*b + a*b^2)*sinh(x)^4 + 8*a^3 + 28*a^2*b + 20*a*b^2 - 24*(a^3 + 3
*a^2*b + 2*a*b^2)*cosh(x)^2 - 24*(a^3 + 3*a^2*b + 2*a*b^2 - 3*(a^2*b + a*b
^2)*cosh(x)^2)*sinh(x)^2 + 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^
2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 3*b^
2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cos
h(x)^4 - 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 - b^2 + 6*(b^2*cosh(x)^5 - 2*b^2
*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a^2 + a*b)*log((b^2*cosh(x)^4 + 4*
b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b
^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(
x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(
x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x)^4 + 4*b*cosh(x)*si
nh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b
)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 48*((a^2
*b + a*b^2)*cosh(x)^3 - (a^3 + 3*a^2*b + 2*a*b^2)*cosh(x))*sinh(x))/((a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*cosh(x)*sinh(x)^5 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^6 -
 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 - 3*(a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3 - 5*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^2)*sinh(x)^4
 - a^4 - 3*a^3*b - 3*a^2*b^2 - a*b^3 + 4*(5*(a^4 + 3*a^3*b + 3*a^2*b^2 ...
```

3.17.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a + b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{a + b\cosh^2(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*cosh(x)**2),x)
```

output
```
Integral(csch(x)**4/(a + b*cosh(x)**2), x)
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(49) = 98.

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh^2(x)}\,dx = -\frac{b^2\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}(a^2+2ab+b^2)} - \frac{2\left(6(a+2b)e^{(-2x)}-3be^{(-4x)}-2a-5b\right)}{3\left(a^2+2ab+b^2-3(a^2+2ab+b^2)e^{(-2x)}+3(a^2+2ab+b^2)e^{(-4x)}-(a^2+2ab+b^2)e^{(-6x)}\right)}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*b^2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a
+ b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2)) - 2/3*(6*(
a + 2*b)*e^(-2*x) - 3*b*e^(-4*x) - 2*a - 5*b)/(a^2 + 2*a*b + b^2 - 3*(a^2
+ 2*a*b + b^2)*e^(-2*x) + 3*(a^2 + 2*a*b + b^2)*e^(-4*x) - (a^2 + 2*a*b +
b^2)*e^(-6*x))
```

3.17.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}(x)^4}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.17.9 Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.15

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh^2(x)}\,dx = \frac{2\,b}{(a+b)^2\,\left(\mathrm{e}^{2\,x}-1\right)} - \frac{4}{(a+b)\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{8}{3\,(a+b)\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)} - \frac{b^2\,\ln\left(\frac{4\,b^2\,\left(2\,a\,b+8\,a^2\,\mathrm{e}^{2\,x}+b^2\,\mathrm{e}^{2\,x}+b^2+8\,a\,b\,\mathrm{e}^{2\,x}\right)}{a\,(a+b)^5}-\frac{8\,b^2\,\left(b+4\,a\,\mathrm{e}^{2\,x}+2\,b\,\mathrm{e}^{2\,x}\right)}{\sqrt{a}\,(a+b)^{9/2}}\right)}{2\,\sqrt{a}\,(a+b)^{5/2}} + \frac{b^2\,\ln\left(\frac{8\,b^2\,\left(b+4\,a\,\mathrm{e}^{2\,x}+2\,b\,\mathrm{e}^{2\,x}\right)}{\sqrt{a}\,(a+b)^{9/2}}+\frac{4\,b^2\,\left(2\,a\,b+8\,a^2\,\mathrm{e}^{2\,x}+b^2\,\mathrm{e}^{2\,x}+b^2+8\,a\,b\,\mathrm{e}^{2\,x}\right)}{a\,(a+b)^5}\right)}{2\,\sqrt{a}\,(a+b)^{5/2}}$$

input
```
int(1/(sinh(x)^4*(a + b*cosh(x)^2)),x)
```

output
```
(2*b)/((a + b)^2*(exp(2*x) - 1)) - 4/((a + b)*(exp(4*x) - 2*exp(2*x) + 1))
 - 8/(3*(a + b)*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (b^2*log((4*b^
2*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*(a +
b)^5) - (8*b^2*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(1/2)*(a + b)^(9/2)))
)/(2*a^(1/2)*(a + b)^(5/2)) + (b^2*log((8*b^2*(b + 4*a*exp(2*x) + 2*b*exp(
2*x)))/(a^(1/2)*(a + b)^(9/2)) + (4*b^2*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(
2*x) + b^2 + 8*a*b*exp(2*x)))/(a*(a + b)^5)))/(2*a^(1/2)*(a + b)^(5/2))
```

3.18 $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx$

3.18.1 Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{7/2}} - \frac{(a^2+3ab+3b^2)\coth(x)}{(a+b)^3} + \frac{(2a+3b)\coth^3(x)}{3(a+b)^2} - \frac{\coth^5(x)}{5(a+b)}$$

output
```
-(a^2+3*a*b+3*b^2)*coth(x)/(a+b)^3+1/3*(2*a+3*b)*coth(x)^3/(a+b)^2-1/5*cot
h(x)^5/(a+b)-b^3*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/(a+b)^(7/2)/a^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{7/2}} - \frac{\coth(x)\left(8a^2+26ab+33b^2-(4a^2+13ab+9b^2)\operatorname{csch}^2(x)+3(a+b)^2\operatorname{csch}^4(x)\right)}{15(a+b)^3}$$

input
```
Integrate[Csch[x]^6/(a + b*Cosh[x]^2),x]
```

output
```
-((b^3*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(7/2))) -
(Coth[x]*(8*a^2 + 26*a*b + 33*b^2 - (4*a^2 + 13*a*b + 9*b^2)*Csch[x]^2 + 3
*(a + b)^2*Csch[x]^4))/(15*(a + b)^3)
```

3.18.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 3670, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{\cos\left(\frac{\pi}{2}+ix\right)^6\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\cos\left(ix+\frac{\pi}{2}\right)^6\left(b\sin\left(ix+\frac{\pi}{2}\right)^2+a\right)}dx$$

$$\Big\downarrow \ 3670$$

$$-\int \frac{\left(1-\coth^2(x)\right)^3}{a-(a+b)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \ 300$$

$$-\int \left(\frac{\coth^4(x)}{a+b}-\frac{(2a+3b)\coth^2(x)}{(a+b)^2}+\frac{a^2+3ba+3b^2}{(a+b)^3}+\frac{b^3}{(a+b)^3\left(a-(a+b)\coth^2(x)\right)}\right)d\coth(x)$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(a^2+3ab+3b^2\right)\coth(x)}{(a+b)^3}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}}-\frac{\coth^5(x)}{5(a+b)}+\frac{(2a+3b)\coth^3(x)}{3(a+b)^2}$$

input
```
Int[Csch[x]^6/(a + b*Cosh[x]^2),x]
```

output
```
-((b^3*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2))) -
((a^2 + 3*a*b + 3*b^2)*Coth[x])/(a + b)^3 + ((2*a + 3*b)*Coth[x]^3)/(3*(a
+ b)^2) - Coth[x]^5/(5*(a + b))
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3670
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Su
bst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
 + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(77) = 154$.

Time = 54.96 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.79

method	result
default	$-\frac{\frac{a^2\tanh\left(\frac{x}{2}\right)^5}{5}+\frac{2ab\tanh\left(\frac{x}{2}\right)^5}{5}+\frac{b^2\tanh\left(\frac{x}{2}\right)^5}{5}-\frac{5a^2\tanh\left(\frac{x}{2}\right)^3}{3}-\frac{14ab\tanh\left(\frac{x}{2}\right)^3}{3}-3b^2\tanh\left(\frac{x}{2}\right)^3+10a^2\tanh\left(\frac{x}{2}\right)+32ab\tanh\left(\frac{x}{2}\right)+38b^2\,\mathrm{ta}}{32(a+b)^3}$
risch	$-\frac{2(15b^2e^{8x}-30ab\,e^{6x}-90b^2e^{6x}+80a^2e^{4x}+230ab\,e^{4x}+240b^2e^{4x}-40a^2e^{2x}-130b\,e^{2x}a-150b^2e^{2x}+8a^2+26ab+33b^2)}{15(e^{2x}-1)^5(a+b)^3}+\frac{b^3\ln\left(e^{2x}+\right.}{}$

input
```
int(csch(x)^6/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/(a+b)^3*(1/5*a^2*tanh(1/2*x)^5+2/5*a*b*tanh(1/2*x)^5+1/5*b^2*tanh(1/
2*x)^5-5/3*a^2*tanh(1/2*x)^3-14/3*a*b*tanh(1/2*x)^3-3*b^2*tanh(1/2*x)^3+10
*a^2*tanh(1/2*x)+32*a*b*tanh(1/2*x)+38*b^2*tanh(1/2*x))-1/160/(a+b)/tanh(1
/2*x)^5-1/96*(-5*a-9*b)/(a+b)^2/tanh(1/2*x)^3-1/32/(a+b)^3*(10*a^2+32*a*b+
38*b^2)/tanh(1/2*x)+2*b^3/(a+b)^3*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)
*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^(1/2)*
ln((a+b)^(1/2)*tanh(1/2*x)^2-2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2)))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2408 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 4977, normalized size of antiderivative = 55.92

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[-1/30*(60*(a^2*b^2 + a*b^3)*cosh(x)^8 + 480*(a^2*b^2 + a*b^3)*cosh(x)*sin
h(x)^7 + 60*(a^2*b^2 + a*b^3)*sinh(x)^8 - 120*(a^3*b + 4*a^2*b^2 + 3*a*b^3
)*cosh(x)^6 - 120*(a^3*b + 4*a^2*b^2 + 3*a*b^3 - 14*(a^2*b^2 + a*b^3)*cosh
(x)^2)*sinh(x)^6 + 240*(14*(a^2*b^2 + a*b^3)*cosh(x)^3 - 3*(a^3*b + 4*a^2*
b^2 + 3*a*b^3)*cosh(x))*sinh(x)^5 + 40*(8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24
*a*b^3)*cosh(x)^4 + 40*(105*(a^2*b^2 + a*b^3)*cosh(x)^4 + 8*a^4 + 31*a^3*b
 + 47*a^2*b^2 + 24*a*b^3 - 45*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sin
h(x)^4 + 32*a^4 + 136*a^3*b + 236*a^2*b^2 + 132*a*b^3 + 160*(21*(a^2*b^2 +
 a*b^3)*cosh(x)^5 - 15*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 + (8*a^4 +
31*a^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x))*sinh(x)^3 - 40*(4*a^4 + 17*a^3*
b + 28*a^2*b^2 + 15*a*b^3)*cosh(x)^2 + 40*(42*(a^2*b^2 + a*b^3)*cosh(x)^6
- 45*(a^3*b + 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 - 4*a^4 - 17*a^3*b - 28*a^2*b
^2 - 15*a*b^3 + 6*(8*a^4 + 31*a^3*b + 47*a^2*b^2 + 24*a*b^3)*cosh(x)^2)*si
nh(x)^2 - 15*(b^3*cosh(x)^10 + 10*b^3*cosh(x)*sinh(x)^9 + b^3*sinh(x)^10 -
 5*b^3*cosh(x)^8 + 10*b^3*cosh(x)^6 + 5*(9*b^3*cosh(x)^2 - b^3)*sinh(x)^8
+ 40*(3*b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x)^7 - 10*b^3*cosh(x)^4 + 10*(21
*b^3*cosh(x)^4 - 14*b^3*cosh(x)^2 + b^3)*sinh(x)^6 + 4*(63*b^3*cosh(x)^5 -
 70*b^3*cosh(x)^3 + 15*b^3*cosh(x))*sinh(x)^5 + 5*b^3*cosh(x)^2 + 10*(21*b
^3*cosh(x)^6 - 35*b^3*cosh(x)^4 + 15*b^3*cosh(x)^2 - b^3)*sinh(x)^4 + 40*(
3*b^3*cosh(x)^7 - 7*b^3*cosh(x)^5 + 5*b^3*cosh(x)^3 - b^3*cosh(x))*sinh...
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(csch(x)**6/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.45

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = \frac{b^3\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)a}} - \frac{2\,\left(15\,b^2e^{(-8\,x)}+8\,a^2+26\,ab+33\,b^2-10\,(4\,a^2+13\,ab+15\,b^2)e^{\cdots}\right.}{15\,(a^3+3\,a^2b+3\,ab^2+b^3-5\,(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-2\,x)}+10\,(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,x)}-10\cdots}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/2*b^3*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a +
 b + 2*sqrt((a + b)*a)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*a))
 - 2/15*(15*b^2*e^(-8*x) + 8*a^2 + 26*a*b + 33*b^2 - 10*(4*a^2 + 13*a*b +
15*b^2)*e^(-2*x) + 10*(8*a^2 + 23*a*b + 24*b^2)*e^(-4*x) - 30*(a*b + 3*b^2
)*e^(-6*x))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*e^(-2*x) + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*x) - 10*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*e^(-6*x) + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-8*
x) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-10*x))
```

3.18.8 Giac [F]

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{csch}(x)^6}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.74

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh^2(x)}\,dx = \frac{4\,(b^2+a\,b)}{(a+b)^3\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)} - \frac{16}{(a+b)\,\left(6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1\right)} - \frac{2\,b^2}{(a+b)^3\,\left(e^{2\,x}-1\right)} - \frac{32}{5\,(a+b)\,\left(5\,e^{2\,x}-10\,e^{4\,x}+10\,e^{6\,x}-5\,e^{8\,x}+e^{10\,x}-1\right)} - \frac{8\,(4\,a+3\,b)}{3\,(a+b)^2\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} + \frac{b^3\,\ln\left(\frac{4\,b^4\,(2\,a\,b+8\,a^2\,e^{2\,x}+b^2\,e^{2\,x}+b^2+8\,a\,b\,e^{2\,x})}{a\,(a+b)^7}-\frac{8\,b^4\,(b+4\,a\,e^{2\,x}+2\,b\,e^{2\,x})}{\sqrt{a}\,(a+b)^{13/2}}\right)}{2\,\sqrt{a}\,(a+b)^{7/2}} - \frac{b^3\,\ln\left(\frac{8\,b^4\,(b+4\,a\,e^{2\,x}+2\,b\,e^{2\,x})}{\sqrt{a}\,(a+b)^{13/2}}+\frac{4\,b^4\,(2\,a\,b+8\,a^2\,e^{2\,x}+b^2\,e^{2\,x}+b^2+8\,a\,b\,e^{2\,x})}{a\,(a+b)^7}\right)}{2\,\sqrt{a}\,(a+b)^{7/2}}$$

input
```
int(1/(sinh(x)^6*(a + b*cosh(x)^2)),x)
```

output
```
(4*(a*b + b^2))/((a + b)^3*(exp(4*x) - 2*exp(2*x) + 1)) - 16/((a + b)*(6*e
xp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) - (2*b^2)/((a + b)^3*(e
xp(2*x) - 1)) - 32/(5*(a + b)*(5*exp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*
exp(8*x) + exp(10*x) - 1)) - (8*(4*a + 3*b))/(3*(a + b)^2*(3*exp(2*x) - 3*
exp(4*x) + exp(6*x) - 1)) + (b^3*log((4*b^4*(2*a*b + 8*a^2*exp(2*x) + b^2*
exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a*(a + b)^7) - (8*b^4*(b + 4*a*exp(2*x)
 + 2*b*exp(2*x)))/(a^(1/2)*(a + b)^(13/2))))/(2*a^(1/2)*(a + b)^(7/2)) - (
b^3*log((8*b^4*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(1/2)*(a + b)^(13/2))
 + (4*b^4*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/
(a*(a + b)^7)))/(2*a^(1/2)*(a + b)^(7/2))
```

3.19 $\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx$

3.19.1 Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = \frac{\arctan\left(\frac{1+\sqrt[3]{6}\cosh(x)}{\sqrt{3}}\right)}{2\sqrt[3]{2}3^{5/6}} - \frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}} + \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cosh(x)+3^{2/3}\cosh^2(x)\right)}{12\sqrt[3]{6}}$$

output
```
1/12*arctan(1/3*(1+6^(1/3)*cosh(x))*3^(1/2))*2^(2/3)*3^(1/6)-1/36*ln(2^(2/
3)-3^(1/3)*cosh(x))*6^(2/3)+1/72*ln(2*2^(1/3)+2^(2/3)*3^(1/3)*cosh(x)+3^(2
/3)*cosh(x)^2)*6^(2/3)
```

3.19.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = \frac{1}{72}\left(6\ 2^{2/3}\sqrt[6]{3}\arctan\left(\frac{1+\sqrt[3]{6}\cosh(x)}{\sqrt{3}}\right) + 6^{2/3}\left(-2\log\left(2-\sqrt[3]{6}\cosh(x)\right)+\log\left(4+2\sqrt[3]{6}\cosh(x)+6^{2/3}\cosh^2(x)\right)\right)\right)$$

input
```
Integrate[Sinh[x]/(4 - 3*Cosh[x]^3),x]
```

output
```
(6*2^(2/3)*3^(1/6)*ArcTan[(1 + 6^(1/3)*Cosh[x])/Sqrt[3]] + 6^(2/3)*(-2*Log
[2 - 6^(1/3)*Cosh[x]] + Log[4 + 2*6^(1/3)*Cosh[x] + 6^(2/3)*Cosh[x]^2]))/7
2
```

3.19.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 26, 3702, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(\frac{\pi}{2}+ix\right)}{4-3\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix+\frac{\pi}{2}\right)}{4-3\sin\left(ix+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3702}$$

$$\int \frac{1}{4-3\cosh^3(x)}d\cosh(x)$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{\sqrt[3]{3}\cosh(x)+2\ 2^{2/3}}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{6\sqrt[3]{2}}+\frac{\int \frac{1}{2^{2/3}-\sqrt[3]{3}\cosh(x)}d\cosh(x)}{6\sqrt[3]{2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{\sqrt[3]{3}\cosh(x)+2\ 2^{2/3}}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3\int \frac{1}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{\sqrt[3]{2}}+\frac{\int \frac{2^{2/3}\sqrt[3]{3}\left(\sqrt[3]{6}\cosh(x)+1\right)}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{2\sqrt[3]{3}}}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{1}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{\sqrt[3]{2}}+\frac{\int \frac{\sqrt[3]{6}\cosh(x)+1}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{\sqrt[3]{2}}}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\int \frac{\sqrt[3]{6}\cosh(x)+1}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{\sqrt[3]{2}}-3^{2/3}\int \frac{1}{-\left(\sqrt[3]{6}\cosh(x)+1\right)^2-3}d\left(\sqrt[3]{6}\cosh(x)+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\int \frac{\sqrt[3]{6}\cosh(x)+1}{3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}}d\cosh(x)}{\sqrt[3]{2}}+\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{6}\cosh(x)+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\sqrt[6]{3}\arctan\left(\frac{\sqrt[3]{6}\cosh(x)+1}{\sqrt{3}}\right)+\frac{\log\left(3^{2/3}\cosh^2(x)+2^{2/3}\sqrt[3]{3}\cosh(x)+2\sqrt[3]{2}\right)}{2\sqrt[3]{3}}}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cosh(x)\right)}{6\sqrt[3]{6}}$$

input
```
Int[Sinh[x]/(4 - 3*Cosh[x]^3),x]
```

output
```
-1/6*Log[2^(2/3) - 3^(1/3)*Cosh[x]]/6^(1/3) + (3^(1/6)*ArcTan[(1 + 6^(1/3)
*Cosh[x])/Sqrt[3]] + Log[2*2^(1/3) + 2^(2/3)*3^(1/3)*Cosh[x] + 3^(2/3)*Cos
h[x]^2]/(2*3^(1/3)))/(6*2^(1/3))
```

3.19. $\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.19.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.27

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(1296\_Z^3+1)} \_R\ln\left(24\_R\,\mathrm{e}^{x}+\mathrm{e}^{2x}+1\right)$
derivativedivides	$-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cosh(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cosh(x)^2+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cosh(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}+\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cosh(x)}{2}+\right.}{3}\right.}{12}$
default	$-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cosh(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cosh(x)^2+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cosh(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}+\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cosh(x)}{2}+\right.}{3}\right.}{12}$

input
```
int(sinh(x)/(4-3*cosh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(24*_R*exp(x)+exp(2*x)+1),_R=RootOf(1296*_Z^3+1))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.11

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = \frac{1}{12} \cdot 6^{\frac{1}{6}}\sqrt{2}(-1)^{\frac{1}{3}} \arctan\left(\frac{1}{12}\right.$$
$$\cdot 6^{\frac{1}{6}}\left(6^{\frac{2}{3}}\sqrt{2}(-1)^{\frac{2}{3}}\cosh(x)^3 + 6^{\frac{2}{3}}\sqrt{2}(-1)^{\frac{2}{3}}\sinh(x)^3 + \left(3\cdot 6^{\frac{2}{3}}\sqrt{2}(-1)^{\frac{2}{3}}\cosh(x) + 4\cdot 6^{\frac{1}{3}}\sqrt{2}\right)\sinh(x)^2 + 4\cdot\right.$$
$$-\frac{1}{12}$$
$$\cdot 6^{\frac{1}{6}}\sqrt{2}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{12}\cdot 6^{\frac{1}{6}}\left(6^{\frac{2}{3}}\sqrt{2}(-1)^{\frac{2}{3}}\cosh(x) + 6^{\frac{2}{3}}\sqrt{2}(-1)^{\frac{2}{3}}\sinh(x) + 2\cdot 6^{\frac{1}{3}}\sqrt{2}\right)\right)$$
$$-\frac{1}{72}$$
$$\cdot 6^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\left(2\cdot 6^{\frac{2}{3}}(-1)^{\frac{1}{3}}\cosh(x) - 3\cosh(x)^2 - 3\sinh(x)^2 - 4\cdot 6^{\frac{1}{3}}(-1)^{\frac{2}{3}} - 3\right)}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}\right)$$
$$+\frac{1}{36}\cdot 6^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{2\left(6^{\frac{2}{3}}(-1)^{\frac{1}{3}} + 3\cosh(x)\right)}{\cosh(x) - \sinh(x)}\right)$$

input
```
integrate(sinh(x)/(4-3*cosh(x)^3),x, algorithm="fricas")
```

output
```
1/12*6^(1/6)*sqrt(2)*(-1)^(1/3)*arctan(1/12*6^(1/6)*(6^(2/3)*sqrt(2)*(-1)^
(2/3)*cosh(x)^3 + 6^(2/3)*sqrt(2)*(-1)^(2/3)*sinh(x)^3 + (3*6^(2/3)*sqrt(2
)*(-1)^(2/3)*cosh(x) + 4*6^(1/3)*sqrt(2))*sinh(x)^2 + 4*6^(1/3)*sqrt(2)*co
sh(x)^2 + (6^(2/3)*sqrt(2)*(-1)^(2/3) - 16*sqrt(2)*(-1)^(1/3))*cosh(x) + (
3*6^(2/3)*sqrt(2)*(-1)^(2/3)*cosh(x)^2 + 6^(2/3)*sqrt(2)*(-1)^(2/3) + 8*6^
(1/3)*sqrt(2)*cosh(x) - 16*sqrt(2)*(-1)^(1/3))*sinh(x) + 2*6^(1/3)*sqrt(2)
)) - 1/12*6^(1/6)*sqrt(2)*(-1)^(1/3)*arctan(1/12*6^(1/6)*(6^(2/3)*sqrt(2)*
(-1)^(2/3)*cosh(x) + 6^(2/3)*sqrt(2)*(-1)^(2/3)*sinh(x) + 2*6^(1/3)*sqrt(2
))) - 1/72*6^(2/3)*(-1)^(1/3)*log(-2*(2*6^(2/3)*(-1)^(1/3)*cosh(x) - 3*cos
h(x)^2 - 3*sinh(x)^2 - 4*6^(1/3)*(-1)^(2/3) - 3)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)) + 1/36*6^(2/3)*(-1)^(1/3)*log(2*(6^(2/3)*(-1)^(1/3) +
3*cosh(x))/(cosh(x) - sinh(x)))
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = -\frac{6^{\frac{2}{3}}\log\left(\cosh(x)-\frac{6^{\frac{2}{3}}}{3}\right)}{36} + \frac{6^{\frac{2}{3}}\log\left(36\cosh^2(x)+12\cdot 6^{\frac{2}{3}}\cosh(x)+24\cdot\sqrt[3]{6}\right)}{72} + \frac{2^{\frac{2}{3}}\cdot\sqrt[6]{3}\,\mathrm{atan}\left(\frac{\sqrt[3]{2}\cdot 3^{\frac{5}{6}}\cosh(x)}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(sinh(x)/(4-3*cosh(x)**3),x)
```

output
```
-6**(2/3)*log(cosh(x) - 6**(2/3)/3)/36 + 6**(2/3)*log(36*cosh(x)**2 + 12*6
**(2/3)*cosh(x) + 24*6**(1/3))/72 + 2**(2/3)*3**(1/6)*atan(2**(1/3)*3**(5/
6)*cosh(x)/3 + sqrt(3)/3)/12
```

3.19.7 Maxima [F]

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = \int -\frac{\sinh(x)}{3\cosh(x)^3-4}\,dx$$

input
```
integrate(sinh(x)/(4-3*cosh(x)^3),x, algorithm="maxima")
```

output
```
-integrate(sinh(x)/(3*cosh(x)^3 - 4), x)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx = \frac{1}{12}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{4}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{2}{3}}\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}+e^{(-x)}+e^{x}\right)\right) + \frac{1}{72}\cdot 36^{\frac{1}{3}}\log\left(\left(e^{(-x)}+e^{x}\right)^2+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(e^{(-x)}+e^{x}\right)+4\left(\frac{4}{3}\right)^{\frac{2}{3}}\right) - \frac{1}{12}\left(\frac{4}{3}\right)^{\frac{1}{3}}\log\left(\left|-2\left(\frac{4}{3}\right)^{\frac{1}{3}}+e^{(-x)}+e^{x}\right|\right)$$

input
```
integrate(sinh(x)/(4-3*cosh(x)^3),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*(4/3)^(1/3)*arctan(1/4*sqrt(3)*(4/3)^(2/3)*((4/3)^(1/3) + e^(
-x) + e^x)) + 1/72*36^(1/3)*log((e^(-x) + e^x)^2 + 2*(4/3)^(1/3)*(e^(-x) +
 e^x) + 4*(4/3)^(2/3)) - 1/12*(4/3)^(1/3)*log(abs(-2*(4/3)^(1/3) + e^(-x)
+ e^x))
```

3.19.9 Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.09

$$\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx$$

$$= -\frac{6^{2/3}\ln\left(\frac{256\,e^{2x}}{81}-\frac{128\,e^{x}}{27}+\frac{6^{2/3}\left(\frac{256\,e^{2x}}{9}-\frac{2048\,e^{x}}{27}+\frac{6^{2/3}\left(256\,e^{2x}-\frac{2048\,e^{x}}{3}+256\right)}{36}+\frac{256}{9}\right)}{36}+\frac{256}{81}\right)}{36}$$

$$-\frac{6^{2/3}\ln\left(\frac{256\,e^{2x}}{81}-\frac{128\,e^{x}}{27}+\frac{6^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{256\,e^{2x}}{9}-\frac{2048\,e^{x}}{27}+\frac{6^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(256\,e^{2x}-\frac{2048\,e^{x}}{3}+256\right)}{36}+\frac{256}{9}\right)}{36}+\frac{256}{81}\right)\left(-\right.}{36}$$

$$+\frac{6^{2/3}\ln\left(\frac{256\,e^{2x}}{81}-\frac{128\,e^{x}}{27}-\frac{6^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{256\,e^{2x}}{9}-\frac{2048\,e^{x}}{27}-\frac{6^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(256\,e^{2x}-\frac{2048\,e^{x}}{3}+256\right)}{36}+\frac{256}{9}\right)}{36}+\frac{256}{81}\right)\left(\frac{1}{2}+\right.}{36}$$

input
```
int(-sinh(x)/(3*cosh(x)^3 - 4),x)
```

output
```
(6^(2/3)*log((256*exp(2*x))/81 - (128*exp(x))/27 - (6^(2/3)*((3^(1/2)*1i)/
2 + 1/2)*((256*exp(2*x))/9 - (2048*exp(x))/27 - (6^(2/3)*((3^(1/2)*1i)/2 +
 1/2)*(256*exp(2*x) - (2048*exp(x))/3 + 256))/36 + 256/9))/36 + 256/81)*((
3^(1/2)*1i)/2 + 1/2))/36 - (6^(2/3)*log((256*exp(2*x))/81 - (128*exp(x))/2
7 + (6^(2/3)*((3^(1/2)*1i)/2 - 1/2)*((256*exp(2*x))/9 - (2048*exp(x))/27 +
 (6^(2/3)*((3^(1/2)*1i)/2 - 1/2)*(256*exp(2*x) - (2048*exp(x))/3 + 256))/3
6 + 256/9))/36 + 256/81)*((3^(1/2)*1i)/2 - 1/2))/36 - (6^(2/3)*log((256*ex
p(2*x))/81 - (128*exp(x))/27 + (6^(2/3)*((256*exp(2*x))/9 - (2048*exp(x))/
27 + (6^(2/3)*(256*exp(2*x) - (2048*exp(x))/3 + 256))/36 + 256/9))/36 + 25
6/81))/36
```

3.19. $\int \frac{\sinh(x)}{4-3\cosh^3(x)}\,dx$

3.20 $\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx$

3.20.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx = -\frac{a^3\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{(a^2-ab+b^2)\sinh(x)}{b^3}-\frac{(a-2b)\sinh^3(x)}{3b^2}+\frac{\sinh^5(x)}{5b}$$

output
```
(a^2-a*b+b^2)*sinh(x)/b^3-1/3*(a-2*b)*sinh(x)^3/b^2+1/5*sinh(x)^5/b-a^3*ar
ctan(sinh(x)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx = \frac{a^3\arctan\left(\frac{\sqrt{a+b}\operatorname{csch}(x)}{\sqrt{b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{(8a^2-6ab+5b^2)\sinh(x)}{8b^3}-\frac{(4a-5b)\sinh(3x)}{48b^2}+\frac{\sinh(5x)}{80b}$$

input
```
Integrate[Cosh[x]^7/(a + b*Cosh[x]^2),x]
```

output
```
(a^3*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/(b^(7/2)*Sqrt[a + b]) + ((8*a^
2 - 6*a*b + 5*b^2)*Sinh[x])/(8*b^3) - ((4*a - 5*b)*Sinh[3*x])/(48*b^2) + S
inh[5*x]/(80*b)
```

3.20.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^7}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{\left(\sinh^2(x)+1\right)^3}{a+b\sinh^2(x)+b}d\sinh(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(-\frac{a^3}{b^3\left(a+b\sinh^2(x)+b\right)}+\frac{a^2-ab+b^2}{b^3}-\frac{(a-2b)\sinh^2(x)}{b^2}+\frac{\sinh^4(x)}{b}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{\left(a^2-ab+b^2\right)\sinh(x)}{b^3}-\frac{(a-2b)\sinh^3(x)}{3b^2}+\frac{\sinh^5(x)}{5b}$$

input
```
Int[Cosh[x]^7/(a + b*Cosh[x]^2),x]
```

output
```
-((a^3*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b])) + ((a
^2 - a*b + b^2)*Sinh[x])/b^3 - ((a - 2*b)*Sinh[x]^3)/(3*b^2) + Sinh[x]^5/(
5*b)
```

3.20.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(66) = 132$.

Time = 1.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.64

method	result
risch	$\frac{e^{5x}}{160b} + \frac{5\,e^{3x}}{96b} - \frac{e^{3x}a}{24b^2} + \frac{e^{x}a^2}{2b^3} - \frac{3a\,e^{x}}{8b^2} + \frac{5\,e^{x}}{16b} - \frac{e^{-x}a^2}{2b^3} + \frac{3a\,e^{-x}}{8b^2} - \frac{5\,e^{-x}}{16b} - \frac{5\,e^{-3x}}{96b} + \frac{e^{-3x}a}{24b^2} - \frac{e^{-5x}}{160b} - \frac{a^3\ln\left(e^{2x}+\right.}{2\sqrt{-}}$
default	$-\frac{2a^3\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}+2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}+\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}-2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}\right)}{b^3} - \frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)+1\right)^5} + \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} - \frac{-7b+}{8b^2\left(\tanh\left(\right.\right.}$

input
```
int(cosh(x)^7/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/160/b*exp(5*x)+5/96/b*exp(3*x)-1/24/b^2*exp(3*x)*a+1/2/b^3*exp(x)*a^2-3/
8*a/b^2*exp(x)+5/16/b*exp(x)-1/2/b^3*exp(-x)*a^2+3/8*a/b^2*exp(-x)-5/16/b*
exp(-x)-5/96/b*exp(-3*x)+1/24/b^2*exp(-3*x)*a-1/160/b*exp(-5*x)-1/2/(-a*b-
b^2)^(1/2)*a^3/b^3*ln(exp(2*x)+2*(a+b)/(-a*b-b^2)^(1/2)*exp(x)-1)+1/2/(-a*
b-b^2)^(1/2)*a^3/b^3*ln(exp(2*x)-2*(a+b)/(-a*b-b^2)^(1/2)*exp(x)-1)
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. $2(66) = 132$.

Time = 0.28 (sec) , antiderivative size = 2508, normalized size of antiderivative = 32.15

$$\int \frac{\cosh^7(x)}{a + b\cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^7/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/480*(3*(a*b^3 + b^4)*cosh(x)^10 + 30*(a*b^3 + b^4)*cosh(x)*sinh(x)^9 +
3*(a*b^3 + b^4)*sinh(x)^10 - 5*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^8 - 5*(
4*a^2*b^2 - a*b^3 - 5*b^4 - 27*(a*b^3 + b^4)*cosh(x)^2)*sinh(x)^8 + 40*(9*
(a*b^3 + b^4)*cosh(x)^3 - (4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x))*sinh(x)^7 +
 30*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^6 + 10*(63*(a*b^3 + b^4)
*cosh(x)^4 + 24*a^3*b + 6*a^2*b^2 - 3*a*b^3 + 15*b^4 - 14*(4*a^2*b^2 - a*b
^3 - 5*b^4)*cosh(x)^2)*sinh(x)^6 + 4*(189*(a*b^3 + b^4)*cosh(x)^5 - 70*(4*
a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^3 + 45*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b
^4)*cosh(x))*sinh(x)^5 - 30*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^
4 + 10*(63*(a*b^3 + b^4)*cosh(x)^6 - 35*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x
)^4 - 24*a^3*b - 6*a^2*b^2 + 3*a*b^3 - 15*b^4 + 45*(8*a^3*b + 2*a^2*b^2 -
a*b^3 + 5*b^4)*cosh(x)^2)*sinh(x)^4 - 3*a*b^3 - 3*b^4 + 40*(9*(a*b^3 + b^4
)*cosh(x)^7 - 7*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^5 + 15*(8*a^3*b + 2*a^
2*b^2 - a*b^3 + 5*b^4)*cosh(x)^3 - 3*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)
*cosh(x))*sinh(x)^3 + 5*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^2 + 5*(27*(a*b
^3 + b^4)*cosh(x)^8 - 28*(4*a^2*b^2 - a*b^3 - 5*b^4)*cosh(x)^6 + 90*(8*a^3
*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^4 + 4*a^2*b^2 - a*b^3 - 5*b^4 - 36
*(8*a^3*b + 2*a^2*b^2 - a*b^3 + 5*b^4)*cosh(x)^2)*sinh(x)^2 - 240*(a^3*cos
h(x)^5 + 5*a^3*cosh(x)^4*sinh(x) + 10*a^3*cosh(x)^3*sinh(x)^2 + 10*a^3*cos
h(x)^2*sinh(x)^3 + 5*a^3*cosh(x)*sinh(x)^4 + a^3*sinh(x)^5)*sqrt(-a*b -...
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^7(x)}{a + b\cosh^2(x)}\, dx = \text{Timed out}$$

input
```
integrate(cosh(x)**7/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.20.7 Maxima [F]

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\cosh(x)^7}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(cosh(x)^7/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/480*(3*b^2*e^(10*x) - 3*b^2 - 5*(4*a*b - 5*b^2)*e^(8*x) + 30*(8*a^2 - 6*
a*b + 5*b^2)*e^(6*x) - 30*(8*a^2 - 6*a*b + 5*b^2)*e^(4*x) + 5*(4*a*b - 5*b
^2)*e^(2*x))*e^(-5*x)/b^3 - 1/128*integrate(256*(a^3*e^(3*x) + a^3*e^x)/(b
^4*e^(4*x) + b^4 + 2*(2*a*b^3 + b^4)*e^(2*x)), x)
```

3.20.8 Giac [F]

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\cosh(x)^7}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(cosh(x)^7/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.20.9 Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.76

$$\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{5x}}{160\,b} - \frac{e^{-5x}}{160\,b} - \frac{e^{-x}\,(8\,a^2-6\,a\,b+5\,b^2)}{16\,b^3} + \frac{\left(2\,\mathrm{atan}\left(\frac{\left(b^9\,\sqrt{b^8+a\,b^7}+a\,b^8\,\sqrt{b^8+a\,b^7}\right)\left(e^x\left(\frac{2\,a^7}{b^{11}\,(a+b)^2\,\sqrt{a^6}}-\frac{4\,\left(2\,a^4\,b^4\,\sqrt{a^6}+2\,a^5\,b^3\,\sqrt{a^6}\right)}{a^3\,b^8\,(a+b)\,\sqrt{b^7\,(a+b)}\,\sqrt{b^8+a\,b^7}}\right)-\frac{2\,a^7\,e^{3x}}{b^{11}\,(a+b)^2\,\sqrt{a^6}}\right)}{4\,a^4}\right) - 2\,\mathrm{atan}\left(\frac{a^3\,e \cdots}{2\,b \cdots}\right.\cdots}{2\,\sqrt{b^8+a\,b^7}} + \frac{e^{-3x}\,(4\,a-5\,b)}{96\,b^2} - \frac{e^{3x}\,(4\,a-5\,b)}{96\,b^2} + \frac{e^x\,(8\,a^2-6\,a\,b+5\,b^2)}{16\,b^3}$$

input
```
int(cosh(x)^7/(a + b*cosh(x)^2),x)
```

output
```
exp(5*x)/(160*b) - exp(-5*x)/(160*b) - (exp(-x)*(8*a^2 - 6*a*b + 5*b^2))/(
16*b^3) + ((2*atan(((b^9*(a*b^7 + b^8)^(1/2) + a*b^8*(a*b^7 + b^8)^(1/2))*
(exp(x)*((2*a^7)/(b^11*(a + b)^2*(a^6)^(1/2)) - (4*(2*a^4*b^4*(a^6)^(1/2)
+ 2*a^5*b^3*(a^6)^(1/2)))/(a^3*b^8*(a + b)*(b^7*(a + b))^(1/2)*(a*b^7 + b^
8)^(1/2))) - (2*a^7*exp(3*x))/(b^11*(a + b)^2*(a^6)^(1/2))))/(4*a^4)) - 2*
atan((a^3*exp(x)*(b^7*(a + b))^(1/2))/(2*b^3*(a + b)*(a^6)^(1/2))))*(a^6)^
(1/2))/(2*(a*b^7 + b^8)^(1/2)) + (exp(-3*x)*(4*a - 5*b))/(96*b^2) - (exp(3
*x)*(4*a - 5*b))/(96*b^2) + (exp(x)*(8*a^2 - 6*a*b + 5*b^2))/(16*b^3)
```

3.20. $\int \frac{\cosh^7(x)}{a+b\cosh^2(x)}\,dx$

3.21 $\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx$

3.21.1 Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{(8a^2-4ab+3b^2)\,x}{8b^3} - \frac{a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^3\sqrt{a+b}} - \frac{(4a-3b)\cosh(x)\sinh(x)}{8b^2} + \frac{\cosh^3(x)\sinh(x)}{4b}$$

output
```
1/8*(8*a^2-4*a*b+3*b^2)*x/b^3-1/8*(4*a-3*b)*cosh(x)*sinh(x)/b^2+1/4*cosh(x
)^3*sinh(x)/b-a^(5/2)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/b^3/(a+b)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{4(8a^2-4ab+3b^2)\,x - \frac{32a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - 8(a-b)b\sinh(2x) + b^2\sinh(4x)}{32b^3}$$

input
```
Integrate[Cosh[x]^6/(a + b*Cosh[x]^2),x]
```

output
```
(4*(8*a^2 - 4*a*b + 3*b^2)*x - (32*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[
a + b]])/Sqrt[a + b] - 8*(a - b)*b*Sinh[2*x] + b^2*Sinh[4*x])/(32*b^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3666, 372, 440, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^6}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 3666}$$

$$-\int \frac{\coth^6(x)}{\left(1-\coth^2(x)\right)^3\left(a-(a+b)\coth^2(x)\right)}d\coth(x)$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\coth^3(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\int \frac{\coth^2(x)\left((a-3b)\coth^2(x)+3a\right)}{\left(1-\coth^2(x)\right)^2\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{4b}$$

$$\Big\downarrow \text{ 440}$$

$$\frac{\coth^3(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{-\frac{\int -\frac{\left(4a^2-ba+3b^2\right)\coth^2(x)+a(4a-3b)}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{(4a-3b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\coth^3(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\int \frac{\left(4a^2-ba+3b^2\right)\coth^2(x)+a(4a-3b)}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{(4a-3b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\coth^3(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\frac{8a^3\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{\left(8a^2-4ab+3b^2\right)\int \frac{1}{1-\coth^2(x)}d\coth(x)}{b}}{2b}-\frac{(4a-3b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\coth^3(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\frac{8a^3\int\frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{(8a^2-4ab+3b^2)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{(4a-3b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\coth^3(x)}{4b\left(1-\coth^2(x)\right)^2}-\frac{\frac{\frac{8a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{(8a^2-4ab+3b^2)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{(4a-3b)\coth(x)}{2b(1-\coth^2(x))}}{4b}$$

input
```
Int[Cosh[x]^6/(a + b*Cosh[x]^2),x]
```

output
```
Coth[x]^3/(4*b*(1 - Coth[x]^2)^2) - ((-(((8*a^2 - 4*a*b + 3*b^2)*ArcTanh[C
oth[x]])/b) + (8*a^(5/2)*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(b*Sqrt[a
 + b]))/(2*b) - ((4*a - 3*b)*Coth[x])/(2*b*(1 - Coth[x]^2)))/(4*b)
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(74) = 148$.

Time = 0.84 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.93

method	result
risch	$\frac{x\,a^{2}}{b^{3}}-\frac{a x}{2 b^{2}}+\frac{3 x}{8 b}+\frac{{\mathrm e}^{4 x}}{64 b}-\frac{a\,{\mathrm e}^{2 x}}{8 b^{2}}+\frac{{\mathrm e}^{2 x}}{8 b}+\frac{a\,{\mathrm e}^{-2 x}}{8 b^{2}}-\frac{{\mathrm e}^{-2 x}}{8 b}-\frac{{\mathrm e}^{-4 x}}{64 b}+\frac{\sqrt{a(a+b)}\,a^{2}\ln\left({\mathrm e}^{2 x}+\frac{2\sqrt{a(a+b)}+2 a+b}{b}\right)}{2(a+b) b^{3}}-\frac{\sqrt{a(a+}$
default	$\frac{2 a^{3}\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(-\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}-\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}\right)}{b^{3}}-\frac{1}{4 b\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{1}{2 b(\tanh(\frac{}{}}$

input
```
int(cosh(x)^6/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^3*a^2-1/2*a*x/b^2+3/8*x/b+1/64/b*exp(4*x)-1/8/b^2*a*exp(2*x)+1/8/b*exp
(2*x)+1/8/b^2*a*exp(-2*x)-1/8/b*exp(-2*x)-1/64/b*exp(-4*x)+1/2*(a*(a+b))^(
1/2)/(a+b)*a^2/b^3*ln(exp(2*x)+(2*(a*(a+b))^(1/2)+2*a+b)/b)-1/2*(a*(a+b))^
(1/2)/(a+b)*a^2/b^3*ln(exp(2*x)-(2*(a*(a+b))^(1/2)-2*a-b)/b)
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 1245, normalized size of antiderivative = 14.15

$$\int \frac{\cosh^6(x)}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^6/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 - 8*(a*b -
b^2)*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 - 2*a*b + 2*b^2)*sinh(x)^6 + 8*(8*a^2
- 4*a*b + 3*b^2)*x*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 - 6*(a*b - b^2)*cosh(x))
*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 - 60*(a*b - b^2)*cosh(x)^2 + 4*(8*a^2 - 4
*a*b + 3*b^2)*x)*sinh(x)^4 + 8*(7*b^2*cosh(x)^5 - 20*(a*b - b^2)*cosh(x)^3
 + 4*(8*a^2 - 4*a*b + 3*b^2)*x*cosh(x))*sinh(x)^3 + 8*(a*b - b^2)*cosh(x)^
2 + 4*(7*b^2*cosh(x)^6 - 30*(a*b - b^2)*cosh(x)^4 + 12*(8*a^2 - 4*a*b + 3*
b^2)*x*cosh(x)^2 + 2*a*b - 2*b^2)*sinh(x)^2 + 32*(a^2*cosh(x)^4 + 4*a^2*co
sh(x)^3*sinh(x) + 6*a^2*cosh(x)^2*sinh(x)^2 + 4*a^2*cosh(x)*sinh(x)^3 + a^
2*sinh(x)^4)*sqrt(a/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3
+ b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b +
 b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*c
osh(x))*sinh(x) + 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x)
 + (a*b + b^2)*sinh(x)^2 + 2*a^2 + 3*a*b + b^2)*sqrt(a/(a + b)))/(b*cosh(x
)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b
*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh
(x) + b)) - b^2 + 8*(b^2*cosh(x)^7 - 6*(a*b - b^2)*cosh(x)^5 + 4*(8*a^2 -
4*a*b + 3*b^2)*x*cosh(x)^3 + 2*(a*b - b^2)*cosh(x))*sinh(x))/(b^3*cosh(x)^
4 + 4*b^3*cosh(x)^3*sinh(x) + 6*b^3*cosh(x)^2*sinh(x)^2 + 4*b^3*cosh(x)*si
nh(x)^3 + b^3*sinh(x)^4), 1/64*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7...
```

3.21.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(x)}{a + b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**6/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(74) = 148$.

Time = 0.32 (sec) , antiderivative size = 651, normalized size of antiderivative = 7.40

$$\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx = -\frac{15\,(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}b} - \frac{5\,\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{32\,\sqrt{(a+b)a}} - \frac{3\,(2\,a+b)x}{2\,b^2} + \frac{15\,x}{16\,b} - \frac{\left(4\,(2\,a+b)e^{(-2\,x)}-b\right)e^{(4\,x)}}{64\,b^2} + \frac{3\,e^{(2\,x)}}{16\,b} - \frac{3\,e^{(-2\,x)}}{16\,b} + \frac{\left(4\,(2\,a+b)e^{(2\,x)}-b\right)e^{(-4\,x)}}{64\,b^2} + \frac{3\,(2\,a+b)\log\left(be^{(4\,x)}+2\,(2\,a+b)e^{(2\,x)}+b\right)}{16\,b^2} - \frac{3\,(2\,a+b)\log\left(2\,(2\,a+b)e^{(-2\,x)}+be^{(-4\,x)}+b\right)}{16\,b^2} + \frac{3\,(8\,a^2+8\,ab+b^2)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}b^2} - \frac{3\,(8\,a^2+8\,ab+b^2)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{64\,\sqrt{(a+b)a}b^2} + \frac{(16\,a^2+16\,ab+3\,b^2)x}{8\,b^3} - \frac{(16\,a^2+16\,ab+3\,b^2)\log\left(be^{(4\,x)}+2\,(2\,a+b)e^{(2\,x)}+b\right)}{64\,b^3} + \frac{(16\,a^2+16\,ab+3\,b^2)\log\left(2\,(2\,a+b)e^{(-2\,x)}+be^{(-4\,x)}+b\right)}{64\,b^3} - \frac{(32\,a^3+48\,a^2b+18\,ab^2+b^3)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{128\,\sqrt{(a+b)a}b^3} + \frac{(32\,a^3+48\,a^2b+18\,ab^2+b^3)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{128\,\sqrt{(a+b)a}b^3}$$

input
```
integrate(cosh(x)^6/(a+b*cosh(x)^2),x, algorithm="maxima")
```

3.21. $\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx$

output
```
-15/64*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x)
+ 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 5/32*log((b*e^(-2*x)
 + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a))
)/sqrt((a + b)*a) - 3/2*(2*a + b)*x/b^2 + 15/16*x/b - 1/64*(4*(2*a + b)*e^
(-2*x) - b)*e^(4*x)/b^2 + 3/16*e^(2*x)/b - 3/16*e^(-2*x)/b + 1/64*(4*(2*a
+ b)*e^(2*x) - b)*e^(-4*x)/b^2 + 3/16*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b
)*e^(2*x) + b)/b^2 - 3/16*(2*a + b)*log(2*(2*a + b)*e^(-2*x) + b*e^(-4*x)
+ b)/b^2 + 3/64*(8*a^2 + 8*a*b + b^2)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a
 + b)*a))/(b*e^(2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2)
 - 3/64*(8*a^2 + 8*a*b + b^2)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a
))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) + 1/8
*(16*a^2 + 16*a*b + 3*b^2)*x/b^3 - 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(b*e^
(4*x) + 2*(2*a + b)*e^(2*x) + b)/b^3 + 1/64*(16*a^2 + 16*a*b + 3*b^2)*log(
2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^3 - 1/128*(32*a^3 + 48*a^2*b + 18
*a*b^2 + b^3)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2
*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^3) + 1/128*(32*a^3 + 48*a^
2*b + 18*a*b^2 + b^3)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^
(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^3)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^6(x)}{a + b\cosh^2(x)}\, dx$$
$$= -\frac{a^3 \arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^3} + \frac{be^{(4x)} - 8\,ae^{(2x)} + 8\,be^{(2x)}}{64\,b^2} + \frac{(8\,a^2 - 4\,ab + 3\,b^2)x}{8\,b^3} - \frac{\left(48\,a^2e^{(4x)} - 24\,abe^{(4x)} + 18\,b^2e^{(4x)} - 8\,abe^{(2x)} + 8\,b^2e^{(2x)} + b^2\right)e^{(-4x)}}{64\,b^3}$$

input
```
integrate(cosh(x)^6/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
-a^3*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*
b^3) + 1/64*(b*e^(4*x) - 8*a*e^(2*x) + 8*b*e^(2*x))/b^2 + 1/8*(8*a^2 - 4*a
*b + 3*b^2)*x/b^3 - 1/64*(48*a^2*e^(4*x) - 24*a*b*e^(4*x) + 18*b^2*e^(4*x)
 - 8*a*b*e^(2*x) + 8*b^2*e^(2*x) + b^2)*e^(-4*x)/b^3
```

3.21.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

$$\int \frac{\cosh^6(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{4x}}{64b} - \frac{e^{-4x}}{64b} + \frac{x\,(8a^2-4ab+3b^2)}{8b^3} + \frac{e^{-2x}\,(a-b)}{8b^2} - \frac{e^{2x}\,(a-b)}{8b^2} + \frac{a^{5/2}\,\ln\left(\frac{4a^3e^{2x}}{b^4} - \frac{2a^{5/2}\,(b+2a\,e^{2x}+b\,e^{2x})}{b^4\sqrt{a+b}}\right)}{2b^3\sqrt{a+b}} - \frac{a^{5/2}\,\ln\left(\frac{4a^3e^{2x}}{b^4} + \frac{2a^{5/2}\,(b+2a\,e^{2x}+b\,e^{2x})}{b^4\sqrt{a+b}}\right)}{2b^3\sqrt{a+b}}$$

input
```
int(cosh(x)^6/(a + b*cosh(x)^2),x)
```

output
```
exp(4*x)/(64*b) - exp(-4*x)/(64*b) + (x*(8*a^2 - 4*a*b + 3*b^2))/(8*b^3) +
 (exp(-2*x)*(a - b))/(8*b^2) - (exp(2*x)*(a - b))/(8*b^2) + (a^(5/2)*log((
4*a^3*exp(2*x))/b^4 - (2*a^(5/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(b^4*(a
+ b)^(1/2))))/(2*b^3*(a + b)^(1/2)) - (a^(5/2)*log((4*a^3*exp(2*x))/b^4 +
(2*a^(5/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(b^4*(a + b)^(1/2))))/(2*b^3*(
a + b)^(1/2))
```

3.22 $\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx$

3.22.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx=\frac{a^2\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sinh(x)}{b^2}+\frac{\sinh^3(x)}{3b}$$

output
```
-(a-b)*sinh(x)/b^2+1/3*sinh(x)^3/b+a^2*arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))
/b^(5/2)/(a+b)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx=-\frac{a^2\arctan\left(\frac{\sqrt{a+b}\operatorname{csch}(x)}{\sqrt{b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(4a-3b)\sinh(x)}{4b^2}+\frac{\sinh(3x)}{12b}$$

input
```
Integrate[Cosh[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
-((a^2*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/(b^(5/2)*Sqrt[a + b])) - ((4
*a - 3*b)*Sinh[x])/(4*b^2) + Sinh[3*x]/(12*b)
```

3.22.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3665, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^5}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{\left(\sinh^2(x)+1\right)^2}{a+b\sinh^2(x)+b}d\sinh(x)$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{a^2}{b^2\left(a+b\sinh^2(x)+b\right)}-\frac{a-b}{b^2}+\frac{\sinh^2(x)}{b}\right)d\sinh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sinh(x)}{b^2}+\frac{\sinh^3(x)}{3b}$$

input

```
Int[Cosh[x]^5/(a + b*Cosh[x]^2),x]
```

output

```
(a^2*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) - ((a -
b)*Sinh[x])/b^2 + Sinh[x]^3/(3*b)
```

3.22.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(46) = 92$.

Time = 0.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.61

method	result
risch	$\frac{e^{3x}}{24b} - \frac{a\,e^{x}}{2b^{2}} + \frac{3\,e^{x}}{8b} + \frac{a\,e^{-x}}{2b^{2}} - \frac{3\,e^{-x}}{8b} - \frac{e^{-3x}}{24b} - \frac{a^{2}\ln\left(e^{2x} - \frac{2(a+b)e^{x}}{\sqrt{-ab-b^{2}}} - 1\right)}{2\sqrt{-ab-b^{2}}\,b^{2}} + \frac{a^{2}\ln\left(e^{2x} + \frac{2(a+b)e^{x}}{\sqrt{-ab-b^{2}}} - 1\right)}{2\sqrt{-ab-b^{2}}\,b^{2}}$
default	$\frac{2a^{2}\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}+2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}} + \frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}-2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}\right)}{b^{2}} - \frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}} + \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}} - \frac{-a+b}{b^{2}\left(\tanh\left(\frac{x}{2}\right)-\right.}$

input
```
int(cosh(x)^5/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/24/b*exp(3*x)-1/2*a/b^2*exp(x)+3/8/b*exp(x)+1/2*a/b^2*exp(-x)-3/8/b*exp(
-x)-1/24/b*exp(-3*x)-1/2/(-a*b-b^2)^(1/2)*a^2/b^2*ln(exp(2*x)-2*(a+b)/(-a*
b-b^2)^(1/2)*exp(x)-1)+1/2/(-a*b-b^2)^(1/2)*a^2/b^2*ln(exp(2*x)+2*(a+b)/(-
a*b-b^2)^(1/2)*exp(x)-1)
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 1184, normalized size of antiderivative = 21.14

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/24*((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^
2 + b^3)*sinh(x)^6 - 3*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^4 - 3*(4*a^2*b +
a*b^2 - 3*b^3 - 5*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*
cosh(x)^3 - 3*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x))*sinh(x)^3 - a*b^2 - b^3 +
 3*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^2 + 3*(5*(a*b^2 + b^3)*cosh(x)^4 + 4*
a^2*b + a*b^2 - 3*b^3 - 6*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^2)*sinh(x)^2 -
 12*(a^2*cosh(x)^3 + 3*a^2*cosh(x)^2*sinh(x) + 3*a^2*cosh(x)*sinh(x)^2 + a
^2*sinh(x)^3)*sqrt(-a*b - b^2)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
b*sinh(x)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh
(x)^2 + 4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*(cosh(x)^3 + 3*c
osh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(-
a*b - b^2) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*
a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3
+ (2*a + b)*cosh(x))*sinh(x) + b)) + 6*((a*b^2 + b^3)*cosh(x)^5 - 2*(4*a^2
*b + a*b^2 - 3*b^3)*cosh(x)^3 + (4*a^2*b + a*b^2 - 3*b^3)*cosh(x))*sinh(x)
)/((a*b^3 + b^4)*cosh(x)^3 + 3*(a*b^3 + b^4)*cosh(x)^2*sinh(x) + 3*(a*b^3
+ b^4)*cosh(x)*sinh(x)^2 + (a*b^3 + b^4)*sinh(x)^3), 1/24*((a*b^2 + b^3)*c
osh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 - 3
*(4*a^2*b + a*b^2 - 3*b^3)*cosh(x)^4 - 3*(4*a^2*b + a*b^2 - 3*b^3 - 5*(a*b
^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 - 3*(4*a^...
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**5/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.22.7 Maxima [F]

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\cosh\left(x\right)^{5}}{b\cosh\left(x\right)^{2}+a}\,dx$$

input
```
integrate(cosh(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/24*(b*e^(6*x) - 3*(4*a - 3*b)*e^(4*x) + 3*(4*a - 3*b)*e^(2*x) - b)*e^(-3
*x)/b^2 + 1/32*integrate(64*(a^2*e^(3*x) + a^2*e^x)/(b^3*e^(4*x) + b^3 + 2
*(2*a*b^2 + b^3)*e^(2*x)), x)
```

3.22.8 Giac [F]

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\cosh\left(x\right)^{5}}{b\cosh\left(x\right)^{2}+a}\,dx$$

input
```
integrate(cosh(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.22.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.34

$$\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx=\frac{\mathrm{e}^{3\,x}}{24\,b}-\frac{\mathrm{e}^{-3\,x}}{24\,b}+\frac{\mathrm{e}^{-x}\,\left(4\,a-3\,b\right)}{8\,b^2}+\frac{\sqrt{a^4}\,\Bigg(2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^x\,\sqrt{b^5\,\left(a+b\right)}}{2\,b^2\,\left(a+b\right)\,\sqrt{a^4}}\right)-2\,\mathrm{atan}\Bigg(\Bigg(\left(\frac{b^7\,\sqrt{b^6+a\,b^5}}{4}+\frac{a\,b^6\,\sqrt{b^6+a\,b^5}}{4}\right)\,\Bigg(\mathrm{e}^x\,\Bigg(\frac{2\,a^2}{b^8\,{\left(a+b\right)}^2\,\sqrt{a^4}}-\frac{4\,\big(2\,a^3\,b^3\,\sqrt{a^4}+2\,a^4\,b^{\cdots}}{a^5\,b^6\,\left(a+b\right)\,\sqrt{b^5\,\left(a+b\right)}\,\cdots}}{2\,\sqrt{b^6+a\,b^5}}-\frac{\mathrm{e}^x\,\left(4\,a-3\,b\right)}{8\,b^2}$$

input
```
int(cosh(x)^5/(a + b*cosh(x)^2),x)
```

output
```
exp(3*x)/(24*b) - exp(-3*x)/(24*b) + (exp(-x)*(4*a - 3*b))/(8*b^2) + ((a^4
)^(1/2)*(2*atan((a^2*exp(x)*(b^5*(a + b))^(1/2))/(2*b^2*(a + b)*(a^4)^(1/2
))) - 2*atan(((b^7*(a*b^5 + b^6)^(1/2))/4 + (a*b^6*(a*b^5 + b^6)^(1/2))/4)
*(exp(x)*((2*a^2)/(b^8*(a + b)^2*(a^4)^(1/2)) - (4*(2*a^3*b^3*(a^4)^(1/2)
+ 2*a^4*b^2*(a^4)^(1/2)))/(a^5*b^6*(a + b)*(b^5*(a + b))^(1/2)*(a*b^5 + b^
6)^(1/2))) - (2*a^2*exp(3*x))/(b^8*(a + b)^2*(a^4)^(1/2))))))/(2*(a*b^5 +
b^6)^(1/2)) - (exp(x)*(4*a - 3*b))/(8*b^2)
```

3.22. $\int \frac{\cosh^5(x)}{a+b\cosh^2(x)}\,dx$

3.23 $\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx$

3.23.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx=-\frac{(2a-b)x}{2b^2}+\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}}+\frac{\cosh(x)\sinh(x)}{2b}$$

output
```
-1/2*(2*a-b)*x/b^2+1/2*cosh(x)*sinh(x)/b+a^(3/2)*arctanh(a^(1/2)*tanh(x)/(
a+b)^(1/2))/b^2/(a+b)^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx=\frac{2(-2a+b)x+\frac{4a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+b\sinh(2x)}{4b^2}$$

input
```
Integrate[Cosh[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(2*(-2*a + b)*x + (4*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/Sqrt[
a + b] + b*Sinh[2*x])/(4*b^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3666, 372, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 3666}$$

$$\int \frac{\coth^4(x)}{\left(1-\coth^2(x)\right)^2\left(a-(a+b)\coth^2(x)\right)}d\coth(x)$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\int \frac{(a-b)\coth^2(x)+a}{\left(1-\coth^2(x)\right)\left(a-(a+b)\coth^2(x)\right)}d\coth(x)}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{2a^2\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{(2a-b)\int \frac{1}{1-\coth^2(x)}d\coth(x)}{b}}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2a^2\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}-\frac{(2a-b)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}-\frac{(2a-b)\text{arctanh}(\coth(x))}{b}}{2b}-\frac{\coth(x)}{2b\left(1-\coth^2(x)\right)}$$

input
```
Int[Cosh[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(-(((2*a - b)*ArcTanh[Coth[x]])/b) + (2*a^(3/2)*ArcTanh[(Sqrt[a + b]*Coth[
x])/Sqrt[a]])/(b*Sqrt[a + b]))/(2*b) - Coth[x]/(2*b*(1 - Coth[x]^2))
```

3.23.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(47) = 94$.

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{ax}{b^2}+\frac{x}{2b}+\frac{e^{2x}}{8b}-\frac{e^{-2x}}{8b}+\frac{\sqrt{a(a+b)}\,a\ln\left(e^{2x}-\frac{2\sqrt{a(a+b)}-2a-b}{b}\right)}{2(a+b)b^2}-\frac{\sqrt{a(a+b)}\,a\ln\left(e^{2x}+\frac{2\sqrt{a(a+b)}+2a+b}{b}\right)}{2(a+b)b^2}$
default	$-\frac{2a^2\left(-\frac{\ln\left(\sqrt{a+b}\tanh\left(\frac{x}{2}\right)^2+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\ln\left(-\sqrt{a+b}\tanh\left(\frac{x}{2}\right)^2+2\tanh\left(\frac{x}{2}\right)\sqrt{a}-\sqrt{a+b}\right)}{4\sqrt{a}\sqrt{a+b}}\right)}{b^2}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{}{2b(\tanh}$

input
```
int(cosh(x)^4/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-a*x/b^2+1/2*x/b+1/8/b*exp(2*x)-1/8/b*exp(-2*x)+1/2*(a*(a+b))^(1/2)/(a+b)*
a/b^2*ln(exp(2*x)-(2*(a*(a+b))^(1/2)-2*a-b)/b)-1/2*(a*(a+b))^(1/2)/(a+b)*a
/b^2*ln(exp(2*x)+(2*(a*(a+b))^(1/2)+2*a+b)/b)
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 573, normalized size of antiderivative = 9.71

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx$$
$$=\left[\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-4\,(2\,a-b)x\cosh(x)^2+2\,\left(3\,b\cosh(x)^2-2\,(2\,a-b)x\right)}{}\right.$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/8*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a - b)*x*co
sh(x)^2 + 2*(3*b*cosh(x)^2 - 2*(2*a - b)*x)*sinh(x)^2 + 4*(a*cosh(x)^2 + 2
*a*cosh(x)*sinh(x) + a*sinh(x)^2)*sqrt(a/(a + b))*log((b^2*cosh(x)^4 + 4*b
^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x
)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b +
 b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 + 2*a^2 + 3*a*b + b^2)*sqrt(
a/(a + b)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a +
b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2
*a + b)*cosh(x))*sinh(x) + b)) + 4*(b*cosh(x)^3 - 2*(2*a - b)*x*cosh(x))*s
inh(x) - b)/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2), 1/8*(
b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 4*(2*a - b)*x*cosh(x)^
2 + 2*(3*b*cosh(x)^2 - 2*(2*a - b)*x)*sinh(x)^2 + 8*(a*cosh(x)^2 + 2*a*cos
h(x)*sinh(x) + a*sinh(x)^2)*sqrt(-a/(a + b))*arctan(1/2*(b*cosh(x)^2 + 2*b
*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a/(a + b))/a) + 4*(b*cosh(
x)^3 - 2*(2*a - b)*x*cosh(x))*sinh(x) - b)/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*
sinh(x) + b^2*sinh(x)^2)]
```

3.23.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**4/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.88

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx = -\frac{(2a+b)\log\left(\frac{be^{(2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{4\sqrt{(a+b)a}b} - \frac{3\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{16\sqrt{(a+b)a}} - \frac{(2a+b)x}{b^2} + \frac{x}{b} + \frac{e^{(2x)}}{8b} - \frac{e^{(-2x)}}{8b} + \frac{(2a+b)\log\left(be^{(4x)}+2(2a+b)e^{(2x)}+b\right)}{8b^2} - \frac{(2a+b)\log\left(2(2a+b)e^{(-2x)}+be^{(-4x)}+b\right)}{8b^2} + \frac{(8a^2+8ab+b^2)\log\left(\frac{be^{(2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{32\sqrt{(a+b)a}b^2} - \frac{(8a^2+8ab+b^2)\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{32\sqrt{(a+b)a}b^2}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) +
2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 3/16*log((b*e^(-2*x) +
 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/
sqrt((a + b)*a) - (2*a + b)*x/b^2 + x/b + 1/8*e^(2*x)/b - 1/8*e^(-2*x)/b +
 1/8*(2*a + b)*log(b*e^(4*x) + 2*(2*a + b)*e^(2*x) + b)/b^2 - 1/8*(2*a + b
)*log(2*(2*a + b)*e^(-2*x) + b*e^(-4*x) + b)/b^2 + 1/32*(8*a^2 + 8*a*b + b
^2)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) + 2*a + b + 2
*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b^2) - 1/32*(8*a^2 + 8*a*b + b^2)*log(
(b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*b^2)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx = \frac{a^2\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^2} - \frac{(2a-b)x}{2b^2} + \frac{e^{(2x)}}{8b} + \frac{\left(4ae^{(2x)}-2be^{(2x)}-b\right)e^{(-2x)}}{8b^2}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
a^2*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b
^2) - 1/2*(2*a - b)*x/b^2 + 1/8*e^(2*x)/b + 1/8*(4*a*e^(2*x) - 2*b*e^(2*x)
 - b)*e^(-2*x)/b^2
```

3.23.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.41

$$\int \frac{\cosh^4(x)}{a+b\cosh^2(x)}\,dx = \frac{e^{2x}}{8b} - \frac{e^{-2x}}{8b} - \frac{x(2a-b)}{2b^2} + \frac{a^{3/2}\ln\left(-\frac{4a^2e^{2x}}{b^3} - \frac{2a^{3/2}(b+2ae^{2x}+be^{2x})}{b^3\sqrt{a+b}}\right)}{2b^2\sqrt{a+b}} - \frac{a^{3/2}\ln\left(\frac{2a^{3/2}(b+2ae^{2x}+be^{2x})}{b^3\sqrt{a+b}} - \frac{4a^2e^{2x}}{b^3}\right)}{2b^2\sqrt{a+b}}$$

input
```
int(cosh(x)^4/(a + b*cosh(x)^2),x)
```

output
```
exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (x*(2*a - b))/(2*b^2) + (a^(3/2)*log(-
(4*a^2*exp(2*x))/b^3 - (2*a^(3/2)*(b + 2*a*exp(2*x) + b*exp(2*x)))/(b^3*(a
 + b)^(1/2))))/(2*b^2*(a + b)^(1/2)) - (a^(3/2)*log((2*a^(3/2)*(b + 2*a*ex
p(2*x) + b*exp(2*x)))/(b^3*(a + b)^(1/2)) - (4*a^2*exp(2*x))/b^3))/(2*b^2*
(a + b)^(1/2))
```

3.24 $\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx$

3.24.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx=-\frac{a\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sinh(x)}{b}$$

output
```
sinh(x)/b-a*arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx=-\frac{a\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sinh(x)}{b}$$

input
```
Integrate[Cosh[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
-((a*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sinh[
x]/b
```

3.24.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3665, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\int \frac{\sinh^2(x)+1}{a+b\sinh^2(x)+b}d\sinh(x)$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sinh(x)}{b}-\frac{a\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sinh(x)}{b}-\frac{a\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

input
```
Int[Cosh[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
-((a*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sinh[
x]/b
```

3.24.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(30) = 60$.

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

method	result	size
default	$-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2a\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}+2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}+\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}-2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}\right)}{b}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	101
risch	$\frac{e^{x}}{2b}-\frac{e^{-x}}{2b}-\frac{a\ln\left(e^{2x}+\frac{2(a+b)e^{x}}{\sqrt{-ab-b^{2}}}-1\right)}{2\sqrt{-ab-b^{2}}\,b}+\frac{a\ln\left(e^{2x}-\frac{2(a+b)e^{x}}{\sqrt{-ab-b^{2}}}-1\right)}{2\sqrt{-ab-b^{2}}\,b}$	106

input
```
int(cosh(x)^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(tanh(1/2*x)+1)-2*a/b*(1/2/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*tanh(1/2
*x)*(a+b)^(1/2)+2*a^(1/2))/b^(1/2))+1/2/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*
tanh(1/2*x)*(a+b)^(1/2)-2*a^(1/2))/b^(1/2)))-1/b/(tanh(1/2*x)-1)
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 498, normalized size of antiderivative = 13.11

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx$$
$$= \left[\frac{(ab+b^2)\cosh(x)^2 + 2(ab+b^2)\cosh(x)\sinh(x) + (ab+b^2)\sinh(x)^2 - \sqrt{-ab-b^2}(a\cosh(x) + a\sinh}{}\right.$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*
sinh(x)^2 - sqrt(-a*b - b^2)*(a*cosh(x) + a*sinh(x))*log((b*cosh(x)^4 + 4*
b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(
x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x
) + 4*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sin
h(x) - cosh(x))*sqrt(-a*b - b^2) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(
x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) - a*b - b^2)/((a*
b^2 + b^3)*cosh(x) + (a*b^2 + b^3)*sinh(x)), 1/2*((a*b + b^2)*cosh(x)^2 +
2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 - 2*sqrt(a*b + b^2)*
(a*cosh(x) + a*sinh(x))*arctan(1/2*(b*cosh(x)^3 + 3*b*cosh(x)*sinh(x)^2 +
b*sinh(x)^3 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^2 + 4*a + 3*b)*sinh(x))/s
qrt(a*b + b^2)) - 2*sqrt(a*b + b^2)*(a*cosh(x) + a*sinh(x))*arctan(1/2*sqr
t(a*b + b^2)*(cosh(x) + sinh(x))/(a + b)) - a*b - b^2)/((a*b^2 + b^3)*cosh
(x) + (a*b^2 + b^3)*sinh(x))]
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.24.7 Maxima [F]

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\cosh{(x)}^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*x) - 1)*e^(-x)/b - 1/8*integrate(16*(a*e^(3*x) + a*e^x)/(b^2*e^(
4*x) + b^2 + 2*(2*a*b + b^2)*e^(2*x)), x)
```

3.24.8 Giac [F]

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\cosh{(x)}^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.24.9 Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.37

$$\int \frac{\cosh^3(x)}{a+b\cosh^2(x)}\,dx = \frac{e^x}{2\,b} - \frac{e^{-x}}{2\,b} - \frac{\left(2\,\mathrm{atan}\left(\frac{a^3\,e^x\,\sqrt{b^3\,(a+b)}}{2\,b\,(a+b)\,{(a^2)}^{3/2}}\right) - 2\,\mathrm{atan}\left(\left(\frac{b^5\,\sqrt{b^4+a\,b^3}}{4} + \frac{a\,b^4\,\sqrt{b^4+a\,b^3}}{4}\right)\left(e^x\left(\frac{2\,a^3}{b^5\,{(a+b)}^2\,{(a^2)}^{3/2}} - \frac{4\left(2\,b^2\,{(a^2)}^{3/2}+2\,a\,b\,(a^2)\right.}{a^3\,b^4\,(a+b)\,\sqrt{b^3\,(a+b)}\,\sqrt{b^4}}\right.\right.\right.}{2\,\sqrt{b^4+a\,b^3}}$$

input
```
int(cosh(x)^3/(a + b*cosh(x)^2),x)
```

output
```
exp(x)/(2*b) - exp(-x)/(2*b) - ((2*atan((a^3*exp(x)*(b^3*(a + b))^(1/2))/(
2*b*(a + b)*(a^2)^(3/2))) - 2*atan(((b^5*(a*b^3 + b^4)^(1/2))/4 + (a*b^4*(
a*b^3 + b^4)^(1/2))/4)*(exp(x)*((2*a^3)/(b^5*(a + b)^2*(a^2)^(3/2)) - (4*(
2*b^2*(a^2)^(3/2) + 2*a*b*(a^2)^(3/2)))/(a^3*b^4*(a + b)*(b^3*(a + b))^(1/
2)*(a*b^3 + b^4)^(1/2))) - (2*a^3*exp(3*x))/(b^5*(a + b)^2*(a^2)^(3/2)))))
*(a^2)^(1/2))/(2*(a*b^3 + b^4)^(1/2))
```

3.25 $\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx$

3.25.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx=\frac{x}{b}-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}$$

output
```
x/b-arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))*a^(1/2)/b/(a+b)^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx=\frac{x-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{b}$$

input
```
Integrate[Cosh[x]^2/(a + b*Cosh[x]^2),x]
```

output
```
(x - (Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/Sqrt[a + b])/b
```

3.25.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3650, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \; 3650$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\cosh^2(x)+a}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{x}{b}-\frac{a\int \frac{1}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx}{b}$$

$$\Big\downarrow \; 3660$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{b}$$

$$\Big\downarrow \; 221$$

$$\frac{x}{b}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}$$

input
```
Int[Cosh[x]^2/(a + b*Cosh[x]^2),x]
```

output
```
x/b - (Sqrt[a]*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(b*Sqrt[a + b])
```

3.25.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(31) = 62$.

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.36

method	result
risch	$\frac{x}{b}+\frac{\sqrt{a(a+b)}\,\ln\left(\mathrm{e}^{2x}+\frac{2\sqrt{a(a+b)}+2a+b}{b}\right)}{2(a+b)b}-\frac{\sqrt{a(a+b)}\,\ln\left(\mathrm{e}^{2x}-\frac{2\sqrt{a(a+b)}-2a-b}{b}\right)}{2(a+b)b}$
default	$\frac{2a\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}-2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-\right.}{b}$

input
```
int(cosh(x)^2/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x/b+1/2*(a*(a+b))^(1/2)/(a+b)/b*ln(exp(2*x)+(2*(a*(a+b))^(1/2)+2*a+b)/b)-1
/2*(a*(a+b))^(1/2)/(a+b)/b*ln(exp(2*x)-(2*(a*(a+b))^(1/2)-2*a-b)/b)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 8.13

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx$$
$$= \left[\frac{\sqrt{\frac{a}{a+b}}\log\left(\frac{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\,(2\,ab+b^2)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+2\,ab+b^2\right)\sinh(x)^2+8\,a^2+8\,ab+b^2+4\,(\cdots}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\,(\cdots}\right.\cdots}{2\cdots}\right.$$
$$\left.-\frac{\sqrt{-\frac{a}{a+b}}\arctan\left(\frac{\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2+2\,a+b\right)\sqrt{-\frac{a}{a+b}}}{2\,a}\right)-x}{b}\right]$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a/(a + b))*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*s
inh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*s
inh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))
*sinh(x) + 4*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b
 + b^2)*sinh(x)^2 + 2*a^2 + 3*a*b + b^2)*sqrt(a/(a + b)))/(b*cosh(x)^4 + 4
*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x
)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b
)) + 2*x)/b, -(sqrt(-a/(a + b))*arctan(1/2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh
(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a/(a + b))/a) - x)/b]
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**2/(a+b*cosh(x)**2),x)
```

output
```
Timed out
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.08

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{(2\,a+b)\log\left(\frac{be^{(2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}b} - \frac{\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}} + \frac{x}{b}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/4*(2*a + b)*log((b*e^(2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(2*x) +
2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*b) - 1/4*log((b*e^(-2*x) +
2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/s
qrt((a + b)*a) + x/b
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{a\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b} + \frac{x}{b}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
-a*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b)
 + x/b
```

3.25.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 376, normalized size of antiderivative = 9.64

$$\int \frac{\cosh^2(x)}{a+b\cosh^2(x)}\,dx = \frac{x}{b} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(b^5\sqrt{-b^3-a\,b^2}+a\,b^4\sqrt{-b^3-a\,b^2}\right)\left(\mathrm{e}^{2\,x}\left(\frac{2\left(8\,a^{5/2}\sqrt{-b^3-a\,b^2}+\sqrt{a}\,b^2\sqrt{-b^3-a\,b^2}+8\,a^{3/2}\,b\sqrt{-b^3-a\,b^2}\right)\left(8\,a^2+8\,a\,b+b^2\right)}{b^8\,(a+b)^2\sqrt{-b^3-a\,b^2}}+\frac{4\sqrt{a}\,(4\,a\cdots}{b^7\,(a+\cdots}\right.\right.}{4\,a}\right.}{\sqrt{-b^3-a\,b^2}}$$

input
```
int(cosh(x)^2/(a + b*cosh(x)^2),x)
```

output
```
x/b + (a^(1/2)*atan(((b^5*(- a*b^2 - b^3)^(1/2) + a*b^4*(- a*b^2 - b^3)^(1
/2))*(exp(2*x)*((2*(8*a^(5/2)*(- a*b^2 - b^3)^(1/2) + a^(1/2)*b^2*(- a*b^2
 - b^3)^(1/2) + 8*a^(3/2)*b*(- a*b^2 - b^3)^(1/2))*(8*a*b + 8*a^2 + b^2))/
(b^8*(a + b)^2*(- a*b^2 - b^3)^(1/2)) + (4*a^(1/2)*(4*a + 2*b)*(4*a*b^3 +
8*a^3*b + 12*a^2*b^2))/(b^7*(a + b)*(-b^2*(a + b))^(1/2)*(- a*b^2 - b^3)^(
1/2))) + (2*(a^(1/2)*b^2*(- a*b^2 - b^3)^(1/2) + 2*a^(3/2)*b*(- a*b^2 - b^
3)^(1/2))*(8*a*b + 8*a^2 + b^2))/(b^8*(a + b)^2*(- a*b^2 - b^3)^(1/2)) + (
4*a^(1/2)*(2*a*b^3 + 2*a^2*b^2)*(4*a + 2*b))/(b^7*(a + b)*(-b^2*(a + b))^(
1/2)*(- a*b^2 - b^3)^(1/2))))/(4*a))/(- a*b^2 - b^3)^(1/2)
```

3.26 $\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx$

3.26.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

output
```
arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))/b^(1/2)/(a+b)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

input
```
Integrate[Cosh[x]/(a + b*Cosh[x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

3.26.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3665, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\int \frac{1}{a+b\sinh^2(x)+b}d\sinh(x)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

input
```
Int[Cosh[x]/(a + b*Cosh[x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

3.26.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(21) = 42$.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

method	result	size
default	$\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}+2\sqrt{a}}{2\sqrt{b}}\right)}{\sqrt{a+b}\,\sqrt{b}}+\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}-2\sqrt{a}}{2\sqrt{b}}\right)}{\sqrt{a+b}\,\sqrt{b}}$	66
risch	$-\frac{\ln\left(\mathrm{e}^{2x}-\frac{2(a+b)\mathrm{e}^{x}}{\sqrt{-ab-b^{2}}}-1\right)}{2\sqrt{-ab-b^{2}}}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2(a+b)\mathrm{e}^{x}}{\sqrt{-ab-b^{2}}}-1\right)}{2\sqrt{-ab-b^{2}}}$	82

input
```
int(cosh(x)/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*tanh(1/2*x)*(a+b)^(1/2)+2*a^(1/2))/b^(
1/2))+1/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*tanh(1/2*x)*(a+b)^(1/2)-2*a^(1/2
))/b^(1/2))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 337, normalized size of antiderivative = 11.62

$$\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx = \left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2\,(2\,a+3\,b)\cosh(x)^2+2\,\left(3\,b\cosh(x)^2-2\,a-3\,b\right)\sinh(x)^2+4\,\left(b\cosh(x)^3-\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(x)^2+2\,\left(3\right.}\right.}{2\,(ab+b^2)}\right.$$

input
```
integrate(cosh(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b - b^2)*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x
)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 +
4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*(cosh(x)^3 + 3*cosh(x)*s
inh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(-a*b - b^
2) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*c
osh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a +
 b)*cosh(x))*sinh(x) + b))/(a*b + b^2), (sqrt(a*b + b^2)*arctan(1/2*(b*cos
h(x)^3 + 3*b*cosh(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + 3*b)*cosh(x) + (3*b*
cosh(x)^2 + 4*a + 3*b)*sinh(x))/sqrt(a*b + b^2)) + sqrt(a*b + b^2)*arctan(
1/2*sqrt(a*b + b^2)*(cosh(x) + sinh(x))/(a + b)))/(a*b + b^2)]
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55498 vs. $2(27) = 54$.

Time = 106.06 (sec) , antiderivative size = 55498, normalized size of antiderivative = 1913.72

$$\int \frac{\cosh(x)}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)**2),x)
```

output
```
Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (tanh(x/2)/(2*b) - 1
/(2*b*tanh(x/2)), Eq(a, -b)), (sinh(x)/a, Eq(b, 0)), (13*a**6*b*sqrt(a/(a
+ b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-sqrt(a/(a + b) - b/(a + b) +
 2*sqrt(-a*b)/(a + b)) + tanh(x/2))/(2*a**7*b*sqrt(a/(a + b) - b/(a + b) -
 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b))
- 130*a**6*b**2*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/
(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 24*a**6*b*sqrt(-a*b)*sqrt(a/
(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2
*sqrt(-a*b)/(a + b)) + 858*a**5*b**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-
a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 416*a**
5*b**2*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(
a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 858*a**4*b**4*sqrt(a/(a +
b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt
(-a*b)/(a + b)) + 1144*a**4*b**3*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2
*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
858*a**3*b**5*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a
 + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 858*a**2*b**6*sqrt(a/(a + b) -
 b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*
b)/(a + b)) - 1144*a**2*b**5*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2*sqr
t(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 1...
```

3.26.7 Maxima [F]

$$\int \frac{\cosh(x)}{a + b\cosh^2(x)}\,dx = \int \frac{\cosh(x)}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/(b*cosh(x)^2 + a), x)
```

3.26.8 Giac [F]

$$\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\cosh(x)}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.26.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.00

$$\int \frac{\cosh(x)}{a+b\cosh^2(x)}\,dx = -\frac{\ln\left(-\frac{4\,(a-a\,e^{2\,x})}{b^2\,(a+b)}-\frac{8\,a\,e^x}{(-b)^{5/2}\,\sqrt{a+b}}\right)-\ln\left(\frac{8\,a\,e^x}{(-b)^{5/2}\,\sqrt{a+b}}-\frac{4\,(a-a\,e^{2\,x})}{b^2\,(a+b)}\right)}{2\,\sqrt{-b}\,\sqrt{a+b}}$$

input
```
int(cosh(x)/(a + b*cosh(x)^2),x)
```

output
```
-(log(- (4*(a - a*exp(2*x)))/(b^2*(a + b)) - (8*a*exp(x))/((-b)^(5/2)*(a +
 b)^(1/2))) - log((8*a*exp(x))/((-b)^(5/2)*(a + b)^(1/2)) - (4*(a - a*exp(
2*x)))/(b^2*(a + b))))/(2*(-b)^(1/2)*(a + b)^(1/2))
```

3.27 $\int \frac{1}{a+b\cosh^2(x)}\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{1}{a+b\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

output
```
arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(1/2)/(a+b)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cosh^2(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Integrate[(a + b*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

3.27.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^2(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$
$$\Big\downarrow \; 3660$$
$$\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)$$
$$\Big\downarrow \; 221$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}}$$

input
```
Int[(a + b*Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b])
```

3.27.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(21) = 42$.

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

method	result	size
default	$\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{2\sqrt{a}\,\sqrt{a+b}}-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}-2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{2\sqrt{a}\,\sqrt{a+b}}$	78
risch	$\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}+2a^{2}+2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}}$	128

input
```
int(1/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)
+(a+b)^(1/2))-1/2/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2-2*tanh(
1/2*x)*a^(1/2)+(a+b)^(1/2))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 10.10

$$\int \frac{1}{a+b\cosh^2(x)}\,dx$$
$$=\left[\frac{\log\left(\frac{b^{2}\cosh(x)^{4}+4\,b^{2}\cosh(x)\sinh(x)^{3}+b^{2}\sinh(x)^{4}+2\,(2\,ab+b^{2})\cosh(x)^{2}+2\left(3\,b^{2}\cosh(x)^{2}+2\,ab+b^{2}\right)\sinh(x)^{2}+8\,a^{2}+8\,ab+b^{2}+4\left(b^{2}\cosh(\right.}{b\cosh(x)^{4}+4\,b\cosh(x)\sinh(x)^{3}+b\sinh(x)^{4}+2\,(2\,a+b)\cosh(x)^{2}+2\left(3\,b\cosh(x)^{2}+2\,a+b\right)\mathrm{s}}\right.}{2\sqrt{a^{2}+ab}}\right.$$

input
```
integrate(1/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a
*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2
+ 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(
b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2
+ 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(
x))*sinh(x) + b))/sqrt(a^2 + a*b), sqrt(-a^2 - a*b)*arctan(1/2*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a^2 + a
*b))/(a^2 + a*b)]
```

3.27.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(27) = 54$.

Time = 24.28 (sec) , antiderivative size = 10924, normalized size of antiderivative = 376.69

$$\int \frac{1}{a + b\cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)**2),x)
```

output
```
Piecewise((zoo*tanh(x/2)/(tanh(x/2)**2 + 1), Eq(a, 0) & Eq(b, 0)), (-tanh(
x/2)/(2*b) - 1/(2*b*tanh(x/2)), Eq(a, -b)), (2*tanh(x/2)/(b*(tanh(x/2)**2
+ 1)), Eq(a, 0)), (-a**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b)
)*log(-sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tanh(x/2))/(2*
a**4*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b
/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(a/(a + b) - b/(a + b) -
2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) +
 8*a**3*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt
(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(a/(a +
b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt
(-a*b)/(a + b)) + 2*a*b**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a*b**2*sqrt(-a*
b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(
a + b) + 2*sqrt(-a*b)/(a + b))) + a**3*sqrt(a/(a + b) - b/(a + b) - 2*sqrt
(-a*b)/(a + b))*log(sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) + t
anh(x/2))/(2*a**4*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(
a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(a/(a + b) -
 b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*
b)/(a + b)) + 8*a**3*sqrt(-a*b)*sqrt(a/(a + b) - b/(a + b) - 2*sqrt(-a*b)/
(a + b))*sqrt(a/(a + b) - b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b...
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{1}{a+b\cosh^2(x)}\,dx = -\frac{\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}}$$

input
```
integrate(1/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b
+ 2*sqrt((a + b)*a)))/sqrt((a + b)*a)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{a+b\cosh^2(x)}\,dx = \frac{\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}}$$

input
```
integrate(1/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/sqrt(-a^2 - a*b)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.21

$$\int \frac{1}{a+b\cosh^2(x)}\,dx =$$
$$-\frac{\operatorname{atan}\left(\frac{b^2\,\mathrm{e}^{2\,x}\,{\left(-a^2-b\,a\right)}^{3/2}\,\left(\frac{4\,\left(4\,a+2\,b\right)\,\left(8\,a^3+12\,a^2\,b+4\,a\,b^2\right)}{b^5\,{\left(-a^2-b\,a\right)}^{3/2}\,\sqrt{-a\,\left(a+b\right)}}+\frac{2\,\left(8\,a^2+8\,a\,b+b^2\right)\,\left(8\,a^2\,\sqrt{-a^2-b\,a}+b^2\,\sqrt{-a^2-b\,a}+8\,a\,b\,\sqrt{-a^2-b\,a}\right)}{a\,b^5\,\left(a+b\right)\,{\left(-a^2-b\,a\right)}^{3/2}}\right)}{4}+\frac{(2\,a^2\,b+}{b^3}\right.}{\sqrt{-a^2-b\,a}}$$

input
```
int(1/(a + b*cosh(x)^2),x)
```

output
```
-atan((b^2*exp(2*x)*(- a*b - a^2)^(3/2)*((4*(4*a + 2*b)*(4*a*b^2 + 12*a^2*
b + 8*a^3))/(b^5*(- a*b - a^2)^(3/2)*(-a*(a + b))^(1/2)) + (2*(8*a*b + 8*a
^2 + b^2)*(8*a^2*(- a*b - a^2)^(1/2) + b^2*(- a*b - a^2)^(1/2) + 8*a*b*(-
a*b - a^2)^(1/2)))/(a*b^5*(a + b)*(- a*b - a^2)^(3/2))))/4 + ((2*a*b^2 + 2
*a^2*b)*(4*a + 2*b))/(b^3*(-a*(a + b))^(1/2)) + ((b^2*(- a*b - a^2)^(1/2)
+ 2*a*b*(- a*b - a^2)^(1/2))*(8*a*b + 8*a^2 + b^2))/(2*a*b^3*(a + b)))/(-
a*b - a^2)^(1/2)
```

3.27. $\int \frac{1}{a+b\cosh^2(x)}\,dx$

3.28 $\int \frac{\text{sech}(x)}{a+b\cosh^2(x)}\,dx$

3.28.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{\text{sech}(x)}{a+b\cosh^2(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

output
```
arctan(sinh(x))/a-arctan(sinh(x)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/(a+b)^(1/2
)
```

3.28.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{\text{sech}(x)}{a+b\cosh^2(x)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\,\text{csch}(x)}{\sqrt{b}}\right)}{a\sqrt{a+b}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sech[x]/(a + b*Cosh[x]^2),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/(a*Sqrt[a + b]) + (2*ArcTa
n[Tanh[x/2]])/a
```

3.28.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3665, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{ 3665}$$

$$\int \frac{1}{(\sinh^2(x)+1)\,(a+b\sinh^2(x)+b)}d\sinh(x)$$

$$\Big\downarrow \text{ 303}$$

$$\frac{\int \frac{1}{\sinh^2(x)+1}d\sinh(x)}{a}-\frac{b\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{a}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{b\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{a}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

input
```
Int[Sech[x]/(a + b*Cosh[x]^2),x]
```

output
```
ArcTan[Sinh[x]]/a - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(a*Sqr
t[a + b])
```

3.28.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(33) = 66$.

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.07

method	result	size
default	$\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2b\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}+2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}+\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}-2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}\right)}{a}$	85
risch	$\frac{i\ln\left(e^{x}+i\right)}{a}-\frac{i\ln\left(e^{x}-i\right)}{a}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2x}-\frac{2\sqrt{-(a+b)b}\,e^{x}}{b}-1\right)}{2(a+b)a}-\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2x}+\frac{2\sqrt{-(a+b)b}\,e^{x}}{b}-1\right)}{2(a+b)a}$	106

input
```
int(sech(x)/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/a*arctan(tanh(1/2*x))-2*b/a*(1/2/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*tanh(
1/2*x)*(a+b)^(1/2)+2*a^(1/2))/b^(1/2))+1/2/(a+b)^(1/2)/b^(1/2)*arctan(1/2*
(2*tanh(1/2*x)*(a+b)^(1/2)-2*a^(1/2))/b^(1/2)))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(33) = 66$.

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 8.78

$$\int \frac{\operatorname{sech}(x)}{a + b\cosh^2(x)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a+b}}\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4-2\,(2\,a+3\,b)\cosh(x)^2+2\,\left(3\,b\cosh(x)^2-2\,a-3\,b\right)\sinh(x)^2+4\,\left(b\cosh(x)^3-(2\,a+3\right.}{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(2\,a+b)\cosh(}\right.}{}\right.$$
$$-\frac{\sqrt{\frac{b}{a+b}}\arctan\left(\frac{1}{2}\sqrt{\frac{b}{a+b}}(\cosh(x)+\sinh(x))\right)+\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left(b\cosh(x)^3+3\,b\cosh(x)\sinh(x)^2+b\sinh(x)^3+(4\,a+3\,b\right.}{2\,b}\right.}{a}$$

input
```
integrate(sech(x)/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/(a + b))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x
)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 +
4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*((a + b)*cosh(x)^3 + 3*(
a + b)*cosh(x)*sinh(x)^2 + (a + b)*sinh(x)^3 - (a + b)*cosh(x) + (3*(a + b
)*cosh(x)^2 - a - b)*sinh(x))*sqrt(-b/(a + b)) + b)/(b*cosh(x)^4 + 4*b*cos
h(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 +
2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 4
*arctan(cosh(x) + sinh(x)))/a, -(sqrt(b/(a + b))*arctan(1/2*sqrt(b/(a + b)
)*(cosh(x) + sinh(x))) + sqrt(b/(a + b))*arctan(1/2*(b*cosh(x)^3 + 3*b*cos
h(x)*sinh(x)^2 + b*sinh(x)^3 + (4*a + 3*b)*cosh(x) + (3*b*cosh(x)^2 + 4*a
+ 3*b)*sinh(x))*sqrt(b/(a + b))/b) - 2*arctan(cosh(x) + sinh(x)))/a]
```

3.28.6 Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*cosh(x)**2),x)
```

output
```
Integral(sech(x)/(a + b*cosh(x)**2), x)
```

3.28.7 Maxima [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
2*arctan(e^x)/a - 2*integrate((b*e^(3*x) + b*e^x)/(a*b*e^(4*x) + a*b + 2*(
2*a^2 + a*b)*e^(2*x)), x)
```

3.28.8 Giac [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.28.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 5.07

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh^2(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{e^x\,\left(16\,{\left(a^2\right)}^{3/2}+9\,b^2\,\sqrt{a^2}+24\,a\,b\,\sqrt{a^2}\right)}{16\,a^3+24\,a^2\,b+9\,a\,b^2}\right)}{\sqrt{a^2}} - \frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,e^x\,\sqrt{a^2\,(a+b)}}{2\,a\,(a+b)}\right)+2\,\mathrm{atan}\left(\frac{4\,a^4\,e^x+8\,a^3\,b\,e^x+4\,a^2\,b^2\,e^x-b\,e^x\,\sqrt{a^2\,(a+b)}\,\sqrt{a^3+b\,a^2}+b\,e^{3\,x}\,\sqrt{a^2\,(a+b)}\,\sqrt{a^3+b\,a^2}}{\sqrt{b}\,\sqrt{a^2\,(a+b)}\,(2\,a^2+2\,b\,a)}\right)\right)}{2\,\sqrt{a^3+b\,a^2}}$$

input
```
int(1/(cosh(x)*(a + b*cosh(x)^2)),x)
```

output
```
(2*atan((exp(x)*(16*(a^2)^(3/2) + 9*b^2*(a^2)^(1/2) + 24*a*b*(a^2)^(1/2)))
/(9*a*b^2 + 24*a^2*b + 16*a^3)))/(a^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*e
xp(x)*(a^2*(a + b))^(1/2))/(2*a*(a + b))) + 2*atan((4*a^4*exp(x) + 8*a^3*b
*exp(x) + 4*a^2*b^2*exp(x) - b*exp(x)*(a^2*(a + b))^(1/2)*(a^2*b + a^3)^(1
/2) + b*exp(3*x)*(a^2*(a + b))^(1/2)*(a^2*b + a^3)^(1/2))/(b^(1/2)*(a^2*(a
 + b))^(1/2)*(2*a*b + 2*a^2)))))/(2*(a^2*b + a^3)^(1/2))
```

3.29 $\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx$

3.29.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tanh(x)}{a}$$

output
```
-b*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(3/2)/(a+b)^(1/2)+tanh(x)/a
```

3.29.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tanh(x)}{a}$$

input
```
Integrate[Sech[x]^2/(a + b*Cosh[x]^2),x]
```

output
```
-((b*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b])) + Tanh
[x]/a
```

3.29.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3666, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$-\int \frac{(1-\coth^2(x))\tanh^2(x)}{a-(a+b)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{359}$$

$$\frac{\tanh(x)}{a}-\frac{b\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\tanh(x)}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}$$

input
```
Int[Sech[x]^2/(a + b*Cosh[x]^2),x]
```

output
```
-((b*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b])) + Tanh
[x]/a
```

3.29.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(30) = 60$.

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.82

method	result	size
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{a\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)}+\frac{2b\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(-\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}-\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}\right)}{a}$	107
risch	$-\frac{2}{a(1+e^{2x})}+\frac{b\ln\left(e^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}+2a^{2}+2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}\,a}-\frac{b\ln\left(e^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}\,a}$	149

input
```
int(sech(x)^2/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/a*tanh(1/2*x)/(1+tanh(1/2*x)^2)+2*b/a*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)
^(1/2)*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^
(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)-(a+b)^(1/2)))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 457, normalized size of antiderivative = 12.03

$$\int \frac{\operatorname{sech}^2(x)}{a + b\cosh^2(x)}\,dx$$
$$= \left[\frac{\left(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 + b\right)\sqrt{a^2+ab}\log\left(\frac{b^2\cosh(x)^4+4b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2}{b\cosh}\right.}{2\left(a^3+a^2b+\left(a^3+a^2\right.\right.} \right.$$
$$-\frac{\left(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 + b\right)\sqrt{-a^2-ab}\arctan\left(\frac{\left(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x\right.}{2(a^2+ab)}\right.}{a^3+a^2b+(a^3+a^2b)\cosh(x)^2+2(a^3+a^2b)\cosh(x)\sinh(x)+(a^3+a^2b)\sin}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a^2 + a*b
)*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b
+ b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8
*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) + 4*(b*cosh
(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*c
osh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2
*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))
*sinh(x) + b)) - 4*a^2 - 4*a*b)/(a^3 + a^2*b + (a^3 + a^2*b)*cosh(x)^2 + 2
*(a^3 + a^2*b)*cosh(x)*sinh(x) + (a^3 + a^2*b)*sinh(x)^2), -((b*cosh(x)^2
+ 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(-a^2 - a*b)*arctan(1/2*(b*co
sh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(-a^2 - a*b)/(a
^2 + a*b)) + 2*a^2 + 2*a*b)/(a^3 + a^2*b + (a^3 + a^2*b)*cosh(x)^2 + 2*(a^
3 + a^2*b)*cosh(x)*sinh(x) + (a^3 + a^2*b)*sinh(x)^2)]
```

3.29.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(sech(x)**2/(a+b*cosh(x)**2),x)
```

output
```
Integral(sech(x)**2/(a + b*cosh(x)**2), x)
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx = \frac{b\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}a} + \frac{2}{ae^{(-2x)}+a}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/2*b*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a + b
 + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*a) + 2/(a*e^(-2*x) + a)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx = -\frac{b\arctan\left(\frac{be^{(2x)}+2a+b}{2\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a} - \frac{2}{a(e^{(2x)}+1)}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
-b*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a)
 - 2/(a*(e^(2*x) + 1))
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.84

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh^2(x)}\,dx = \frac{b\ln\left(\frac{4\,e^{2\,x}}{a}-\frac{2\,\left(b+2\,a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^{3/2}\,\sqrt{a+b}}\right)}{2\,a^{3/2}\,\sqrt{a+b}}-\frac{2}{a\,\left(e^{2\,x}+1\right)}-\frac{b\ln\left(\frac{4\,e^{2\,x}}{a}+\frac{2\,\left(b+2\,a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^{3/2}\,\sqrt{a+b}}\right)}{2\,a^{3/2}\,\sqrt{a+b}}$$

input
```
int(1/(cosh(x)^2*(a + b*cosh(x)^2)),x)
```

output
```
(b*log((4*exp(2*x))/a - (2*(b + 2*a*exp(2*x) + b*exp(2*x)))/(a^(3/2)*(a +
b)^(1/2))))/(2*a^(3/2)*(a + b)^(1/2)) - 2/(a*(exp(2*x) + 1)) - (b*log((4*e
xp(2*x))/a + (2*(b + 2*a*exp(2*x) + b*exp(2*x)))/(a^(3/2)*(a + b)^(1/2))))
/(2*a^(3/2)*(a + b)^(1/2))
```

3.30 $\int \frac{\text{sech}^3(x)}{a+b\cosh^2(x)}\,dx$

3.30.1 Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\text{sech}^3(x)}{a+b\cosh^2(x)}\,dx=\frac{(a-2b)\arctan(\sinh(x))}{2a^2}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{\text{sech}(x)\tanh(x)}{2a}$$

output
```
1/2*(a-2*b)*arctan(sinh(x))/a^2+b^(3/2)*arctan(sinh(x)*b^(1/2)/(a+b)^(1/2)
)/a^2/(a+b)^(1/2)+1/2*sech(x)*tanh(x)/a
```

3.30.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{\text{sech}^3(x)}{a+b\cosh^2(x)}\,dx=\frac{-\frac{2b^{3/2}\arctan\left(\frac{\sqrt{a+b}\,\text{csch}(x)}{\sqrt{b}}\right)}{\sqrt{a+b}}+2(a-2b)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+a\,\text{sech}(x)\tanh(x)}{2a^2}$$

input
```
Integrate[Sech[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
((-2*b^(3/2)*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/Sqrt[a + b] + 2*(a - 2
*b)*ArcTan[Tanh[x/2]] + a*Sech[x]*Tanh[x])/(2*a^2)
```

3.30.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3665, 316, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1}{\left(\sinh^2(x)+1\right)^2\left(a+b\sinh^2(x)+b\right)}d\sinh(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\sinh(x)}{2a\left(\sinh^2(x)+1\right)}-\frac{\int -\frac{b\sinh^2(x)+a-b}{\left(\sinh^2(x)+1\right)\left(b\sinh^2(x)+a+b\right)}d\sinh(x)}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\sinh^2(x)+a-b}{\left(\sinh^2(x)+1\right)\left(b\sinh^2(x)+a+b\right)}d\sinh(x)}{2a}+\frac{\sinh(x)}{2a\left(\sinh^2(x)+1\right)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2b^2\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{a}+\frac{(a-2b)\int \frac{1}{\sinh^2(x)+1}d\sinh(x)}{a}}{2a}+\frac{\sinh(x)}{2a\left(\sinh^2(x)+1\right)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{2b^2\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{a}+\frac{(a-2b)\arctan(\sinh(x))}{a}}{2a}+\frac{\sinh(x)}{2a\left(\sinh^2(x)+1\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-2b)\arctan(\sinh(x))}{a}}{2a}+\frac{\sinh(x)}{2a\left(\sinh^2(x)+1\right)}$$

input
```
Int[Sech[x]^3/(a + b*Cosh[x]^2),x]
```

output
```
(((a - 2*b)*ArcTan[Sinh[x]])/a + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[
a + b]])/(a*Sqrt[a + b]))/(2*a) + Sinh[x]/(2*a*(1 + Sinh[x]^2))
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.30. $\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx$

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(47) = 94$.

Time = 0.84 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.08

method	result
default	$\frac{\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)^{3}}{2}+\frac{a\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{2}}+(a-2b)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}+\frac{2b^{2}\left(\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}+2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}+\frac{\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)\sqrt{a+b}-2\sqrt{a}}{2\sqrt{b}}\right)}{2\sqrt{a+b}\,\sqrt{b}}\right)}{a^{2}}$
risch	$\frac{e^{x}\left(e^{2x}-1\right)}{\left(1+e^{2x}\right)^{2}a}+\frac{i\ln\left(e^{x}+i\right)}{2a}-\frac{ib\ln\left(e^{x}+i\right)}{a^{2}}-\frac{i\ln\left(e^{x}-i\right)}{2a}+\frac{ib\ln\left(e^{x}-i\right)}{a^{2}}+\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2x}+\frac{2\sqrt{-(a+b)b}\,e^{x}}{b}-1\right)}{2(a+b)a^{2}}-\frac{\sqrt{-(a+b)b}}{}$

input
```
int(sech(x)^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/a^2*((-1/2*a*tanh(1/2*x)^3+1/2*a*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+1/2*(a
-2*b)*arctan(tanh(1/2*x)))+2*b^2/a^2*(1/2/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(
2*tanh(1/2*x)*(a+b)^(1/2)+2*a^(1/2))/b^(1/2))+1/2/(a+b)^(1/2)/b^(1/2)*arct
an(1/2*(2*tanh(1/2*x)*(a+b)^(1/2)-2*a^(1/2))/b^(1/2)))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 963, normalized size of antiderivative = 16.32

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*cosh(x)^3 + 6*a*cosh(x)*sinh(x)^2 + 2*a*sinh(x)^3 + (b*cosh(x)^4
 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2
+ b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-b/(a + b))
*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*(2*a + 3*b)*co
sh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2 + 4*(b*cosh(x)^3 - (2*a
+ 3*b)*cosh(x))*sinh(x) + 4*((a + b)*cosh(x)^3 + 3*(a + b)*cosh(x)*sinh(x)
^2 + (a + b)*sinh(x)^3 - (a + b)*cosh(x) + (3*(a + b)*cosh(x)^2 - a - b)*s
inh(x))*sqrt(-b/(a + b)) + b)/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sin
h(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4
*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 2*((a - 2*b)*cosh(x)^4
+ 4*(a - 2*b)*cosh(x)*sinh(x)^3 + (a - 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x
)^2 + 2*(3*(a - 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 + 4*((a - 2*b)*cosh(x)
^3 + (a - 2*b)*cosh(x))*sinh(x) + a - 2*b)*arctan(cosh(x) + sinh(x)) - 2*a
*cosh(x) + 2*(3*a*cosh(x)^2 - a)*sinh(x))/(a^2*cosh(x)^4 + 4*a^2*cosh(x)*s
inh(x)^3 + a^2*sinh(x)^4 + 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 + a^2)*sin
h(x)^2 + a^2 + 4*(a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x)), (a*cosh(x)^3 + 3*
a*cosh(x)*sinh(x)^2 + a*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 +
 b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh
(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(b/(a + b))*arctan(1/2*sqrt(b/(a + b))
*(cosh(x) + sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x...
```

3.30.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(sech(x)**3/(a+b*cosh(x)**2),x)
```

output
```
Integral(sech(x)**3/(a + b*cosh(x)**2), x)
```

3.30.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\operatorname{sech}(x)^3}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
(e^(3*x) - e^x)/(a*e^(4*x) + 2*a*e^(2*x) + a) + (a - 2*b)*arctan(e^x)/a^2
+ 8*integrate(1/4*(b^2*e^(3*x) + b^2*e^x)/(a^2*b*e^(4*x) + a^2*b + 2*(2*a^
3 + a^2*b)*e^(2*x)), x)
```

3.30.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx=\int \frac{\operatorname{sech}(x)^3}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.30.9 Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 447, normalized size of antiderivative = 7.58

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{e^x\left(a^7\,(a^4)^{3/2}-12\,b^3\,(a^4)^{5/2}-18\,b^7\,(a^4)^{3/2}+36\,a^2\,b^5\,(a^4)^{3/2}-30\,a^3\,b^4\,(a^4)^{3/2}+21\,a^5\,b^2\,(a^4)^{3/2}+9\,a\,b^6\,(a^4)^{3/2}-8\,a^6\,b\,(a^4)^{3/2}\right.}{a^{12}\sqrt{a^2-4\,a\,b+4\,b^2}-6\,a^{11}\,b\sqrt{a^2-4\,a\,b+4\,b^2}+9\,a^6\,b^6\sqrt{a^2-4\,a\,b+4\,b^2}-18\,a^8\,b^4\sqrt{a^2-4\,a\,b+4\,b^2}+6\,a^9\,b^3\sqrt{a^2-4\,a\,b+4\,b^2}+9\,a^{10}\,b^2\sqrt{a^2-}}\right.}{\sqrt{a^4}} - \frac{2\,e^x}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}+\frac{e^x}{a\,\left(e^{2\,x}+1\right)} - \frac{(-b)^{3/2}\,\ln\left(\frac{64\,\left(e^{2\,x}-1\right)\,\left(a^3-3\,a^2\,b+3\,b^3\right)}{a^5\,(a+b)^2}-\frac{128\,e^x\,\left(a^3-3\,a^2\,b+3\,b^3\right)}{a^5\,\sqrt{-b}\,(a+b)^{3/2}}\right)}{2\,a^2\,\sqrt{a+b}} + \frac{(-b)^{3/2}\,\ln\left(\frac{64\,\left(e^{2\,x}-1\right)\,\left(a^3-3\,a^2\,b+3\,b^3\right)}{a^5\,(a+b)^2}+\frac{128\,e^x\,\left(a^3-3\,a^2\,b+3\,b^3\right)}{a^5\,\sqrt{-b}\,(a+b)^{3/2}}\right)}{2\,a^2\,\sqrt{a+b}}$$

input
```
int(1/(cosh(x)^3*(a + b*cosh(x)^2)),x)
```

output
```
(atan((exp(x)*(a^7*(a^4)^(3/2) - 12*b^3*(a^4)^(5/2) - 18*b^7*(a^4)^(3/2) +
 36*a^2*b^5*(a^4)^(3/2) - 30*a^3*b^4*(a^4)^(3/2) + 21*a^5*b^2*(a^4)^(3/2)
+ 9*a*b^6*(a^4)^(3/2) - 8*a^6*b*(a^4)^(3/2)))/(a^12*(a^2 - 4*a*b + 4*b^2)^
(1/2) - 6*a^11*b*(a^2 - 4*a*b + 4*b^2)^(1/2) + 9*a^6*b^6*(a^2 - 4*a*b + 4*
b^2)^(1/2) - 18*a^8*b^4*(a^2 - 4*a*b + 4*b^2)^(1/2) + 6*a^9*b^3*(a^2 - 4*a
*b + 4*b^2)^(1/2) + 9*a^10*b^2*(a^2 - 4*a*b + 4*b^2)^(1/2)))*(a^2 - 4*a*b
+ 4*b^2)^(1/2))/(a^4)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1)) +
 exp(x)/(a*(exp(2*x) + 1)) - ((-b)^(3/2)*log((64*(exp(2*x) - 1)*(a^3 - 3*a
^2*b + 3*b^3))/(a^5*(a + b)^2) - (128*exp(x)*(a^3 - 3*a^2*b + 3*b^3))/(a^5
*(-b)^(1/2)*(a + b)^(3/2))))/(2*a^2*(a + b)^(1/2)) + ((-b)^(3/2)*log((64*(
exp(2*x) - 1)*(a^3 - 3*a^2*b + 3*b^3))/(a^5*(a + b)^2) + (128*exp(x)*(a^3
- 3*a^2*b + 3*b^3))/(a^5*(-b)^(1/2)*(a + b)^(3/2))))/(2*a^2*(a + b)^(1/2))
```

3.30. $\int \frac{\operatorname{sech}^3(x)}{a+b\cosh^2(x)}\,dx$

3.31 $\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx$

3.31.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx=\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tanh(x)}{a^2}-\frac{\tanh^3(x)}{3a}$$

output
```
b^2*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(5/2)/(a+b)^(1/2)+(a-b)*tanh(x)
/a^2-1/3*tanh(x)^3/a
```

3.31.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx=\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(2a-3b+a\operatorname{sech}^2(x))\tanh(x)}{3a^2}$$

input
```
Integrate[Sech[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(b^2*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + ((2*a
 - 3*b + a*Sech[x]^2)*Tanh[x])/(3*a^2)
```

3.31.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3666, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{3666}$$

$$\int \frac{\tanh^4(x)\left(1-\coth^2(x)\right)^2}{a-(a+b)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{b^2}{a^2\left(a-(a+b)\coth^2(x)\right)}+\frac{(b-a)\tanh^2(x)}{a^2}+\frac{\tanh^4(x)}{a}\right)d\coth(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tanh(x)}{a^2}-\frac{\tanh^3(x)}{3a}$$

input
```
Int[Sech[x]^4/(a + b*Cosh[x]^2),x]
```

output
```
(b^2*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) + ((a -
 b)*Tanh[x])/a^2 - Tanh[x]^3/(3*a)
```

3.31.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3666
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m + 1
)/f   Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1))
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &
& IntegerQ[p]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(45) = 90$.

Time = 1.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.58

method	result
default	$-\frac{2\left(\left(-a+b\right)\tanh\left(\frac{x}{2}\right)^{5}+\left(-\frac{2a}{3}+2b\right)\tanh\left(\frac{x}{2}\right)^{3}+\left(-a+b\right)\tanh\left(\frac{x}{2}\right)\right)}{a^{2}\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}-\frac{2b^{2}\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(-\sqrt{a+b}\right.}{}\right.}{a^{2}}$
risch	$-\frac{2\left(-3b\,e^{4x}+6a\,e^{2x}-6b\,e^{2x}+2a-3b\right)}{3\left(1+e^{2x}\right)^{3}a^{2}}+\frac{b^{2}\ln\left(e^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}\,a^{2}}-\frac{b^{2}\ln\left(e^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}+2a^{2}+2ab}{b\sqrt{a^{2}+ab}}\right.}{2\sqrt{a^{2}+ab}\,a^{2}}$

input
```
int(sech(x)^4/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*((-a+b)*tanh(1/2*x)^5+(-2/3*a+2*b)*tanh(1/2*x)^3+(-a+b)*tanh(1/2*x)
)/(1+tanh(1/2*x)^2)^3-2*b^2/a^2*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*t
anh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^(1/2)*ln
(-(a+b)^(1/2)*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1/2)-(a+b)^(1/2)))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 1377, normalized size of antiderivative = 25.04

$$\int \frac{\operatorname{sech}^4(x)}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b + a*b^2)*cosh(x)^4 + 48*(a^2*b + a*b^2)*cosh(x)*sinh(x)^3
+ 12*(a^2*b + a*b^2)*sinh(x)^4 - 8*a^3 + 4*a^2*b + 12*a*b^2 - 24*(a^3 - a*
b^2)*cosh(x)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2
+ 3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(
x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*co
sh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2
+ b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x))
*sinh(x))*sqrt(a^2 + a*b)*log((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b
^2*sinh(x)^4 + 2*(2*a*b + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^
2)*sinh(x)^2 + 8*a^2 + 8*a*b + b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh
(x))*sinh(x) - 4*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a +
b)*sqrt(a^2 + a*b))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2
*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x
)^3 + (2*a + b)*cosh(x))*sinh(x) + b)) + 48*((a^2*b + a*b^2)*cosh(x)^3 - (
a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 + a^3*b)*cosh(x)^6 + 6*(a^4 + a^3*b)*
cosh(x)*sinh(x)^5 + (a^4 + a^3*b)*sinh(x)^6 + 3*(a^4 + a^3*b)*cosh(x)^4 +
3*(a^4 + a^3*b + 5*(a^4 + a^3*b)*cosh(x)^2)*sinh(x)^4 + a^4 + a^3*b + 4*(5
*(a^4 + a^3*b)*cosh(x)^3 + 3*(a^4 + a^3*b)*cosh(x))*sinh(x)^3 + 3*(a^4 + a
^3*b)*cosh(x)^2 + 3*(5*(a^4 + a^3*b)*cosh(x)^4 + a^4 + a^3*b + 6*(a^4 + a^
3*b)*cosh(x)^2)*sinh(x)^2 + 6*((a^4 + a^3*b)*cosh(x)^5 + 2*(a^4 + a^3*b...
```

3.31.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a + b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{a + b\cosh^2(x)}\,dx$$

input
```
integrate(sech(x)**4/(a+b*cosh(x)**2),x)
```

output
```
Integral(sech(x)**4/(a + b*cosh(x)**2), x)
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx = -\frac{b^2\log\left(\frac{be^{(-2x)}+2a+b-2\sqrt{(a+b)a}}{be^{(-2x)}+2a+b+2\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}a^2} + \frac{2\left(6(a-b)e^{(-2x)}-3be^{(-4x)}+2a-3b\right)}{3\left(3a^2e^{(-2x)}+3a^2e^{(-4x)}+a^2e^{(-6x)}+a^2\right)}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*b^2*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x) + 2*a
+ b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2/3*(6*(a - b)*e^(-2*x)
- 3*b*e^(-4*x) + 2*a - 3*b)/(3*a^2*e^(-2*x) + 3*a^2*e^(-4*x) + a^2*e^(-6*x
) + a^2)
```

3.31.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)^4}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.35

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx = \frac{8}{3a\left(3e^{2x}+3e^{4x}+e^{6x}+1\right)} - \frac{4}{a\left(2e^{2x}+e^{4x}+1\right)} + \frac{2b}{a^2\left(e^{2x}+1\right)} - \frac{b^2\ln\left(\frac{4b^2\left(2ab+8a^2e^{2x}+b^2e^{2x}+b^2+8abe^{2x}\right)}{a^5(a+b)} - \frac{8b^2\left(b+4ae^{2x}+2be^{2x}\right)}{a^{9/2}\sqrt{a+b}}\right)}{2a^{5/2}\sqrt{a+b}} + \frac{b^2\ln\left(\frac{8b^2\left(b+4ae^{2x}+2be^{2x}\right)}{a^{9/2}\sqrt{a+b}} + \frac{4b^2\left(2ab+8a^2e^{2x}+b^2e^{2x}+b^2+8abe^{2x}\right)}{a^5(a+b)}\right)}{2a^{5/2}\sqrt{a+b}}$$

input
```
int(1/(cosh(x)^4*(a + b*cosh(x)^2)),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - 4/(a*(2*exp(2*x) + exp(
4*x) + 1)) + (2*b)/(a^2*(exp(2*x) + 1)) - (b^2*log((4*b^2*(2*a*b + 8*a^2*e
xp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b*exp(2*x)))/(a^5*(a + b)) - (8*b^2*(b
+ 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(9/2)*(a + b)^(1/2))))/(2*a^(5/2)*(a +
b)^(1/2)) + (b^2*log((8*b^2*(b + 4*a*exp(2*x) + 2*b*exp(2*x)))/(a^(9/2)*(a
 + b)^(1/2)) + (4*b^2*(2*a*b + 8*a^2*exp(2*x) + b^2*exp(2*x) + b^2 + 8*a*b
*exp(2*x)))/(a^5*(a + b))))/(2*a^(5/2)*(a + b)^(1/2))
```

3.31. $\int \frac{\operatorname{sech}^4(x)}{a+b\cosh^2(x)}\,dx$

3.32 $\int \frac{\text{sech}^5(x)}{a+b\cosh^2(x)}\,dx$

3.32.1 Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \frac{\text{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \frac{(3a^2-4ab+8b^2)\arctan(\sinh(x))}{8a^3} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}} + \frac{(3a-4b)\text{sech}(x)\tanh(x)}{8a^2} + \frac{\text{sech}^3(x)\tanh(x)}{4a}$$

output
```
1/8*(3*a^2-4*a*b+8*b^2)*arctan(sinh(x))/a^3-b^(5/2)*arctan(sinh(x)*b^(1/2)
/(a+b)^(1/2))/a^3/(a+b)^(1/2)+1/8*(3*a-4*b)*sech(x)*tanh(x)/a^2+1/4*sech(x
)^3*tanh(x)/a
```

3.32.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{\text{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \frac{\frac{8b^{5/2}\arctan\left(\frac{\sqrt{a+b}\,\text{csch}(x)}{\sqrt{b}}\right)}{\sqrt{a+b}} + 2(3a^2-4ab+8b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + a(3a-4b)\text{sech}(x)\tanh(x) + 2a^2\text{sech}^3(x}{8a^3}$$

input
```
Integrate[Sech[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
((8*b^(5/2)*ArcTan[(Sqrt[a + b]*Csch[x])/Sqrt[b]])/Sqrt[a + b] + 2*(3*a^2
- 4*a*b + 8*b^2)*ArcTan[Tanh[x/2]] + a*(3*a - 4*b)*Sech[x]*Tanh[x] + 2*a^2
*Sech[x]^3*Tanh[x])/(8*a^3)
```

3.32.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3665, 316, 25, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^5(x)}{a+b\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^5\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)}dx$$

$$\Big\downarrow \text{3665}$$

$$\int \frac{1}{\left(\sinh^2(x)+1\right)^3\left(a+b\sinh^2(x)+b\right)}d\sinh(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}-\frac{\int -\frac{3b\sinh^2(x)+3a-b}{\left(\sinh^2(x)+1\right)^2\left(b\sinh^2(x)+a+b\right)}d\sinh(x)}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b\sinh^2(x)+3a-b}{\left(\sinh^2(x)+1\right)^2\left(b\sinh^2(x)+a+b\right)}d\sinh(x)}{4a}+\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{(3a-4b)\sinh(x)}{2a\left(\sinh^2(x)+1\right)}-\frac{\int -\frac{3a^2-ba+4b^2+(3a-4b)b\sinh^2(x)}{\left(\sinh^2(x)+1\right)\left(b\sinh^2(x)+a+b\right)}d\sinh(x)}{2a}}{4a}+\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3a^2-ba+4b^2+(3a-4b)b\sinh^2(x)}{\left(\sinh^2(x)+1\right)\left(b\sinh^2(x)+a+b\right)}d\sinh(x)}{2a}+\frac{(3a-4b)\sinh(x)}{2a(\sinh^2(x)+1)}}{4a}+\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\int \frac{1}{\sinh^2(x)+1}d\sinh(x)}{a}-\frac{8b^3\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{a}}{2a}+\frac{(3a-4b)\sinh(x)}{2a(\sinh^2(x)+1)}}{4a}+\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\arctan(\sinh(x))}{a}-\frac{8b^3\int \frac{1}{b\sinh^2(x)+a+b}d\sinh(x)}{a}}{2a}+\frac{(3a-4b)\sinh(x)}{2a(\sinh^2(x)+1)}}{4a}+\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\left(3a^2-4ab+8b^2\right)\arctan(\sinh(x))}{a}-\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\sinh(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}+\frac{(3a-4b)\sinh(x)}{2a(\sinh^2(x)+1)}}{4a}+\frac{\sinh(x)}{4a\left(\sinh^2(x)+1\right)^2}$$

input
```
Int[Sech[x]^5/(a + b*Cosh[x]^2),x]
```

output
```
Sinh[x]/(4*a*(1 + Sinh[x]^2)^2) + ((((3*a^2 - 4*a*b + 8*b^2)*ArcTan[Sinh[x
]])/a - (8*b^(5/2)*ArcTan[(Sqrt[b]*Sinh[x])/Sqrt[a + b]])/(a*Sqrt[a + b]))
/(2*a) + ((3*a - 4*b)*Sinh[x])/(2*a*(1 + Sinh[x]^2)))/(4*a)
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.32. $\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx$

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.32. $\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx$

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(76) = 152$.

Time = 1.74 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

method	result
default	$\frac{\frac{2\left(\left(-\frac{5}{8}a^2+\frac{1}{2}ab\right)\tanh\left(\frac{x}{2}\right)^7+\left(\frac{3}{8}a^2+\frac{1}{2}ab\right)\tanh\left(\frac{x}{2}\right)^5+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tanh\left(\frac{x}{2}\right)^3+\left(\frac{5}{8}a^2-\frac{1}{2}ab\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^4}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}}{a^3}$
risch	$\frac{e^x\left(3a\,e^{6x}-4b\,e^{6x}+11a\,e^{4x}-4b\,e^{4x}-11a\,e^{2x}+4b\,e^{2x}-3a+4b\right)}{4\left(1+e^{2x}\right)^4a^2}+\frac{3i\ln(e^x+i)}{8a}-\frac{ib\ln(e^x+i)}{2a^2}+\frac{i\ln(e^x+i)b^2}{a^3}-\frac{3i\ln(e^x-i)}{8a}+ib$

input
```
int(sech(x)^5/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/a^3*(((-5/8*a^2+1/2*a*b)*tanh(1/2*x)^7+(3/8*a^2+1/2*a*b)*tanh(1/2*x)^5+(
-3/8*a^2-1/2*a*b)*tanh(1/2*x)^3+(5/8*a^2-1/2*a*b)*tanh(1/2*x))/(1+tanh(1/2
*x)^2)^4+1/8*(3*a^2-4*a*b+8*b^2)*arctan(tanh(1/2*x)))-2*b^3/a^3*(1/2/(a+b)
^(1/2)/b^(1/2)*arctan(1/2*(2*tanh(1/2*x)*(a+b)^(1/2)+2*a^(1/2))/b^(1/2))+1
/2/(a+b)^(1/2)/b^(1/2)*arctan(1/2*(2*tanh(1/2*x)*(a+b)^(1/2)-2*a^(1/2))/b^
(1/2)))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1673 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 3239, normalized size of antiderivative = 35.99

$$\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^5/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
[1/4*((3*a^2 - 4*a*b)*cosh(x)^7 + 7*(3*a^2 - 4*a*b)*cosh(x)*sinh(x)^6 + (3
*a^2 - 4*a*b)*sinh(x)^7 + (11*a^2 - 4*a*b)*cosh(x)^5 + (21*(3*a^2 - 4*a*b)
*cosh(x)^2 + 11*a^2 - 4*a*b)*sinh(x)^5 + 5*(7*(3*a^2 - 4*a*b)*cosh(x)^3 +
(11*a^2 - 4*a*b)*cosh(x))*sinh(x)^4 - (11*a^2 - 4*a*b)*cosh(x)^3 + (35*(3*
a^2 - 4*a*b)*cosh(x)^4 + 10*(11*a^2 - 4*a*b)*cosh(x)^2 - 11*a^2 + 4*a*b)*s
inh(x)^3 + (21*(3*a^2 - 4*a*b)*cosh(x)^5 + 10*(11*a^2 - 4*a*b)*cosh(x)^3 -
 3*(11*a^2 - 4*a*b)*cosh(x))*sinh(x)^2 + 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*
sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*si
nh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5
+ 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x
)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4
*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 +
b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*
sinh(x))*sqrt(-b/(a + b))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sin
h(x)^4 - 2*(2*a + 3*b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 - 2*a - 3*b)*sinh(x)^2
 + 4*(b*cosh(x)^3 - (2*a + 3*b)*cosh(x))*sinh(x) - 4*((a + b)*cosh(x)^3 +
3*(a + b)*cosh(x)*sinh(x)^2 + (a + b)*sinh(x)^3 - (a + b)*cosh(x) + (3*(a
+ b)*cosh(x)^2 - a - b)*sinh(x))*sqrt(-b/(a + b)) + b)/(b*cosh(x)^4 + 4*b*
cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2
 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(x) + b...
```

3.32.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(sech(x)**5/(a+b*cosh(x)**2),x)
```

output
```
Integral(sech(x)**5/(a + b*cosh(x)**2), x)
```

3.32.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)^5}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)^5/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
1/4*((3*a - 4*b)*e^(7*x) + (11*a - 4*b)*e^(5*x) - (11*a - 4*b)*e^(3*x) - (
3*a - 4*b)*e^x)/(a^2*e^(8*x) + 4*a^2*e^(6*x) + 6*a^2*e^(4*x) + 4*a^2*e^(2*
x) + a^2) + 1/4*(3*a^2 - 4*a*b + 8*b^2)*arctan(e^x)/a^3 - 32*integrate(1/1
6*(b^3*e^(3*x) + b^3*e^x)/(a^3*b*e^(4*x) + a^3*b + 2*(2*a^4 + a^3*b)*e^(2*
x)), x)
```

3.32.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \int \frac{\operatorname{sech}(x)^5}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(sech(x)^5/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 43.63 (sec) , antiderivative size = 1305, normalized size of antiderivative = 14.50

$$\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(cosh(x)^5*(a + b*cosh(x)^2)),x)
```

output
```
(atan((exp(x)*(243*a^12*(a^6)^(1/2) + 5024*b^6*(a^6)^(3/2) + 18432*b^12*(a
^6)^(1/2) + 6912*a^2*b^10*(a^6)^(1/2) + 30720*a^3*b^9*(a^6)^(1/2) - 26880*
a^4*b^8*(a^6)^(1/2) + 24192*a^5*b^7*(a^6)^(1/2) - 13408*a^7*b^5*(a^6)^(1/2
) + 17160*a^8*b^4*(a^6)^(1/2) - 9540*a^9*b^3*(a^6)^(1/2) + 4563*a^10*b^2*(
a^6)^(1/2) - 9216*a*b^11*(a^6)^(1/2) - 1134*a^11*b*(a^6)^(1/2)))/(81*a^13*
(9*a^4 - 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) - 270*a^12*b*(9*
a^4 - 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 2304*a^3*b^10*(9*
a^4 - 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 3840*a^6*b^7*(9*a
^4 - 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1440*a^7*b^6*(9*a^
4 - 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 864*a^8*b^5*(9*a^4
- 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1600*a^9*b^4*(9*a^4 -
 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1200*a^10*b^3*(9*a^4 -
 24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2) + 945*a^11*b^2*(9*a^4 -
24*a^3*b - 64*a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2)))*(9*a^4 - 24*a^3*b - 64*
a*b^3 + 64*b^4 + 64*a^2*b^2)^(1/2))/(4*(a^6)^(1/2)) - (6*exp(x))/(a*(3*exp
(2*x) + 3*exp(4*x) + exp(6*x) + 1)) + ((b^5)^(1/2)*(2*atan((exp(x)*((2*(48
*b^8*(a^6*b + a^7)^(1/2) + 40*a^3*b^5*(a^6*b + a^7)^(1/2) - 15*a^4*b^4*(a^
6*b + a^7)^(1/2) + 9*a^5*b^3*(a^6*b + a^7)^(1/2)))/(a^11*b*(a + b)*(a*b +
a^2)*(a^6*b + a^7)^(1/2)*(b^5)^(1/2)*(48*a*b^5 - 6*a^5*b + 9*a^6 + 48*b^6
+ 40*a^3*b^3 + 25*a^4*b^2)) - (4*(96*a^4*(b^5)^(3/2) + 18*a^9*(b^5)^(1/...
```

3.32. $\int \frac{\operatorname{sech}^5(x)}{a+b\cosh^2(x)}\,dx$

3.33 $\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx$

3.33.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx=\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\cosh(x)\sinh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

output
```
1/2*(2*a+b)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)-1/2*b
*cosh(x)*sinh(x)/a/(a+b)/(a+b*cosh(x)^2)
```

3.33.2 Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx=\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sinh(2x)}{2a(a+b)(2a+b+b\cosh(2x))}$$

input
```
Integrate[(a + b*Cosh[x]^2)^(-2),x]
```

output
```
((2*a + b)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^(3/2
)) - (b*Sinh[2*x])/(2*a*(a + b)*(2*a + b + b*Cosh[2*x]))
```

3.33.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3663, 25, 27, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)^2}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{2a+b}{b\cosh^2(x)+a}dx}{2a(a+b)}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a+b}{b\cosh^2(x)+a}dx}{2a(a+b)}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2a+b)\int \frac{1}{b\cosh^2(x)+a}dx}{2a(a+b)}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}+\frac{(2a+b)\int \frac{1}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{(2a+b)\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{2a(a+b)}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}$$

input
```
Int[(a + b*Cosh[x]^2)^(-2),x]
```

output
```
((2*a + b)*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2
)) - (b*Cosh[x]*Sinh[x])/(2*a*(a + b)*(a + b*Cosh[x]^2))
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(53) = 106$.

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.63

method	result
default	$-\frac{2\left(\frac{b\tanh\left(\frac{x}{2}\right)^{3}}{2a(a+b)}+\frac{b\tanh\left(\frac{x}{2}\right)}{2a(a+b)}\right)}{\tanh\left(\frac{x}{2}\right)^{4}a+\tanh\left(\frac{x}{2}\right)^{4}b-2\tanh\left(\frac{x}{2}\right)^{2}a+2\tanh\left(\frac{x}{2}\right)^{2}b+a+b}-\frac{(2a+b)\left(-\frac{\ln\left(\sqrt{a+b}\,\tanh\left(\frac{x}{2}\right)^{2}+2\tanh\left(\frac{x}{2}\right)\sqrt{a}+\sqrt{a+b}\right)}{4\sqrt{a}\,\sqrt{a+b}}+\frac{\ln\left(-\sqrt{a+b}\,\ldots\right.}{}\right.}{a(a+b)}$
risch	$\frac{2a\,e^{2x}+b\,e^{2x}+b}{a(a+b)(b\,e^{4x}+4a\,e^{2x}+2b\,e^{2x}+b)}+\frac{\ln\left(e^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)}{2\sqrt{a^{2}+ab}\,(a+b)}+\frac{\ln\left(e^{2x}+\frac{2a\sqrt{a^{2}+ab}+b\sqrt{a^{2}+ab}-2a^{2}-2ab}{b\sqrt{a^{2}+ab}}\right)b}{4\sqrt{a^{2}+ab}\,(a+b)a}-\frac{\ln(\ldots}{}$

input
```
int(1/(a+b*cosh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(1/2*b/a/(a+b)*tanh(1/2*x)^3+1/2*b/a/(a+b)*tanh(1/2*x))/(tanh(1/2*x)^4*
a+tanh(1/2*x)^4*b-2*tanh(1/2*x)^2*a+2*tanh(1/2*x)^2*b+a+b)-(2*a+b)/a/(a+b)
*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*tanh(1/2*x)*a^(1
/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tanh(1/2*x)^2+2*t
anh(1/2*x)*a^(1/2)-(a+b)^(1/2)))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 1239, normalized size of antiderivative = 19.06

$$\int \frac{1}{\left(a+b\cosh^{2}(x)\right)^{2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*b + 4*a*b^2 + 4*(2*a^3 + 3*a^2*b + a*b^2)*cosh(x)^2 + 8*(2*a^3
 + 3*a^2*b + a*b^2)*cosh(x)*sinh(x) + 4*(2*a^3 + 3*a^2*b + a*b^2)*sinh(x)^
2 + ((2*a*b + b^2)*cosh(x)^4 + 4*(2*a*b + b^2)*cosh(x)*sinh(x)^3 + (2*a*b
+ b^2)*sinh(x)^4 + 2*(4*a^2 + 4*a*b + b^2)*cosh(x)^2 + 2*(3*(2*a*b + b^2)*
cosh(x)^2 + 4*a^2 + 4*a*b + b^2)*sinh(x)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2
)*cosh(x)^3 + (4*a^2 + 4*a*b + b^2)*cosh(x))*sinh(x))*sqrt(a^2 + a*b)*log(
(b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a*b + b^2)
*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + 2*a*b + b^2)*sinh(x)^2 + 8*a^2 + 8*a*b +
 b^2 + 4*(b^2*cosh(x)^3 + (2*a*b + b^2)*cosh(x))*sinh(x) - 4*(b*cosh(x)^2
+ 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*a + b)*sqrt(a^2 + a*b))/(b*cosh(x)
^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(2*a + b)*cosh(x)^2 + 2*(3*b*
cosh(x)^2 + 2*a + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + (2*a + b)*cosh(x))*sinh(
x) + b)))/(a^4*b + 2*a^3*b^2 + a^2*b^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cos
h(x)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^4*b + 2*a^
3*b^2 + a^2*b^3)*sinh(x)^4 + 2*(2*a^5 + 5*a^4*b + 4*a^3*b^2 + a^2*b^3)*cos
h(x)^2 + 2*(2*a^5 + 5*a^4*b + 4*a^3*b^2 + a^2*b^3 + 3*(a^4*b + 2*a^3*b^2 +
 a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*cosh(x)^
3 + (2*a^5 + 5*a^4*b + 4*a^3*b^2 + a^2*b^3)*cosh(x))*sinh(x)), 1/2*(2*a^2*
b + 2*a*b^2 + 2*(2*a^3 + 3*a^2*b + a*b^2)*cosh(x)^2 + 4*(2*a^3 + 3*a^2*b +
 a*b^2)*cosh(x)*sinh(x) + 2*(2*a^3 + 3*a^2*b + a*b^2)*sinh(x)^2 + ((2*a...
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)**2)**2,x)
```

output
```
Timed out
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(53) = 106.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx = -\frac{(2\,a+b)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{4\,\sqrt{(a+b)a}(a^2+ab)} - \frac{(2\,a+b)e^{(-2\,x)}+b}{a^2b+ab^2+2\,(2\,a^3+3\,a^2b+ab^2)e^{(-2\,x)}+(a^2b+ab^2)e^{(-4\,x)}}$$

input
```
integrate(1/(a+b*cosh(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*a + b)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a))/(b*e^(-2*x)
+ 2*a + b + 2*sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)) - ((2*a + b)
*e^(-2*x) + b)/(a^2*b + a*b^2 + 2*(2*a^3 + 3*a^2*b + a*b^2)*e^(-2*x) + (a^
2*b + a*b^2)*e^(-4*x))
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx = \frac{(2\,a+b)\arctan\left(\frac{be^{(2\,x)}+2\,a+b}{2\,\sqrt{-a^2-ab}}\right)}{2\,(a^2+ab)\sqrt{-a^2-ab}} + \frac{2\,ae^{(2\,x)}+be^{(2\,x)}+b}{(a^2+ab)(be^{(4\,x)}+4\,ae^{(2\,x)}+2\,be^{(2\,x)}+b)}$$

input
```
integrate(1/(a+b*cosh(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*a + b)*arctan(1/2*(b*e^(2*x) + 2*a + b)/sqrt(-a^2 - a*b))/((a^2 + a
*b)*sqrt(-a^2 - a*b)) + (2*a*e^(2*x) + b*e^(2*x) + b)/((a^2 + a*b)*(b*e^(4
*x) + 4*a*e^(2*x) + 2*b*e^(2*x) + b))
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^2}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(a + b*cosh(x)^2)^2,x)
```

output
```
int(1/(a + b*cosh(x)^2)^2, x)
```

3.34 $\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 107

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx=\frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{b\cosh(x)\sinh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}-\frac{3b(2a+b)\cosh(x)\sinh(x)}{8a^2(a+b)^2\left(a+b\cosh^2(x)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctanh(a^(1/2)*tanh(x)/(a+b)^(1/2))/a^(5/2)/(a+b)
^(5/2)-1/4*b*cosh(x)*sinh(x)/a/(a+b)/(a+b*cosh(x)^2)^2-3/8*b*(2*a+b)*cosh(
x)*sinh(x)/a^2/(a+b)^2/(a+b*cosh(x)^2)
```

3.34.2 Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx=\frac{\frac{(8a^2+8ab+3b^2)\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{\sqrt{a}b(16a^2+16ab+3b^2+3b(2a+b)\cosh(2x))\sinh(2x)}{(a+b)^2(2a+b+b\cosh(2x))^2}}{8a^{5/2}}$$

input
```
Integrate[(a + b*Cosh[x]^2)^(-3),x]
```

output
```
(((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + b]])/(a + b)^
(5/2) - (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 + 3*b*(2*a + b)*Cosh[2*x])*Sin
h[2*x])/((a + b)^2*(2*a + b + b*Cosh[2*x])^2))/(8*a^(5/2))
```

3.34.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)^3}dx$$

$$\Big\downarrow \text{3663}$$

$$-\frac{\int -\frac{-2b\cosh^2(x)+4a+3b}{\left(b\cosh^2(x)+a\right)^2}dx}{4a(a+b)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2b\cosh^2(x)+4a+3b}{\left(b\cosh^2(x)+a\right)^2}dx}{4a(a+b)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}+\frac{\int \frac{-2b\sin\left(ix+\frac{\pi}{2}\right)^2+4a+3b}{\left(b\sin\left(ix+\frac{\pi}{2}\right)^2+a\right)^2}dx}{4a(a+b)}$$

$$\Big\downarrow \text{3652}$$

$$\frac{\frac{\int \frac{8a^2+8ba+3b^2}{b\cosh^2(x)+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sinh(x)\cosh(x)}{2a(a+b)\left(a+b\cosh^2(x)\right)}}{4a(a+b)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\cosh^2(x)+a}dx}{2a(a+b)}-\frac{3b(2a+b)\sinh(x)\cosh(x)}{2a(a+b)(a+b\cosh^2(x))}}{4a(a+b)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}+\frac{-\frac{3b(2a+b)\sinh(x)\cosh(x)}{2a(a+b)(a+b\cosh^2(x))}+\frac{(8a^2+8ab+3b^2)\int \frac{1}{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx}{2a(a+b)}}{4a(a+b)}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\int \frac{1}{a-(a+b)\coth^2(x)}d\coth(x)}{2a(a+b)}-\frac{3b(2a+b)\sinh(x)\cosh(x)}{2a(a+b)(a+b\cosh^2(x))}}{4a(a+b)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(8a^2+8ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\coth(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{3b(2a+b)\sinh(x)\cosh(x)}{2a(a+b)(a+b\cosh^2(x))}}{4a(a+b)}-\frac{b\sinh(x)\cosh(x)}{4a(a+b)\left(a+b\cosh^2(x)\right)^2}$$

input
```
Int[(a + b*Cosh[x]^2)^(-3),x]
```

output
```
-1/4*(b*Cosh[x]*Sinh[x])/(a*(a + b)*(a + b*Cosh[x]^2)^2) + (((8*a^2 + 8*a*
b + 3*b^2)*ArcTanh[(Sqrt[a + b]*Coth[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2
)) - (3*b*(2*a + b)*Cosh[x]*Sinh[x])/(2*a*(a + b)*(a + b*Cosh[x]^2)))/(4*a
*(a + b))
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(93) = 186$.

Time = 0.85 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.47

method	result
default	$-\frac{2\left(\frac{b(8a+3b)\tanh\left(\frac{x}{2}\right)^7}{8a^2(a+b)}-\frac{b\left(8a^2-13ab-9b^2\right)\tanh\left(\frac{x}{2}\right)^5}{8(a+b)^2a^2}-\frac{b\left(8a^2-13ab-9b^2\right)\tanh\left(\frac{x}{2}\right)^3}{8(a+b)^2a^2}+\frac{b(8a+3b)\tanh\left(\frac{x}{2}\right)}{8a^2(a+b)}\right)}{\left(\tanh\left(\frac{x}{2}\right)^4a+\tanh\left(\frac{x}{2}\right)^4b-2\tanh\left(\frac{x}{2}\right)^2a+2\tanh\left(\frac{x}{2}\right)^2b+a+b\right)^2}-\frac{(8a^2+8ab+3b^2)\left(-\frac{\ln(\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{8e^{6x}a^2b+8e^{6x}ab^2+3b^3e^{6x}+48a^3e^{4x}+72a^2be^{4x}+42ab^2e^{4x}+9b^3e^{4x}+40e^{2x}a^2b+40e^{2x}ab^2+9b^3e^{2x}+6ab^2+3b^3}{4a^2(a+b)^2(be^{4x}+4ae^{2x}+2be^{2x}+b)^2}+\frac{\ln\left(e^{2x}+\frac{2a\sqrt{a^2\cdots}}{\cdots}\right.}{2\sqrt{\cdots}}$

input
```
int(1/(a+b*cosh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(1/8*b*(8*a+3*b)/a^2/(a+b)*tanh(1/2*x)^7-1/8*b*(8*a^2-13*a*b-9*b^2)/(a+
b)^2/a^2*tanh(1/2*x)^5-1/8*b*(8*a^2-13*a*b-9*b^2)/(a+b)^2/a^2*tanh(1/2*x)^
3+1/8*b*(8*a+3*b)/a^2/(a+b)*tanh(1/2*x))/(tanh(1/2*x)^4*a+tanh(1/2*x)^4*b-
2*tanh(1/2*x)^2*a+2*tanh(1/2*x)^2*b+a+b)^2-1/4*(8*a^2+8*a*b+3*b^2)/a^2/(a^
2+2*a*b+b^2)*(-1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1/2*x)^2+2*tanh
(1/2*x)*a^(1/2)+(a+b)^(1/2))+1/4/a^(1/2)/(a+b)^(1/2)*ln((a+b)^(1/2)*tanh(1
/2*x)^2-2*tanh(1/2*x)*a^(1/2)+(a+b)^(1/2)))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2478 vs. $2(93) = 186$.

Time = 0.33 (sec) , antiderivative size = 5117, normalized size of antiderivative = 47.82

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.34.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)**2)**3,x)
```

output
```
Timed out
```

3.34.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(93) = 186.

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.21

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx = -\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\log\left(\frac{be^{(-2\,x)}+2\,a+b-2\,\sqrt{(a+b)a}}{be^{(-2\,x)}+2\,a+b+2\,\sqrt{(a+b)a}}\right)}{16\,\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{(a+b)a}} - \frac{6\,ab^2+3\,b^3+\left(40\,a^2b+40\,ab^2+9\,b^3\right)e^{(-2\,x)}+3\,\left(16\,a^3+24\,a^2b+\right.}{4\,\left(a^4b^2+2\,a^3b^3+a^2b^4+4\,\left(2\,a^5b+5\,a^4b^2+4\,a^3b^3+a^2b^4\right)e^{(-2\,x)}+2\,\left(8\,a^6+24\,a^5b+27\,a^4b^2+14\,a^3b^3\right.\right.}$$

input
```
integrate(1/(a+b*cosh(x)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(8*a^2 + 8*a*b + 3*b^2)*log((b*e^(-2*x) + 2*a + b - 2*sqrt((a + b)*a
))/(b*e^(-2*x) + 2*a + b + 2*sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*
sqrt((a + b)*a)) - 1/4*(6*a*b^2 + 3*b^3 + (40*a^2*b + 40*a*b^2 + 9*b^3)*e^
(-2*x) + 3*(16*a^3 + 24*a^2*b + 14*a*b^2 + 3*b^3)*e^(-4*x) + (8*a^2*b + 8*
a*b^2 + 3*b^3)*e^(-6*x))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(2*a^5*b + 5*a
^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-2*x) + 2*(8*a^6 + 24*a^5*b + 27*a^4*b^2
+ 14*a^3*b^3 + 3*a^2*b^4)*e^(-4*x) + 4*(2*a^5*b + 5*a^4*b^2 + 4*a^3*b^3 +
a^2*b^4)*e^(-6*x) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*x))
```

3.34.8 Giac [F]

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx = \int \frac{1}{\left(b\cosh\left(x\right)^2+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\cosh^2(x)\right)^3}\,dx=\int \frac{1}{\left(b\cosh\left(x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(a + b*cosh(x)^2)^3,x)
```

output
```
int(1/(a + b*cosh(x)^2)^3, x)
```

3.35 $\int \frac{1}{1+\cosh^2(x)}\,dx$

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \frac{1}{1+\cosh^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\cosh^2(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[Tanh[x]/Sqrt[2]]/Sqrt[2]
```

3.35.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1+\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$\int \frac{1}{1-2\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{\sqrt{2}}$$

input
```
Int[(1 + Cosh[x]^2)^(-1),x]
```

output
```
ArcTanh[Sqrt[2]*Coth[x]]/Sqrt[2]
```

3.35.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

method	result
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3-2\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}-1\right)\right)}{8}-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right.}{}$

input
```
int(1/(1+cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-1/4*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2))
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 4.40

$$\int \frac{1}{1+\cosh^2(x)}\,dx$$
$$=\frac{1}{4}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh\left(x\right)^{2}-4\left(3\sqrt{2}-4\right)\cosh\left(x\right)\sinh\left(x\right)+3\left(2\sqrt{2}-3\right)\sinh\left(x\right)^{2}+2\sqrt{2}-3}{\cosh\left(x\right)^{2}+\sinh\left(x\right)^{2}+3}\right)$$

input
```
integrate(1/(1+cosh(x)^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*
sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x
)^2 + 3))
```

3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 4.00

$$\int \frac{1}{1+\cosh^2(x)}\,dx = -\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{4} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{4}$$

input
```
integrate(1/(1+cosh(x)**2),x)
```

output
```
-sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/4 + sqrt(2)*log(4*t
anh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/4
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{1}{1+\cosh^2(x)}\,dx = -\frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)$$

input
```
integrate(1/(1+cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{1}{1+\cosh^2(x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)$$

input
```
integrate(1/(1+cosh(x)^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3))
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.33

$$\int \frac{1}{1+\cosh^2(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(-4\,e^{2\,x}-\frac{\sqrt{2}\,(12\,e^{2\,x}+4)}{4}\right)-\ln\left(\frac{\sqrt{2}\,(12\,e^{2\,x}+4)}{4}-4\,e^{2\,x}\right)\right)}{4}$$

input
```
int(1/(cosh(x)^2 + 1),x)
```

output
```
(2^(1/2)*(log(- 4*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/4) - log((2^(1/2)
*(12*exp(2*x) + 4))/4 - 4*exp(2*x))))/4
```

3.36 $\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx$

3.36.1 Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx=\frac{3\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\cosh(x)\sinh(x)}{4\left(1+\cosh^2(x)\right)}$$

output
```
-1/4*cosh(x)*sinh(x)/(1+cosh(x)^2)+3/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2
)
```

3.36.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx=\frac{3\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\sinh(2x)}{4(3+\cosh(2x))}$$

input
```
Integrate[(1 + Cosh[x]^2)^(-2),x]
```

output
```
(3*ArcTanh[Tanh[x]/Sqrt[2]])/(4*Sqrt[2]) - Sinh[2*x]/(4*(3 + Cosh[2*x]))
```

3.36.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3663, 27, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh^2(x)+1)^2} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)^2\right)^2}dx$$

$$\Big\downarrow \ 3663$$

$$-\frac{1}{4}\int -\frac{3}{\cosh^2(x)+1}dx-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{4}\int \frac{1}{\cosh^2(x)+1}dx-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}+\frac{3}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \ 3660$$

$$\frac{3}{4}\int \frac{1}{1-2\coth^2(x)}d\coth(x)-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{3\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{4\sqrt{2}}-\frac{\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}$$

input
```
Int[(1 + Cosh[x]^2)^(-2),x]
```

output
```
(3*ArcTanh[Sqrt[2]*Coth[x]])/(4*Sqrt[2]) - (Cosh[x]*Sinh[x])/(4*(1 + Cosh[
x]^2))
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.71

method	result
risch	$\frac{3\,e^{2x}+1}{2\,e^{4x}+12\,e^{2x}+2}+\frac{3\sqrt{2}\,\ln\left(e^{2x}+3-2\sqrt{2}\right)}{16}-\frac{3\sqrt{2}\,\ln\left(e^{2x}+3+2\sqrt{2}\right)}{16}$
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}}{2}+\frac{\tanh\left(\frac{x}{2}\right)}{2}}{2\left(\tanh\left(\frac{x}{2}\right)^{4}+1\right)}+\frac{3\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}-1\right)\right)}{32}-3\sqrt{2}$

input
```
int(1/(1+cosh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(3*exp(2*x)+1)/(exp(4*x)+6*exp(2*x)+1)+3/16*2^(1/2)*ln(exp(2*x)+3-2*2^
(1/2))-3/16*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 6.11

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx$$
$$= \frac{24\cosh(x)^2 + 3\left(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(x)^4 + 6\left(\sqrt{2}\cosh(x)^2 + \sqrt{2}\right)\sinh(x)^2 \cdots}{16\left(\cosh(x)^4 + 4\cosh(x)\sin \cdots\right.}$$

input
```
integrate(1/(1+cosh(x)^2)^2,x, algorithm="fricas")
```

output
```
1/16*(24*cosh(x)^2 + 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 +
sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 6*sqrt(2)*
cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*l
og(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(
2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^2 + 3)) + 4
8*cosh(x)*sinh(x) + 24*sinh(x)^2 + 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + s
inh(x)^4 + 6*(cosh(x)^2 + 1)*sinh(x)^2 + 6*cosh(x)^2 + 4*(cosh(x)^3 + 3*co
sh(x))*sinh(x) + 1)
```

3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(34) = 68$.

Time = 1.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 6.03

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx = -\frac{3\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^4\left(\frac{x}{2}\right)}{16\tanh^4\left(\frac{x}{2}\right)+16} - \frac{3\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{16\tanh^4\left(\frac{x}{2}\right)+16} + \frac{3\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^4\left(\frac{x}{2}\right)}{16\tanh^4\left(\frac{x}{2}\right)+16} + \frac{3\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{16\tanh^4\left(\frac{x}{2}\right)+16} - \frac{4\tanh^3\left(\frac{x}{2}\right)}{16\tanh^4\left(\frac{x}{2}\right)+16} - \frac{4\tanh\left(\frac{x}{2}\right)}{16\tanh^4\left(\frac{x}{2}\right)+16}$$

input
```
integrate(1/(1+cosh(x)**2)**2,x)
```

output
```
-3*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**4/(16*
tanh(x/2)**4 + 16) - 3*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) +
4)/(16*tanh(x/2)**4 + 16) + 3*sqrt(2)*log(4*tanh(x/2)**2 + 4*sqrt(2)*tanh(
x/2) + 4)*tanh(x/2)**4/(16*tanh(x/2)**4 + 16) + 3*sqrt(2)*log(4*tanh(x/2)*
*2 + 4*sqrt(2)*tanh(x/2) + 4)/(16*tanh(x/2)**4 + 16) - 4*tanh(x/2)**3/(16*
tanh(x/2)**4 + 16) - 4*tanh(x/2)/(16*tanh(x/2)**4 + 16)
```

3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx = -\frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right) - \frac{3e^{(-2x)}+1}{2\left(6e^{(-2x)}+e^{(-4x)}+1\right)}$$

input
```
integrate(1/(1+cosh(x)^2)^2,x, algorithm="maxima")
```

output
```
-3/16*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
- 1/2*(3*e^(-2*x) + 1)/(6*e^(-2*x) + e^(-4*x) + 1)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx = \frac{3}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+\frac{3e^{(2x)}+1}{2\left(e^{(4x)}+6e^{(2x)}+1\right)}$$

input
```
integrate(1/(1+cosh(x)^2)^2,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1
/2*(3*e^(2*x) + 1)/(e^(4*x) + 6*e^(2*x) + 1)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.17

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^2}\,dx = \frac{3\sqrt{2}\,\ln\left(-3\,e^{2x}-\frac{3\sqrt{2}\,\left(12\,e^{2x}+4\right)}{16}\right)}{16} - \frac{3\sqrt{2}\,\ln\left(\frac{3\sqrt{2}\,\left(12\,e^{2x}+4\right)}{16}-3\,e^{2x}\right)}{16}+\frac{\frac{3\,e^{2x}}{2}+\frac{1}{2}}{6\,e^{2x}+e^{4x}+1}$$

input
```
int(1/(cosh(x)^2 + 1)^2,x)
```

output
```
(3*2^(1/2)*log(- 3*exp(2*x) - (3*2^(1/2)*(12*exp(2*x) + 4))/16))/16 - (3*2
^(1/2)*log((3*2^(1/2)*(12*exp(2*x) + 4))/16 - 3*exp(2*x)))/16 + ((3*exp(2*
x))/2 + 1/2)/(6*exp(2*x) + exp(4*x) + 1)
```

3.37 $\int \frac{1}{(1+\cosh^2(x))^3} \, dx$

3.37.1 Optimal result

Integrand size = 8, antiderivative size = 51

$$\int \frac{1}{(1+\cosh^2(x))^3} \, dx = \frac{19\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{32\sqrt{2}} - \frac{\cosh(x)\sinh(x)}{8\left(1+\cosh^2(x)\right)^2} - \frac{9\cosh(x)\sinh(x)}{32\left(1+\cosh^2(x)\right)}$$

output
```
-1/8*cosh(x)*sinh(x)/(1+cosh(x)^2)^2-9/32*cosh(x)*sinh(x)/(1+cosh(x)^2)+19
/64*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+\cosh^2(x))^3} \, dx = \frac{19\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{32\sqrt{2}} - \frac{\sinh(2x)}{4(3+\cosh(2x))^2} - \frac{9\sinh(2x)}{32(3+\cosh(2x))}$$

input
```
Integrate[(1 + Cosh[x]^2)^(-3),x]
```

output
```
(19*ArcTanh[Tanh[x]/Sqrt[2]])/(32*Sqrt[2]) - Sinh[2*x]/(4*(3 + Cosh[2*x])^
2) - (9*Sinh[2*x])/(32*(3 + Cosh[2*x]))
```

3.37.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3663, 25, 3042, 3652, 27, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cosh^2(x)+1\right)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)^2\right)^3}dx$$

$$\Big\downarrow \text{ 3663}$$

$$-\frac{1}{8}\int -\frac{7-2\cosh^2(x)}{\left(\cosh^2(x)+1\right)^2}dx-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{8}\int \frac{7-2\cosh^2(x)}{\left(\cosh^2(x)+1\right)^2}dx-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}+\frac{1}{8}\int \frac{7-2\sin\left(ix+\frac{\pi}{2}\right)^2}{\left(\sin\left(ix+\frac{\pi}{2}\right)^2+1\right)^2}dx$$

$$\Big\downarrow \text{ 3652}$$

$$\frac{1}{8}\left(\frac{1}{4}\int \frac{19}{\cosh^2(x)+1}dx-\frac{9\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}\right)-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\left(\frac{19}{4}\int \frac{1}{\cosh^2(x)+1}dx-\frac{9\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}\right)-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}+\frac{1}{8}\left(-\frac{9\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}+\frac{19}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx\right)$$

$$\Big\downarrow \text{ 3660}$$

$$\frac{1}{8}\left(\frac{19}{4}\int \frac{1}{1-2\coth^2(x)}d\coth(x)-\frac{9\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}\right)-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\left(\frac{19\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{4\sqrt{2}}-\frac{9\sinh(x)\cosh(x)}{4\left(\cosh^2(x)+1\right)}\right)-\frac{\sinh(x)\cosh(x)}{8\left(\cosh^2(x)+1\right)^2}$$

input
```
Int[(1 + Cosh[x]^2)^(-3),x]
```

output
```
-1/8*(Cosh[x]*Sinh[x])/(1 + Cosh[x]^2)^2 + ((19*ArcTanh[Sqrt[2]*Coth[x]])/
(4*Sqrt[2]) - (9*Cosh[x]*Sinh[x])/(4*(1 + Cosh[x]^2)))/8
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3652
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Simp[1/(2*
a*(a + b)*(p + 1))   Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(
p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3663
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Simp[1/(2*a*(p + 1)*(a + b))   Int[(a + b*Sin[e + f*x]^2)^(p +
 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

3.37.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

method	result
risch	$\frac{19\,\mathrm{e}^{6x}+171\,\mathrm{e}^{4x}+89\,\mathrm{e}^{2x}+9}{16(\mathrm{e}^{4x}+6\,\mathrm{e}^{2x}+1)^2}+\frac{19\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3-2\sqrt{2}\right)}{128}-\frac{19\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{128}$
default	$-\frac{\frac{11\tanh\left(\frac{x}{2}\right)^7}{8}+\frac{7\tanh\left(\frac{x}{2}\right)^5}{8}+\frac{7\tanh\left(\frac{x}{2}\right)^3}{8}+\frac{11\tanh\left(\frac{x}{2}\right)}{8}}{4\left(\tanh\left(\frac{x}{2}\right)^4+1\right)^2}+\frac{19\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\,\mathrm{arc}\right.}{256}$

input
```
int(1/(1+cosh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(19*exp(6*x)+171*exp(4*x)+89*exp(2*x)+9)/(exp(4*x)+6*exp(2*x)+1)^2+19
/128*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-19/128*2^(1/2)*ln(exp(2*x)+3+2*2^(1/
2))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(42) = 84.

Time = 0.26 (sec) , antiderivative size = 575, normalized size of antiderivative = 11.27

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+cosh(x)^2)^3,x, algorithm="fricas")
```

output
```
1/128*(152*cosh(x)^6 + 912*cosh(x)*sinh(x)^5 + 152*sinh(x)^6 + 456*(5*cosh
(x)^2 + 3)*sinh(x)^4 + 1368*cosh(x)^4 + 608*(5*cosh(x)^3 + 9*cosh(x))*sinh
(x)^3 + 8*(285*cosh(x)^4 + 1026*cosh(x)^2 + 89)*sinh(x)^2 + 712*cosh(x)^2
+ 19*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8
+ 4*(7*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^6 + 12*sqrt(2)*cosh(x)^6 + 8
*(7*sqrt(2)*cosh(x)^3 + 9*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(
x)^4 + 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4
 + 8*(7*sqrt(2)*cosh(x)^5 + 30*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sin
h(x)^3 + 4*(7*sqrt(2)*cosh(x)^6 + 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x
)^2 + 3*sqrt(2))*sinh(x)^2 + 12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 +
 9*sqrt(2)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x) +
 sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*si
nh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^
2 + 3)) + 16*(57*cosh(x)^5 + 342*cosh(x)^3 + 89*cosh(x))*sinh(x) + 72)/(co
sh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 3)*sinh(x)^6
+ 12*cosh(x)^6 + 8*(7*cosh(x)^3 + 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 +
 90*cosh(x)^2 + 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 + 30*cosh(x)
^3 + 19*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 45*cosh(x)^4 + 57*cosh(x)^2
+ 3)*sinh(x)^2 + 12*cosh(x)^2 + 8*(cosh(x)^7 + 9*cosh(x)^5 + 19*cosh(x)^3
+ 3*cosh(x))*sinh(x) + 1)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(53) = 106$.

Time = 4.10 (sec) , antiderivative size = 428, normalized size of antiderivative = 8.39

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx = -\frac{19\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^8\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} - \frac{38\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^4\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} - \frac{19\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} + \frac{19\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^8\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} + \frac{38\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)\tanh^4\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} + \frac{19\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} - \frac{44\tanh^7\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} - \frac{28\tanh^5\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} - \frac{28\tanh^3\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128} - \frac{44\tanh\left(\frac{x}{2}\right)}{128\tanh^8\left(\frac{x}{2}\right)+256\tanh^4\left(\frac{x}{2}\right)+128}$$

input
```
integrate(1/(1+cosh(x)**2)**3,x)
```

output
```
-19*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**8/(12
8*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 38*sqrt(2)*log(4*tanh(x/2)**2 -
 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**4/(128*tanh(x/2)**8 + 256*tanh(x/2)**
4 + 128) - 19*sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/(128*t
anh(x/2)**8 + 256*tanh(x/2)**4 + 128) + 19*sqrt(2)*log(4*tanh(x/2)**2 + 4*
sqrt(2)*tanh(x/2) + 4)*tanh(x/2)**8/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 +
 128) + 38*sqrt(2)*log(4*tanh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)*tanh(x/2)
**4/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) + 19*sqrt(2)*log(4*tanh(x/
2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 12
8) - 44*tanh(x/2)**7/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 28*tanh
(x/2)**5/(128*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 28*tanh(x/2)**3/(12
8*tanh(x/2)**8 + 256*tanh(x/2)**4 + 128) - 44*tanh(x/2)/(128*tanh(x/2)**8
+ 256*tanh(x/2)**4 + 128)
```

3.37. $\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx$

3.37.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx = -\frac{19}{128}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(-2\,x)}-3}{2\,\sqrt{2}+e^{(-2\,x)}+3}\right) - \frac{89\,e^{(-2\,x)}+171\,e^{(-4\,x)}+19\,e^{(-6\,x)}+9}{16\,\left(12\,e^{(-2\,x)}+38\,e^{(-4\,x)}+12\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}$$

input
```
integrate(1/(1+cosh(x)^2)^3,x, algorithm="maxima")
```

output
```
-19/128*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)
) - 1/16*(89*e^(-2*x) + 171*e^(-4*x) + 19*e^(-6*x) + 9)/(12*e^(-2*x) + 38*
e^(-4*x) + 12*e^(-6*x) + e^(-8*x) + 1)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx = \frac{19}{128}\,\sqrt{2}\log\left(-\frac{2\,\sqrt{2}-e^{(2\,x)}-3}{2\,\sqrt{2}+e^{(2\,x)}+3}\right) + \frac{19\,e^{(6\,x)}+171\,e^{(4\,x)}+89\,e^{(2\,x)}+9}{16\,\left(e^{(4\,x)}+6\,e^{(2\,x)}+1\right)^2}$$

input
```
integrate(1/(1+cosh(x)^2)^3,x, algorithm="giac")
```

output
```
19/128*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) +
 1/16*(19*e^(6*x) + 171*e^(4*x) + 89*e^(2*x) + 9)/(e^(4*x) + 6*e^(2*x) + 1
)^2
```

3.37.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.20

$$\int \frac{1}{\left(1+\cosh^2(x)\right)^3}\,dx = \frac{19\sqrt{2}\,\ln\left(-\frac{19\,\mathrm{e}^{2\,x}}{8}-\frac{19\sqrt{2}\,(12\,\mathrm{e}^{2\,x}+4)}{128}\right)}{128} - \frac{17\,\mathrm{e}^{2\,x}+3}{12\,\mathrm{e}^{2\,x}+38\,\mathrm{e}^{4\,x}+12\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1} - \frac{19\sqrt{2}\,\ln\left(\frac{19\sqrt{2}\,(12\,\mathrm{e}^{2\,x}+4)}{128}-\frac{19\,\mathrm{e}^{2\,x}}{8}\right)}{128} + \frac{\frac{19\,\mathrm{e}^{2\,x}}{16}+\frac{57}{16}}{6\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

input
```
int(1/(cosh(x)^2 + 1)^3,x)
```

output
```
(19*2^(1/2)*log(- (19*exp(2*x))/8 - (19*2^(1/2)*(12*exp(2*x) + 4))/128))/1
28 - (17*exp(2*x) + 3)/(12*exp(2*x) + 38*exp(4*x) + 12*exp(6*x) + exp(8*x)
 + 1) - (19*2^(1/2)*log((19*2^(1/2)*(12*exp(2*x) + 4))/128 - (19*exp(2*x))
/8))/128 + ((19*exp(2*x))/16 + 57/16)/(6*exp(2*x) + exp(4*x) + 1)
```

3.38 $\int \frac{1}{1-\cosh^2(x)}\,dx$

3.38.1 Optimal result

Integrand size = 10, antiderivative size = 2

$$\int \frac{1}{1-\cosh^2(x)}\,dx = \coth(x)$$

output
```
coth(x)
```

3.38.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cosh^2(x)}\,dx = \coth(x)$$

input
```
Integrate[(1 - Cosh[x]^2)^(-1),x]
```

output
```
Coth[x]
```

3.38.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3654, 25, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\int -\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\csc(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$\int \csc(ix)^2dx$$

$$\Big\downarrow \text{4254}$$

$$i\int 1d(-i\coth(x))$$

$$\Big\downarrow \text{24}$$

$$\coth(x)$$

input
```
Int[(1 - Cosh[x]^2)^(-1),x]
```

output
```
Coth[x]
```

3.38.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
parallelrisch	$\coth(x)$	3
risch	$\frac{2}{e^{2x}-1}$	11
default	$\frac{\tanh(\frac{x}{2})}{2}+\frac{1}{2\tanh(\frac{x}{2})}$	16

input
```
int(1/(1-cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
coth(x)
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(2) = 4$.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \frac{1}{1-\cosh^2(x)}\,dx = \frac{2}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}$$

input
```
integrate(1/(1-cosh(x)^2),x, algorithm="fricas")
```

output
```
2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{1-\cosh^2(x)}\,dx = \frac{\tanh\left(\frac{x}{2}\right)}{2} + \frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cosh(x)**2),x)
```

output
```
tanh(x/2)/2 + 1/(2*tanh(x/2))
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \frac{1}{1-\cosh^2(x)}\,dx = -\frac{2}{e^{(-2x)} - 1}$$

input
```
integrate(1/(1-cosh(x)^2),x, algorithm="maxima")
```

output
```
-2/(e^(-2*x) - 1)
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(2) = 4.

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \frac{1}{1-\cosh^2(x)}\,dx = \frac{2}{e^{(2x)}-1}$$

input
```
integrate(1/(1-cosh(x)^2),x, algorithm="giac")
```

output
```
2/(e^(2*x) - 1)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 5.00

$$\int \frac{1}{1-\cosh^2(x)}\,dx = \frac{2}{\mathrm{e}^{2x}-1}$$

input
```
int(-1/(cosh(x)^2 - 1),x)
```

output
```
2/(exp(2*x) - 1)
```

3.39 $\int \frac{1}{(1-\cosh^2(x))^2} dx$

3.39.1 Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{(1-\cosh^2(x))^2} dx = \coth(x) - \frac{\coth^3(x)}{3}$$

output
```
coth(x)-1/3*coth(x)^3
```

3.39.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1}{(1-\cosh^2(x))^2} dx = \frac{2\coth(x)}{3} - \frac{1}{3}\coth(x)\text{csch}^2(x)$$

input
```
Integrate[(1 - Cosh[x]^2)^(-2),x]
```

output
```
(2*Coth[x])/3 - (Coth[x]*Csch[x]^2)/3
```

3.39.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3654, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh^2(x))^2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)^2\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\int \text{csch}^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(ix)^4dx$$

$$\Big\downarrow \text{4254}$$

$$i\int (1-\coth^2(x))\, d(-i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)$$

input
```
Int[(1 - Cosh[x]^2)^(-2),x]
```

output
```
I*((-I)*Coth[x] + (I/3)*Coth[x]^3)
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.39.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
parallelrisch	$\frac{2\coth(x)^3}{3}-\coth(x)\operatorname{csch}(x)^2$	16
risch	$-\frac{4(3\,e^{2x}-1)}{3(e^{2x}-1)^3}$	19
default	$-\frac{\tanh(\frac{x}{2})^3}{24}+\frac{3\tanh(\frac{x}{2})}{8}-\frac{1}{24\tanh(\frac{x}{2})^3}+\frac{3}{8\tanh(\frac{x}{2})}$	32

input
```
int(1/(1-cosh(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*coth(x)^3-coth(x)*csch(x)^2
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 7.64

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^2}\,dx =$$

$$-\frac{8\left(\cosh\left(x\right)+2\,\sinh\left(x\right.\right.}{3\left(\cosh\left(x\right)^{5}+5\,\cosh\left(x\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+\left(10\,\cosh\left(x\right)^{2}-3\right)\sinh\left(x\right)^{3}-3\,\cosh\left(x\right)^{3}+\left(10\,\cosh\left(\right.\right.\right.}$$

input
```
integrate(1/(1-cosh(x)^2)^2,x, algorithm="fricas")
```

output
```
-8/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 +
(10*cosh(x)^2 - 3)*sinh(x)^3 - 3*cosh(x)^3 + (10*cosh(x)^3 - 9*cosh(x))*si
nh(x)^2 + (5*cosh(x)^4 - 9*cosh(x)^2 + 4)*sinh(x) + 2*cosh(x))
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^2}\,dx = -\frac{\tanh^3\left(\frac{x}{2}\right)}{24}+\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{3}{8\tanh\left(\frac{x}{2}\right)}-\frac{1}{24\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cosh(x)**2)**2,x)
```

output
```
-tanh(x/2)**3/24 + 3*tanh(x/2)/8 + 3/(8*tanh(x/2)) - 1/(24*tanh(x/2)**3)
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(9) = 18.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.45

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^2}\,dx = \frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}-\frac{4}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}$$

input
```
integrate(1/(1-cosh(x)^2)^2,x, algorithm="maxima")
```

output
```
4*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) - 4/3/(3*e^(-2*x) - 3*
e^(-4*x) + e^(-6*x) - 1)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^2}\,dx = -\frac{4\left(3\,e^{(2\,x)}-1\right)}{3\left(e^{(2\,x)}-1\right)^3}$$

input
```
integrate(1/(1-cosh(x)^2)^2,x, algorithm="giac")
```

output
```
-4/3*(3*e^(2*x) - 1)/(e^(2*x) - 1)^3
```

3.39.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^2}\,dx = -\frac{4\left(3\,\mathrm{e}^{2\,x}-1\right)}{3\left(\mathrm{e}^{2\,x}-1\right)^3}$$

input
```
int(1/(cosh(x)^2 - 1)^2,x)
```

output
```
-(4*(3*exp(2*x) - 1))/(3*(exp(2*x) - 1)^3)
```

3.40 $\int \frac{1}{\left(1-\cosh^2(x)\right)^3} dx$

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^3} dx = \coth(x) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5}$$

output
```
coth(x)-2/3*coth(x)^3+1/5*coth(x)^5
```

3.40.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^3} dx = \frac{8\coth(x)}{15} - \frac{4}{15}\coth(x)\operatorname{csch}^2(x) + \frac{1}{5}\coth(x)\operatorname{csch}^4(x)$$

input
```
Integrate[(1 - Cosh[x]^2)^(-3),x]
```

output
```
(8*Coth[x])/15 - (4*Coth[x]*Csch[x]^2)/15 + (Coth[x]*Csch[x]^4)/5
```

3.40.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3654, 25, 3042, 25, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh^2(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)^2\right)^3}dx$$

$$\Big\downarrow \text{3654}$$

$$\int -\text{csch}^6(x)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \text{csch}^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$-\int -\csc(ix)^6dx$$

$$\Big\downarrow \text{25}$$

$$\int \csc(ix)^6dx$$

$$\Big\downarrow \text{4254}$$

$$i\int \left(\coth^4(x)-2\coth^2(x)+1\right)d(-i\coth(x))$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{1}{5}i\coth^5(x)+\frac{2}{3}i\coth^3(x)-i\coth(x)\right)$$

input
```
Int[(1 - Cosh[x]^2)^(-3),x]
```

output
```
I*((-I)*Coth[x] + ((2*I)/3)*Coth[x]^3 - (I/5)*Coth[x]^5)
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.40.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{\coth(x)\operatorname{csch}(x)^4(8+\cosh(4x)-6\cosh(2x))}{15}$	21
risch	$\frac{\frac{32\,e^{4x}}{3}-\frac{16\,e^{2x}}{3}+\frac{16}{15}}{\left(e^{2x}-1\right)^5}$	25
default	$\frac{\tanh\left(\frac{x}{2}\right)^5}{160}-\frac{5\tanh\left(\frac{x}{2}\right)^3}{96}+\frac{5\tanh\left(\frac{x}{2}\right)}{16}+\frac{1}{160\tanh\left(\frac{x}{2}\right)^5}-\frac{5}{96\tanh\left(\frac{x}{2}\right)^3}+\frac{5}{16\tanh\left(\frac{x}{2}\right)}$	48

input
```
int(1/(1-cosh(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*coth(x)*csch(x)^4*(8+cosh(4*x)-6*cosh(2*x))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(15) = 30.

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 9.74

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^3}\,dx$$

$$= \frac{}{15\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + \left(28\cosh(x)^2 - 5\right)\sinh(x)^6 - 5\cosh(x)^6 + 2\left(28\cosh\right.}$$

input
```
integrate(1/(1-cosh(x)^2)^3,x, algorithm="fricas")
```

output
```
16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 - 5)/(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 5)*sinh(x)^6 - 5*cosh(x)
^6 + 2*(28*cosh(x)^3 - 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 15*cosh(x
)^2 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 - 25*cosh(x)^3 + 10*co
sh(x))*sinh(x)^3 + (28*cosh(x)^6 - 75*cosh(x)^4 + 60*cosh(x)^2 - 11)*sinh(
x)^2 - 11*cosh(x)^2 + 2*(4*cosh(x)^7 - 15*cosh(x)^5 + 20*cosh(x)^3 - 9*cos
h(x))*sinh(x) + 5)
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 1.57 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^3}\,dx = \frac{\tanh^5\left(\frac{x}{2}\right)}{160} - \frac{5\tanh^3\left(\frac{x}{2}\right)}{96} + \frac{5\tanh\left(\frac{x}{2}\right)}{16} + \frac{5}{16\tanh\left(\frac{x}{2}\right)} - \frac{5}{96\tanh^3\left(\frac{x}{2}\right)} + \frac{1}{160\tanh^5\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cosh(x)**2)**3,x)
```

output
```
tanh(x/2)**5/160 - 5*tanh(x/2)**3/96 + 5*tanh(x/2)/16 + 5/(16*tanh(x/2)) -
 5/(96*tanh(x/2)**3) + 1/(160*tanh(x/2)**5)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(15) = 30$.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 5.84

$$\int \frac{1}{(1-\cosh^2(x))^3}\,dx = \frac{16\,e^{(-2\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} - \frac{32\,e^{(-4\,x)}}{3\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} - \frac{16}{15\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)}$$

input
```
integrate(1/(1-cosh(x)^2)^3,x, algorithm="maxima")
```

output
```
16/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-1
0*x) - 1) - 32/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-
8*x) + e^(-10*x) - 1) - 16/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*
e^(-8*x) + e^(-10*x) - 1)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{(1-\cosh^2(x))^3}\,dx = \frac{16\,(10\,e^{(4\,x)} - 5\,e^{(2\,x)} + 1)}{15\,(e^{(2\,x)} - 1)^5}$$

input
```
integrate(1/(1-cosh(x)^2)^3,x, algorithm="giac")
```

output
```
16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(e^(2*x) - 1)^5
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(1-\cosh^2(x)\right)^3}\,dx = \frac{16\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\left(\mathrm{e}^{2\,x}-1\right)^5}$$

input
```
int(-1/(cosh(x)^2 - 1)^3,x)
```

output
```
(16*(10*exp(4*x) - 5*exp(2*x) + 1))/(15*(exp(2*x) - 1)^5)
```

3.41 $\int \sqrt{a+b\cosh^2(x)}\,dx$

3.41.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \sqrt{a+b\cosh^2(x)}\,dx=-\frac{i\sqrt{a+b\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-\frac{b}{a}\right)}{\sqrt{1+\frac{b\cosh^2(x)}{a}}}$$

output
```
(-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),(-b/a)^(1/2))*(a+b*cosh(x)^2)
^(1/2)/(1+b*cosh(x)^2/a)^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \sqrt{a+b\cosh^2(x)}\,dx=-\frac{i\sqrt{2a+b+b\cosh(2x)}\,E\left(ix\middle|\frac{b}{a+b}\right)}{\sqrt{\frac{2a+b+b\cosh(2x)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[2*a + b + b*Cosh[2*x]]*EllipticE[I*x, b/(a + b)])/Sqrt[(2*a + b
 + b*Cosh[2*x])/(a + b)]
```

3.41.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{a+b\cosh^2(x)}\int \sqrt{\frac{b\cosh^2(x)}{a}+1}dx}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cosh^2(x)}\int \sqrt{\frac{b\sin\left(ix+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{i\sqrt{a+b\cosh^2(x)}E\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}$$

input
```
Int[Sqrt[a + b*Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[a + b*Cosh[x]^2]*EllipticE[Pi/2 + I*x, -(b/a)])/Sqrt[1 + (b*Cos
h[x]^2)/a]
```

3.41.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(47) = 94$.

Time = 1.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

method	result
default	$\frac{\sqrt{\frac{a+b\cosh(x)^2}{a}}\,\sqrt{-\sinh(x)^2}\,\Big(a\,\mathrm{EllipticF}\Big(\cosh(x)\sqrt{-\frac{b}{a}},\sqrt{-\frac{a}{b}}\Big)+b\,\mathrm{EllipticF}\Big(\cosh(x)\sqrt{-\frac{b}{a}},\sqrt{-\frac{a}{b}}\Big)-b\,\mathrm{EllipticE}\Big(\cosh(x)\sqrt{-\frac{b}{a}},\sqrt{-}}{\sqrt{-\frac{b}{a}}\,\sinh(x)\sqrt{a+b\cosh(x)^2}}$

input
```
int((a+b*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*(a*EllipticF(cosh(x)*(-b/a)^(
1/2),(-a/b)^(1/2))+b*EllipticF(cosh(x)*(-b/a)^(1/2),(-a/b)^(1/2))-b*Ellipt
icE(cosh(x)*(-b/a)^(1/2),(-a/b)^(1/2)))/(-b/a)^(1/2)/sinh(x)/(a+b*cosh(x)^
2)^(1/2)
```

3.41.5 Fricas [F]

$$\int \sqrt{a+b\cosh^2(x)}\,dx=\int \sqrt{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cosh(x)^2 + a), x)
```

3.41.6 Sympy [F]

$$\int \sqrt{a+b\cosh^2(x)}\,dx=\int \sqrt{a+b\cosh^2{(x)}}\,dx$$

input
```
integrate((a+b*cosh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*cosh(x)**2), x)
```

3.41.7 Maxima [F]

$$\int \sqrt{a+b\cosh^2(x)}\,dx=\int \sqrt{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x)^2 + a), x)
```

3.41.8 Giac [F]

$$\int \sqrt{a+b\cosh^2(x)}\,dx=\int \sqrt{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate((a+b*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x)^2 + a), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh^2(x)}\,dx=\int \sqrt{b\cosh{(x)}^2+a}\,dx$$

input
```
int((a + b*cosh(x)^2)^(1/2),x)
```

output
```
int((a + b*cosh(x)^2)^(1/2), x)
```

3.42 $\int \sqrt{1+\cosh^2(x)}\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \sqrt{1+\cosh^2(x)}\,dx = -iE\left(\frac{\pi}{2}+ix\middle|-1\right)$$

output
```
(-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),I)
```

3.42.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \sqrt{1+\cosh^2(x)}\,dx = -i\sqrt{2}E\left(ix\middle|\frac{1}{2}\right)$$

input
```
Integrate[Sqrt[1 + Cosh[x]^2],x]
```

output
```
(-I)*Sqrt[2]*EllipticE[I*x, 1/2]
```

3.42.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1+\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3656}$$

$$-iE\left(ix+\frac{\pi}{2}\middle|-1\right)$$

input
```
Int[Sqrt[1 + Cosh[x]^2],x]
```

output
```
(-I)*EllipticE[Pi/2 + I*x, -1]
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

3.42.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(18) = 36.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.41

method	result	size
default	$-\frac{i\sqrt{\left(1+\cosh\left(x\right)^{2}\right)\sinh\left(x\right)^{2}}\,\sqrt{-\sinh\left(x\right)^{2}}\,\left(2\,\mathrm{EllipticF}\left(i\cosh\left(x\right),i\right)-\mathrm{EllipticE}\left(i\cosh\left(x\right),i\right)\right)}{\sqrt{\cosh\left(x\right)^{4}-1}\,\sinh\left(x\right)}$	58

input
```
int((1+cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*((1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)*(2*EllipticF(I*cosh(
x),I)-EllipticE(I*cosh(x),I))/(cosh(x)^4-1)^(1/2)/sinh(x)
```

3.42.5 Fricas [F]

$$\int \sqrt{1+\cosh^2(x)}\,dx = \int \sqrt{\cosh\left(x\right)^{2}+1}\,dx$$

input
```
integrate((1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(cosh(x)^2 + 1), x)
```

3.42.6 Sympy [F]

$$\int \sqrt{1+\cosh^2(x)}\,dx = \int \sqrt{\cosh^{2}\left(x\right)+1}\,dx$$

input
```
integrate((1+cosh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cosh(x)**2 + 1), x)
```

3.42.7 Maxima [F]

$$\int \sqrt{1+\cosh^2(x)}\,dx = \int \sqrt{\cosh(x)^2+1}\,dx$$

input
```
integrate((1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(x)^2 + 1), x)
```

3.42.8 Giac [F]

$$\int \sqrt{1+\cosh^2(x)}\,dx = \int \sqrt{\cosh(x)^2+1}\,dx$$

input
```
integrate((1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(x)^2 + 1), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\cosh^2(x)}\,dx = \int \sqrt{\cosh(x)^2+1}\,dx$$

input
```
int((cosh(x)^2 + 1)^(1/2),x)
```

output
```
int((cosh(x)^2 + 1)^(1/2), x)
```

3.43 $\int \sqrt{1-\cosh^2(x)}\,dx$

3.43.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \sqrt{1-\cosh^2(x)}\,dx = \coth(x)\sqrt{-\sinh^2(x)}$$

output
```
coth(x)*(-sinh(x)^2)^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\cosh^2(x)}\,dx = \coth(x)\sqrt{-\sinh^2(x)}$$

input
```
Integrate[Sqrt[1 - Cosh[x]^2],x]
```

output
```
Coth[x]*Sqrt[-Sinh[x]^2]
```

3.43.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3655, 3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{1-\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \sqrt{-\sinh^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin(ix)^2}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\sqrt{-\sinh^2(x)}\text{csch}(x)\int \sinh(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{-\sinh^2(x)}\text{csch}(x)\int -i\sin(ix)dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\sqrt{-\sinh^2(x)}\text{csch}(x)\int \sin(ix)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\sqrt{-\sinh^2(x)}\coth(x)$$

input
```
Int[Sqrt[1 - Cosh[x]^2],x]
```

output
```
Coth[x]*Sqrt[-Sinh[x]^2]
```

3.43.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.43.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{\cosh(x)\sinh(x)}{\sqrt{-\sinh(x)^2}}$	15
risch	$\frac{\sqrt{-(e^{2x}-1)^2 e^{-2x}}\, e^{2x}}{2\,e^{2x}-2}+\frac{\sqrt{-(e^{2x}-1)^2 e^{-2x}}}{2\,e^{2x}-2}$	58

input
```
int((1-cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)*sinh(x)/(-sinh(x)^2)^(1/2)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \sqrt{1-\cosh^2(x)}\,dx = \frac{\sqrt{-(e^{(4x)} - 2e^{(2x)} + 1)e^{(-2x)}}\cosh(x)e^x}{e^{(2x)} - 1}$$

input
```
integrate((1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-(e^(4*x) - 2*e^(2*x) + 1)*e^(-2*x))*cosh(x)*e^x/(e^(2*x) - 1)
```

3.43.6 Sympy [F]

$$\int \sqrt{1-\cosh^2(x)}\,dx = \int \sqrt{1-\cosh^2(x)}\,dx$$

input
```
integrate((1-cosh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - cosh(x)**2), x)
```

3.43.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sqrt{1-\cosh^2(x)}\,dx = -\frac{1}{2}i\,e^{(-x)} - \frac{1}{2}i\,e^x$$

input
```
integrate((1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*e^(-x) - 1/2*I*e^x
```

3.43.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \sqrt{1-\cosh^2(x)}\,dx = -\frac{1}{2}i\,e^{(-x)}\mathrm{sgn}\left(-e^{(3\,x)}+e^x\right)-\frac{1}{2}i\,e^x\mathrm{sgn}\left(-e^{(3\,x)}+e^x\right)$$

input
```
integrate((1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*e^(-x)*sgn(-e^(3*x) + e^x) - 1/2*I*e^x*sgn(-e^(3*x) + e^x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\cosh^2(x)}\,dx = \coth(x)\,\sqrt{1-\cosh{(x)}^2}$$

input
```
int((1 - cosh(x)^2)^(1/2),x)
```

output
```
coth(x)*(1 - cosh(x)^2)^(1/2)
```

3.44 $\int \sqrt{-1+\cosh^2(x)}\,dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \sqrt{-1+\cosh^2(x)}\,dx = \coth(x)\sqrt{\sinh^2(x)}$$

output
```
coth(x)*(sinh(x)^2)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+\cosh^2(x)}\,dx = \coth(x)\sqrt{\sinh^2(x)}$$

input
```
Integrate[Sqrt[-1 + Cosh[x]^2],x]
```

output
```
Coth[x]*Sqrt[Sinh[x]^2]
```

3.44.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3655, 3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-1+\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sqrt{\sinh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\sin(ix)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sqrt{\sinh^2(x)}\text{csch}(x)\int \sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sinh^2(x)}\text{csch}(x)\int -i\sin(ix)dx$$

$$\Big\downarrow \text{26}$$

$$-i\sqrt{\sinh^2(x)}\text{csch}(x)\int \sin(ix)dx$$

$$\Big\downarrow \text{3118}$$

$$\sqrt{\sinh^2(x)}\coth(x)$$

input
```
Int[Sqrt[-1 + Cosh[x]^2],x]
```

output
```
Coth[x]*Sqrt[Sinh[x]^2]
```

3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.44.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\cosh(x)\sqrt{-\frac{1}{2}+\frac{\cosh(2x)}{2}}}{\sinh(x)}$	14
risch	$\frac{\sqrt{\left(\mathrm{e}^{2x}-1\right)^{2}\mathrm{e}^{-2x}}\,\mathrm{e}^{2x}}{2\,\mathrm{e}^{2x}-2}+\frac{\sqrt{\left(\mathrm{e}^{2x}-1\right)^{2}\mathrm{e}^{-2x}}}{2\,\mathrm{e}^{2x}-2}$	56

input
```
int((cosh(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cosh(x)*(sinh(x)^2)^(1/2)/sinh(x)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \sqrt{-1+\cosh^2(x)}\, dx = \cosh(x)$$

input
```
integrate((-1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
cosh(x)
```

3.44.6 Sympy [F]

$$\int \sqrt{-1+\cosh^2(x)}\, dx = \int \sqrt{\cosh^2(x)-1}\, dx$$

input
```
integrate((-1+cosh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(cosh(x)**2 - 1), x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+\cosh^2(x)}\, dx = -\frac{1}{2}\, e^{(-x)} - \frac{1}{2}\, e^{x}$$

input
```
integrate((-1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x) - 1/2*e^x
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \sqrt{-1+\cosh^2(x)}\,dx = \frac{1}{2}e^{(-x)}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)+\frac{1}{2}e^{x}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)$$

input
```
integrate((-1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*e^(-x)*sgn(e^(3*x) - e^x) + 1/2*e^x*sgn(e^(3*x) - e^x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+\cosh^2(x)}\,dx = \coth(x)\,\sqrt{\cosh(x)^2-1}$$

input
```
int((cosh(x)^2 - 1)^(1/2),x)
```

output
```
coth(x)*(cosh(x)^2 - 1)^(1/2)
```

3.45 $\int \sqrt{-1-\cosh^2(x)}\,dx$

3.45.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \sqrt{-1-\cosh^2(x)}\,dx = -\frac{i\sqrt{-1-\cosh^2(x)}E\left(\frac{\pi}{2}+ix\middle|-1\right)}{\sqrt{1+\cosh^2(x)}}$$

output
```
(-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),I)*(-1-cosh(x)^2)^(1/2)/(1+co
sh(x)^2)^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \sqrt{-1-\cosh^2(x)}\,dx = \frac{i\sqrt{2}\sqrt{3+\cosh(2x)}E\left(ix\middle|\frac{1}{2}\right)}{\sqrt{-3-\cosh(2x)}}$$

input
```
Integrate[Sqrt[-1 - Cosh[x]^2],x]
```

output
```
(I*Sqrt[2]*Sqrt[3 + Cosh[2*x]]*EllipticE[I*x, 1/2])/Sqrt[-3 - Cosh[2*x]]
```

3.45.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3657, 3042, 3656}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\cosh^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-1-\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \text{3657}$$

$$\frac{\sqrt{-\cosh^2(x)-1}\int \sqrt{\cosh^2(x)+1}dx}{\sqrt{\cosh^2(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-\cosh^2(x)-1}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cosh^2(x)+1}}$$

$$\Big\downarrow \text{3656}$$

$$-\frac{i\sqrt{-\cosh^2(x)-1}E\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cosh^2(x)+1}}$$

input
```
Int[Sqrt[-1 - Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[-1 - Cosh[x]^2]*EllipticE[Pi/2 + I*x, -1])/Sqrt[1 + Cosh[x]^2]
```

3.45.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

method	result	size
default	$-\frac{\sqrt{-\left(1+\cosh(x)^2\right)\sinh(x)^2}\,\sqrt{-\sinh(x)^2}\,\sqrt{1+\cosh(x)^2}\,\text{EllipticE}(\cosh(x),i)}{\sqrt{1-\cosh(x)^4}\,\sinh(x)\sqrt{-1-\cosh(x)^2}}$	62

input
```
int((-1-cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)*(1+cosh(x)^2)^(1/2)*E
llipticE(cosh(x),I)/(1-cosh(x)^4)^(1/2)/sinh(x)/(-1-cosh(x)^2)^(1/2)
```

3.45.5 Fricas [F]

$$\int \sqrt{-1-\cosh^2(x)}\,dx = \int \sqrt{-\cosh(x)^2 - 1}\,dx$$

input
```
integrate((-1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*(e^(2*x) - e^x)*integral(4*sqrt(-e^(4*x) - 6*e^(2*x) - 1)*(e^(2*x)
+ 1)/(e^(6*x) - 2*e^(5*x) + 7*e^(4*x) - 12*e^(3*x) + 7*e^(2*x) - 2*e^x + 1
), x) + sqrt(-e^(4*x) - 6*e^(2*x) - 1)*(e^x + 1))/(e^(2*x) - e^x)
```

3.45.6 Sympy [F]

$$\int \sqrt{-1-\cosh^2(x)}\,dx = \int \sqrt{-\cosh^2{(x)} - 1}\,dx$$

input
```
integrate((-1-cosh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-cosh(x)**2 - 1), x)
```

3.45.7 Maxima [F]

$$\int \sqrt{-1-\cosh^2(x)}\,dx = \int \sqrt{-\cosh{(x)}^2 - 1}\,dx$$

input
```
integrate((-1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-cosh(x)^2 - 1), x)
```

3.45.8 Giac [F]

$$\int \sqrt{-1-\cosh^2(x)}\,dx = \int \sqrt{-\cosh{(x)}^2 - 1}\,dx$$

input
```
integrate((-1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-cosh(x)^2 - 1), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-1-\cosh^2(x)}\,dx = \int \sqrt{-\cosh\left(x\right)^2-1}\,dx$$

input
```
int((- cosh(x)^2 - 1)^(1/2),x)
```

output
```
int((- cosh(x)^2 - 1)^(1/2), x)
```

3.46 $\int \left(a+b\cosh^2(x)\right)^{3/2}\,dx$

3.46.1 Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \left(a+b\cosh^2(x)\right)^{3/2}\,dx=-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-\frac{b}{a}\right)}{3\sqrt{1+\frac{b\cosh^2(x)}{a}}}+\frac{ia(a+b)\sqrt{1+\frac{b\cosh^2(x)}{a}}\,\text{EllipticF}\left(\frac{\pi}{2}+ix,-\frac{b}{a}\right)}{3\sqrt{a+b\cosh^2(x)}}+\frac{1}{3}b\cosh(x)\sqrt{a+b\cosh^2(x)}\sinh(x)$$

output
```
1/3*b*cosh(x)*sinh(x)*(a+b*cosh(x)^2)^(1/2)+2/3*(2*a+b)*(-sinh(x)^2)^(1/2)
/sinh(x)*EllipticE(cosh(x),(-b/a)^(1/2))*(a+b*cosh(x)^2)^(1/2)/(1+b*cosh(x
)^2/a)^(1/2)-1/3*a*(a+b)*(-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),(-b/
a)^(1/2))*(1+b*cosh(x)^2/a)^(1/2)/(a+b*cosh(x)^2)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \left(a+b\cosh^2(x)\right)^{3/2}\,dx=\frac{-8i(2a^2+3ab+b^2)\sqrt{\frac{2a+b+b\cosh(2x)}{a+b}}\,E\left(ix\middle|\frac{b}{a+b}\right)+4ia(a+b)\sqrt{\frac{2a+b+b\cosh(2x)}{a+b}}\,\text{EllipticF}\left(\right.}{12\sqrt{2a+b+b\cosh(2x)}}$$

input
```
Integrate[(a + b*Cosh[x]^2)^(3/2),x]
```

output
```
((-8*I)*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b + b*Cosh[2*x])/(a + b)]*Ellipt
icE[I*x, b/(a + b)] + (4*I)*a*(a + b)*Sqrt[(2*a + b + b*Cosh[2*x])/(a + b)
]*EllipticF[I*x, b/(a + b)] + Sqrt[2]*b*(2*a + b + b*Cosh[2*x])*Sinh[2*x])
/(12*Sqrt[2*a + b + b*Cosh[2*x]])
```

3.46.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3659, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\cosh^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2b(2a+b)\cosh^2(x)+a(3a+b)}{\sqrt{b\cosh^2(x)+a}}dx+\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+\frac{1}{3}\int \frac{2b(2a+b)\sin\left(ix+\frac{\pi}{2}\right)^2+a(3a+b)}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{1}{3}\left(2(2a+b)\int \sqrt{b\cosh^2(x)+a}dx-a(a+b)\int \frac{1}{\sqrt{b\cosh^2(x)+a}}dx\right)+\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+ \frac{1}{3}\left(2(2a+b)\int\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}dx-a(a+b)\int\frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}}dx\right)$$

$\downarrow$ 3657

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+ \frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cosh^2(x)}\int\sqrt{\frac{b\cosh^2(x)}{a}+1}dx}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}}dx\right)$$

$\downarrow$ 3042

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+ \frac{1}{3}\left(\frac{2(2a+b)\sqrt{a+b\cosh^2(x)}\int\sqrt{\frac{b\sin\left(ix+\frac{\pi}{2}\right)^2}{a}+1}dx}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}-a(a+b)\int\frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}}dx\right)$$

$\downarrow$ 3656

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+ \frac{1}{3}\left(-a(a+b)\int\frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}}dx-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}E\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}\right)$$

$\downarrow$ 3662

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+ \frac{1}{3}\left(-\frac{a(a+b)\sqrt{\frac{b\cosh^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}dx}{\sqrt{a+b\cosh^2(x)}}-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}E\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}\right)$$

$\downarrow$ 3042

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+ \frac{1}{3}\left(-\frac{a(a+b)\sqrt{\frac{b\cosh^2(x)}{a}+1}\int\frac{1}{\sqrt{\frac{b\sin\left(ix+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cosh^2(x)}}-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}E\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}\right)$$

$\downarrow$ 3661

3.46. $\int\left(a+b\cosh^2(x)\right)^{3/2}\,dx$

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\cosh^2(x)}+\frac{1}{3}\left(\frac{ia(a+b)\sqrt{\frac{b\cosh^2(x)}{a}+1}\,\text{EllipticF}\left(ix+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cosh^2(x)}}-\frac{2i(2a+b)\sqrt{a+b\cosh^2(x)}E\left(ix+\frac{\pi}{2}|-\frac{b}{a}\right)}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}\right)$$

input
```
Int[(a + b*Cosh[x]^2)^(3/2),x]
```

output
```
(((-2*I)*(2*a + b)*Sqrt[a + b*Cosh[x]^2]*EllipticE[Pi/2 + I*x, -(b/a)])/Sq
rt[1 + (b*Cosh[x]^2)/a] + (I*a*(a + b)*Sqrt[1 + (b*Cosh[x]^2)/a]*EllipticF
[Pi/2 + I*x, -(b/a)])/Sqrt[a + b*Cosh[x]^2])/3 + (b*Cosh[x]*Sqrt[a + b*Cos
h[x]^2]*Sinh[x])/3
```

3.46.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(123) = 246$.

Time = 1.33 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.41

method	result
default	$\sqrt{-\frac{b}{a}}\, b^2 \cosh(x)^5+\sqrt{-\frac{b}{a}}\, ab \cosh(x)^3-\sqrt{-\frac{b}{a}}\, b^2 \cosh(x)^3+3a^2\sqrt{\frac{a+b\cosh(x)^2}{a}}\,\sqrt{-\sinh(x)^2}\ \mathrm{EllipticF}\left(\cosh(x)\sqrt{-\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right)+5ab\sqrt{}$

input
```
int((a+b*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-b/a)^(1/2)*b^2*cosh(x)^5+(-b/a)^(1/2)*a*b*cosh(x)^3-(-b/a)^(1/2)*b^
2*cosh(x)^3+3*a^2*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticF(c
osh(x)*(-b/a)^(1/2),(-a/b)^(1/2))+5*a*b*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x
)^2)^(1/2)*EllipticF(cosh(x)*(-b/a)^(1/2),(-a/b)^(1/2))+2*((a+b*cosh(x)^2)
/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticF(cosh(x)*(-b/a)^(1/2),(-a/b)^(1/2))*
b^2-4*a*b*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticE(cosh(x)*(
-b/a)^(1/2),(-a/b)^(1/2))-2*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*E
llipticE(cosh(x)*(-b/a)^(1/2),(-a/b)^(1/2))*b^2-(-b/a)^(1/2)*a*b*cosh(x))/
(-b/a)^(1/2)/sinh(x)/(a+b*cosh(x)^2)^(1/2)
```

3.46.5 Fricas [F]

$$\int \left(a+b\cosh^2(x)\right)^{3/2} dx = \int \left(b\cosh\left(x\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cosh(x)^2 + a)^(3/2), x)
```

3.46.6 Sympy [F]

$$\int \left(a+b\cosh^2(x)\right)^{3/2} dx = \int \left(a+b\cosh^2\left(x\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*cosh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*cosh(x)**2)**(3/2), x)
```

3.46.7 Maxima [F]

$$\int \left(a+b\cosh^2(x)\right)^{3/2} dx = \int \left(b\cosh\left(x\right)^2+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x)^2 + a)^(3/2), x)
```

3.46.8 Giac [F(-2)]

Exception generated.

$$\int \left(a+b\cosh^2(x)\right)^{3/2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((a+b*cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\cosh^2(x)\right)^{3/2}\,dx = \int \left(b\cosh(x)^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*cosh(x)^2)^(3/2),x)
```

output
```
int((a + b*cosh(x)^2)^(3/2), x)
```

3.47 $\int \left(1+\cosh^2(x)\right)^{3/2} dx$

3.47.1 Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \left(1+\cosh^2(x)\right)^{3/2} dx = -2iE\left(\frac{\pi}{2}+ix\middle|-1\right)+\frac{2}{3}i\,\text{EllipticF}\left(\frac{\pi}{2}+ix,-1\right)+\frac{1}{3}\cosh(x)\sqrt{1+\cosh^2(x)}\sinh(x)$$

output
```
2*(-sinh(x)^2)^(1/2)/sinh(x)*EllipticE(cosh(x),I)-2/3*(-sinh(x)^2)^(1/2)/s
inh(x)*EllipticF(cosh(x),I)+1/3*cosh(x)*sinh(x)*(1+cosh(x)^2)^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \left(1+\cosh^2(x)\right)^{3/2} dx = \frac{-24iE\left(ix\middle|\frac{1}{2}\right)+4i\,\text{EllipticF}\left(ix,\frac{1}{2}\right)+\sqrt{3+\cosh(2x)}\sinh(2x)}{6\sqrt{2}}$$

input
```
Integrate[(1 + Cosh[x]^2)^(3/2),x]
```

output
```
((-24*I)*EllipticE[I*x, 1/2] + (4*I)*EllipticF[I*x, 1/2] + Sqrt[3 + Cosh[2
*x]]*Sinh[2*x])/(6*Sqrt[2])
```

3.47.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3656, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cosh^2(x)+1\right)^{3/2} dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(1+\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \; 3659$$

$$\frac{1}{3}\int \frac{2\left(3\cosh^2(x)+2\right)}{\sqrt{\cosh^2(x)+1}}dx+\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}\int \frac{3\cosh^2(x)+2}{\sqrt{\cosh^2(x)+1}}dx+\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}+\frac{2}{3}\int \frac{3\sin\left(ix+\frac{\pi}{2}\right)^2+2}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}}dx$$

$$\Big\downarrow \; 3651$$

$$\frac{2}{3}\left(3\int \sqrt{\cosh^2(x)+1}dx-\int \frac{1}{\sqrt{\cosh^2(x)+1}}dx\right)+\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}+\frac{2}{3}\left(3\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx-\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}}dx\right)$$

$$\Big\downarrow \; 3656$$

$$\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}+\frac{2}{3}\left(-\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}}dx-3iE\left(ix+\frac{\pi}{2}\middle|-1\right)\right)$$

$$\Big\downarrow \text{ 3661}$$

$$\frac{1}{3}\sinh(x)\cosh(x)\sqrt{\cosh^2(x)+1}+\frac{2}{3}\left(i\,\text{EllipticF}\left(ix+\frac{\pi}{2},-1\right)-3iE\left(ix+\frac{\pi}{2}\Big|-1\right)\right)$$

input
```
Int[(1 + Cosh[x]^2)^(3/2),x]
```

output
```
(2*((-3*I)*EllipticE[Pi/2 + I*x, -1] + I*EllipticF[Pi/2 + I*x, -1]))/3 + (
Cosh[x]*Sqrt[1 + Cosh[x]^2]*Sinh[x])/3
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

3.47.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{\left(1+\cosh(x)^{2}\right)\sinh(x)^{2}}\,\left(-\cosh(x)^{5}+10i\sqrt{1+\cosh(x)^{2}}\,\sqrt{-\sinh(x)^{2}}\,\mathrm{EllipticF}(i\cosh(x),i)-6i\sqrt{1+\cosh(x)^{2}}\,\sqrt{-\sinh(x)^{2}}\,\mathrm{Elli}}{3\sqrt{\cosh(x)^{4}-1}\,\sinh(x)\sqrt{1+\cosh(x)^{2}}}$

input
```
int((1+cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-cosh(x)^5+10*I*(1+cosh(x)^2)^(1/2)*
(-sinh(x)^2)^(1/2)*EllipticF(I*cosh(x),I)-6*I*(1+cosh(x)^2)^(1/2)*(-sinh(x
)^2)^(1/2)*EllipticE(I*cosh(x),I)+cosh(x))/(cosh(x)^4-1)^(1/2)/sinh(x)/(1+
cosh(x)^2)^(1/2)
```

3.47.5 Fricas [F]

$$\int \left(1+\cosh^2(x)\right)^{3/2}\,dx=\int \left(\cosh\left(x\right)^{2}+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((cosh(x)^2 + 1)^(3/2), x)
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \left(1+\cosh^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((1+cosh(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.47.7 Maxima [F]

$$\int \left(1+\cosh^2(x)\right)^{3/2} \, dx = \int \left(\cosh\left(x\right)^2+1\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((1+cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((cosh(x)^2 + 1)^(3/2), x)
```

3.47.8 Giac [F]

$$\int \left(1+\cosh^2(x)\right)^{3/2} \, dx = \int \left(\cosh\left(x\right)^2+1\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((1+cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((cosh(x)^2 + 1)^(3/2), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+\cosh^2(x)\right)^{3/2} \, dx = \int \left(\cosh(x)^2+1\right)^{3/2} \, dx$$

input
```
int((cosh(x)^2 + 1)^(3/2),x)
```

output
```
int((cosh(x)^2 + 1)^(3/2), x)
```

3.48 $\int \left(1-\cosh^2(x)\right)^{3/2} \, dx$

3.48.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int \left(1-\cosh^2(x)\right)^{3/2} \, dx=\frac{2}{3}\coth(x)\sqrt{-\sinh^2(x)}+\frac{1}{3}\coth(x)\left(-\sinh^2(x)\right)^{3/2}$$

output
```
1/3*coth(x)*(-sinh(x)^2)^(3/2)+2/3*coth(x)*(-sinh(x)^2)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \left(1-\cosh^2(x)\right)^{3/2} \, dx=-\frac{1}{12}(-9\cosh(x)+\cosh(3x))\text{csch}(x)\sqrt{-\sinh^2(x)}$$

input
```
Integrate[(1 - Cosh[x]^2)^(3/2),x]
```

output
```
-1/12*((-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[-Sinh[x]^2])
```

3.48.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\cosh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(1-\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 3655$$

$$\int \left(-\sinh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(\sin(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 3682$$

$$\frac{2}{3}\int \sqrt{-\sinh^2(x)}dx+\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)+\frac{2}{3}\int \sqrt{\sin(ix)^2}dx$$

$$\Big\downarrow \ 3686$$

$$\frac{2}{3}\sqrt{-\sinh^2(x)}\text{csch}(x)\int \sinh(x)dx+\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)+\frac{2}{3}\sqrt{-\sinh^2(x)}\text{csch}(x)\int -i\sin(ix)dx$$

$$\Big\downarrow \ 26$$

$$\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)-\frac{2}{3}i\sqrt{-\sinh^2(x)}\text{csch}(x)\int \sin(ix)dx$$

$$\Big\downarrow \ 3118$$

$$\frac{1}{3}\left(-\sinh^2(x)\right)^{3/2}\coth(x)+\frac{2}{3}\sqrt{-\sinh^2(x)}\coth(x)$$

input
```
Int[(1 - Cosh[x]^2)^(3/2),x]
```

output
```
(2*Coth[x]*Sqrt[-Sinh[x]^2])/3 + (Coth[x]*(-Sinh[x]^2)^(3/2))/3
```

3.48.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.48.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sinh(x)\cosh(x)\left(\cosh(x)^2-3\right)}{3\sqrt{-\sinh(x)^2}}$	21
risch	$-\frac{e^{4x}\sqrt{-(e^{2x}-1)^2 e^{-2x}}}{24(e^{2x}-1)}+\frac{3\sqrt{-(e^{2x}-1)^2 e^{-2x}}\,e^{2x}}{8(e^{2x}-1)}+\frac{3\sqrt{-(e^{2x}-1)^2 e^{-2x}}}{8(e^{2x}-1)}-\frac{e^{-2x}\sqrt{-(e^{2x}-1)^2 e^{-2x}}}{24(e^{2x}-1)}$	118

input
```
int((1-cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sinh(x)*cosh(x)*(cosh(x)^2-3)/(-sinh(x)^2)^(1/2)
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \left(1-\cosh^2(x)\right)^{3/2}\,dx = -\frac{\left(3\cosh(x)e^x\sinh(x)^2+\left(\cosh(x)^3-9\cosh(x)\right)e^x\right)\sqrt{-(e^{(4x)}-2e^{(2x)}+1)e^{(-2x)}}}{12\left(e^{(2x)}-1\right)}$$

input
```
integrate((1-cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(3*cosh(x)*e^x*sinh(x)^2 + (cosh(x)^3 - 9*cosh(x))*e^x)*sqrt(-(e^(4*
x) - 2*e^(2*x) + 1)*e^(-2*x))/(e^(2*x) - 1)
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \left(1-\cosh^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((1-cosh(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.48.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \left(1-\cosh^2(x)\right)^{3/2}\,dx = \frac{1}{24}i\,e^{(3\,x)} - \frac{3}{8}i\,e^{(-x)} + \frac{1}{24}i\,e^{(-3\,x)} - \frac{3}{8}i\,e^{x}$$

input
```
integrate((1-cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*I*e^(3*x) - 3/8*I*e^(-x) + 1/24*I*e^(-3*x) - 3/8*I*e^x
```

3.48.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \left(1-\cosh^2(x)\right)^{3/2}\,dx = -\frac{1}{24}i\left(9\,e^{(2\,x)}\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right) - \mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)\right)e^{(-3\,x)} + \frac{1}{24}i\,e^{(3\,x)}\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right) - \frac{3}{8}i\,e^{x}\mathrm{sgn}\left(-e^{(3\,x)}+e^{x}\right)$$

input
```
integrate((1-cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*I*(9*e^(2*x)*sgn(-e^(3*x) + e^x) - sgn(-e^(3*x) + e^x))*e^(-3*x) + 1
/24*I*e^(3*x)*sgn(-e^(3*x) + e^x) - 3/8*I*e^x*sgn(-e^(3*x) + e^x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \left(1-\cosh^2(x)\right)^{3/2}\,dx = \int \left(1-\cosh(x)^2\right)^{3/2}\,dx$$

input
```
int((1 - cosh(x)^2)^(3/2),x)
```

output
```
int((1 - cosh(x)^2)^(3/2), x)
```

3.49 $\int \left(-1+\cosh^2(x)\right)^{3/2} dx$

3.49.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \left(-1+\cosh^2(x)\right)^{3/2} dx = -\frac{2}{3}\coth(x)\sqrt{\sinh^2(x)}+\frac{1}{3}\coth(x)\sinh^2(x)^{3/2}$$

output
```
1/3*coth(x)*(sinh(x)^2)^(3/2)-2/3*coth(x)*(sinh(x)^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \left(-1+\cosh^2(x)\right)^{3/2} dx = \frac{1}{12}(-9\cosh(x)+\cosh(3x))\operatorname{csch}(x)\sqrt{\sinh^2(x)}$$

input
```
Integrate[(-1 + Cosh[x]^2)^(3/2),x]
```

output
```
((-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[Sinh[x]^2])/12
```

3.49.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3655, 3042, 3682, 3042, 3686, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cosh^2(x)-1\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-1+\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3655}$$

$$\int \sinh^2(x)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sin(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)-\frac{2}{3}\int \sqrt{\sinh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)-\frac{2}{3}\int \sqrt{-\sin(ix)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)-\frac{2}{3}\sqrt{\sinh^2(x)}\text{csch}(x)\int \sinh(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)-\frac{2}{3}\sqrt{\sinh^2(x)}\text{csch}(x)\int -i\sin(ix)dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x)+\frac{2}{3}i\sqrt{\sinh^2(x)}\text{csch}(x)\int \sin(ix)dx$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{3}\sinh^2(x)^{3/2}\coth(x) - \frac{2}{3}\sqrt{\sinh^2(x)}\coth(x)$$

input
```
Int[(-1 + Cosh[x]^2)^(3/2),x]
```

output
```
(-2*Coth[x]*Sqrt[Sinh[x]^2])/3 + (Coth[x]*(Sinh[x]^2)^(3/2))/3
```

3.49.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.49.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2x)}{2}}\,\cosh(x)\left(\cosh(x)^2-3\right)}{3\sinh(x)}$	21
risch	$\frac{e^{4x}\sqrt{\left(e^{2x}-1\right)^2 e^{-2x}}}{24\,e^{2x}-24}-\frac{3\sqrt{\left(e^{2x}-1\right)^2 e^{-2x}}\,e^{2x}}{8\left(e^{2x}-1\right)}-\frac{3\sqrt{\left(e^{2x}-1\right)^2 e^{-2x}}}{8\left(e^{2x}-1\right)}+\frac{e^{-2x}\sqrt{\left(e^{2x}-1\right)^2 e^{-2x}}}{24\,e^{2x}-24}$	114

input
```
int((cosh(x)^2-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(sinh(x)^2)^(1/2)*cosh(x)*(cosh(x)^2-3)/sinh(x)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \left(-1+\cosh^2(x)\right)^{3/2}\,dx=\frac{1}{12}\cosh\left(x\right)^3+\frac{1}{4}\cosh\left(x\right)\sinh\left(x\right)^2-\frac{3}{4}\cosh\left(x\right)$$

input
```
integrate((-1+cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*cosh(x)^3 + 1/4*cosh(x)*sinh(x)^2 - 3/4*cosh(x)
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \left(-1+\cosh^2(x)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-1+cosh(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \left(-1+\cosh^2(x)\right)^{3/2} dx = -\frac{1}{24}e^{(3x)} + \frac{3}{8}e^{(-x)} - \frac{1}{24}e^{(-3x)} + \frac{3}{8}e^{x}$$

input
```
integrate((-1+cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/24*e^(3*x) + 3/8*e^(-x) - 1/24*e^(-3*x) + 3/8*e^x
```

3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(21) = 42.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \left(-1+\cosh^2(x)\right)^{3/2} dx = -\frac{1}{24}\left(9e^{(2x)}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right) - \operatorname{sgn}\left(e^{(3x)}-e^{x}\right)\right)e^{(-3x)} + \frac{1}{24}e^{(3x)}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right) - \frac{3}{8}e^{x}\operatorname{sgn}\left(e^{(3x)}-e^{x}\right)$$

input
```
integrate((-1+cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*(9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) + 1/24*
e^(3*x)*sgn(e^(3*x) - e^x) - 3/8*e^x*sgn(e^(3*x) - e^x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1+\cosh^2(x)\right)^{3/2} dx = \int \left(\cosh(x)^2-1\right)^{3/2} dx$$

input
```
int((cosh(x)^2 - 1)^(3/2),x)
```

output
```
int((cosh(x)^2 - 1)^(3/2), x)
```

3.50 $\int \left(-1-\cosh^2(x)\right)^{3/2}\,dx$

3.50.1 Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \left(-1-\cosh^2(x)\right)^{3/2}\,dx=\frac{2i\sqrt{-1-\cosh^2(x)}\,E\left(\frac{\pi}{2}+ix\middle|-1\right)}{\sqrt{1+\cosh^2(x)}}+\frac{2i\sqrt{1+\cosh^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+ix,-1\right)}{3\sqrt{-1-\cosh^2(x)}}-\frac{1}{3}\cosh(x)\sqrt{-1-\cosh^2(x)}\sinh(x)$$

output
```
-1/3*cosh(x)*sinh(x)*(-1-cosh(x)^2)^(1/2)-2*(-sinh(x)^2)^(1/2)/sinh(x)*Ell
ipticE(cosh(x),I)*(-1-cosh(x)^2)^(1/2)/(1+cosh(x)^2)^(1/2)-2/3*(-sinh(x)^2
)^(1/2)/sinh(x)*EllipticF(cosh(x),I)*(1+cosh(x)^2)^(1/2)/(-1-cosh(x)^2)^(1
/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

$$\int \left(-1-\cosh^2(x)\right)^{3/2}\,dx=\frac{-48i\sqrt{3+\cosh(2x)}\,E\left(ix\middle|\frac{1}{2}\right)+8i\sqrt{3+\cosh(2x)}\,\text{EllipticF}\left(ix,\frac{1}{2}\right)+6\sinh(2x)+\sinh(}{12\sqrt{2}\sqrt{-3-\cosh(2x)}}$$

input
```
Integrate[(-1 - Cosh[x]^2)^(3/2),x]
```

output
```
((-48*I)*Sqrt[3 + Cosh[2*x]]*EllipticE[I*x, 1/2] + (8*I)*Sqrt[3 + Cosh[2*x
]]*EllipticF[I*x, 1/2] + 6*Sinh[2*x] + Sinh[4*x])/(12*Sqrt[2]*Sqrt[-3 - Co
sh[2*x]])
```

3.50.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3659, 27, 3042, 3651, 3042, 3657, 3042, 3656, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\cosh^2(x)-1\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-1-\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{3659}$$

$$\frac{1}{3}\int \frac{2\left(3\cosh^2(x)+2\right)}{\sqrt{-\cosh^2(x)-1}}dx-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3\cosh^2(x)+2}{\sqrt{-\cosh^2(x)-1}}dx-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+\frac{2}{3}\int \frac{3\sin\left(ix+\frac{\pi}{2}\right)^2+2}{\sqrt{-\sin\left(ix+\frac{\pi}{2}\right)^2-1}}dx$$

$$\Big\downarrow \text{3651}$$

$$\frac{2}{3}\left(-\int \frac{1}{\sqrt{-\cosh^2(x)-1}}dx-3\int \sqrt{-\cosh^2(x)-1}dx\right)-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+$$
$$\frac{2}{3}\left(-\int\frac{1}{\sqrt{-\sin\left(ix+\frac{\pi}{2}\right)^2-1}}dx-3\int\sqrt{-\sin\left(ix+\frac{\pi}{2}\right)^2-1}dx\right)$$

$$\Big\downarrow \text{3657}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+$$
$$\frac{2}{3}\left(-\frac{3\sqrt{-\cosh^2(x)-1}\int\sqrt{\cosh^2(x)+1}dx}{\sqrt{\cosh^2(x)+1}}-\int\frac{1}{\sqrt{-\sin\left(ix+\frac{\pi}{2}\right)^2-1}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+$$
$$\frac{2}{3}\left(-\int\frac{1}{\sqrt{-\sin\left(ix+\frac{\pi}{2}\right)^2-1}}dx-\frac{3\sqrt{-\cosh^2(x)-1}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx}{\sqrt{\cosh^2(x)+1}}\right)$$

$$\Big\downarrow \text{3656}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+$$
$$\frac{2}{3}\left(\frac{3i\sqrt{-\cosh^2(x)-1}E\left(ix+\frac{\pi}{2}|-1\right)}{\sqrt{\cosh^2(x)+1}}-\int\frac{1}{\sqrt{-\sin\left(ix+\frac{\pi}{2}\right)^2-1}}dx\right)$$

$$\Big\downarrow \text{3662}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+$$
$$\frac{2}{3}\left(-\frac{\sqrt{\cosh^2(x)+1}\int\frac{1}{\sqrt{\cosh^2(x)+1}}dx}{\sqrt{-\cosh^2(x)-1}}+\frac{3i\sqrt{-\cosh^2(x)-1}E\left(ix+\frac{\pi}{2}|-1\right)}{\sqrt{\cosh^2(x)+1}}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+$$
$$\frac{2}{3}\left(\frac{3i\sqrt{-\cosh^2(x)-1}E\left(ix+\frac{\pi}{2}|-1\right)}{\sqrt{\cosh^2(x)+1}}-\frac{\sqrt{\cosh^2(x)+1}\int\frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{-\cosh^2(x)-1}}\right)$$

$$\Big\downarrow \text{3661}$$

3.50. $\int\left(-1-\cosh^2(x)\right)^{3/2}dx$

$$-\frac{1}{3}\sinh(x)\cosh(x)\sqrt{-\cosh^2(x)-1}+\frac{2}{3}\left(\frac{i\sqrt{\cosh^2(x)+1}\,\text{EllipticF}\left(ix+\frac{\pi}{2},-1\right)}{\sqrt{-\cosh^2(x)-1}}+\frac{3i\sqrt{-\cosh^2(x)-1}E\left(ix+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cosh^2(x)+1}}\right)$$

input
```
Int[(-1 - Cosh[x]^2)^(3/2),x]
```

output
```
(2*(((3*I)*Sqrt[-1 - Cosh[x]^2]*EllipticE[Pi/2 + I*x, -1])/Sqrt[1 + Cosh[x
]^2] + (I*Sqrt[1 + Cosh[x]^2]*EllipticF[Pi/2 + I*x, -1])/Sqrt[-1 - Cosh[x]
^2]))/3 - (Cosh[x]*Sqrt[-1 - Cosh[x]^2]*Sinh[x])/3
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3651
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Sin[e + f*x]^2], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; Fre
eQ[{a, b, e, f, A, B}, x]
```

rule 3656
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

rule 3657
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)]   Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

rule 3659
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Sim
p[1/(2*p)   Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a
+ b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[
a + b, 0] && GtQ[p, 1]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\sqrt{-\left(1+\cosh(x)^2\right)\sinh(x)^2}\left(-\cosh(x)^5+2\sqrt{-\sinh(x)^2}\,\sqrt{1+\cosh(x)^2}\,\text{EllipticF}(\cosh(x),i)-6\sqrt{-\sinh(x)^2}\,\sqrt{1+\cosh(x)^2}\,\text{Ellipt}\right.}{3\sqrt{1-\cosh(x)^4}\,\sinh(x)\sqrt{-1-\cosh(x)^2}}$

input
```
int((-1-cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-(1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-cosh(x)^5+2*(-sinh(x)^2)^(1/2)*(1+
cosh(x)^2)^(1/2)*EllipticF(cosh(x),I)-6*(-sinh(x)^2)^(1/2)*(1+cosh(x)^2)^(
1/2)*EllipticE(cosh(x),I)+cosh(x))/(1-cosh(x)^4)^(1/2)/sinh(x)/(-1-cosh(x)
^2)^(1/2)
```

3.50.5 Fricas [F]

$$\int \left(-1-\cosh^2(x)\right)^{3/2} dx = \int \left(-\cosh(x)^2 - 1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(24*(e^(4*x) - e^(3*x))*integral(-4/3*sqrt(-e^(4*x) - 6*e^(2*x) - 1)*
(5*e^(2*x) + 2*e^x + 5)/(e^(6*x) - 2*e^(5*x) + 7*e^(4*x) - 12*e^(3*x) + 7*
e^(2*x) - 2*e^x + 1), x) - (e^(5*x) - e^(4*x) + 24*e^(3*x) + 24*e^(2*x) -
e^x + 1)*sqrt(-e^(4*x) - 6*e^(2*x) - 1))/(e^(4*x) - e^(3*x))
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \left(-1-\cosh^2(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((-1-cosh(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int \left(-1-\cosh^2(x)\right)^{3/2} dx = \int \left(-\cosh(x)^2 - 1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-cosh(x)^2 - 1)^(3/2), x)
```

3.50.8 Giac [F]

$$\int \left(-1-\cosh^2(x)\right)^{3/2} dx = \int \left(-\cosh(x)^2-1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-cosh(x)^2 - 1)^(3/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1-\cosh^2(x)\right)^{3/2} dx = \int \left(-\cosh(x)^2-1\right)^{3/2} dx$$

input
```
int((- cosh(x)^2 - 1)^(3/2),x)
```

output
```
int((- cosh(x)^2 - 1)^(3/2), x)
```

3.51 $\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx$

3.51.1 Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx=-\frac{i\sqrt{1+\frac{b\cosh^2(x)}{a}}\,\text{EllipticF}\left(\frac{\pi}{2}+ix,-\frac{b}{a}\right)}{\sqrt{a+b\cosh^2(x)}}$$

output
```
(-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),(-b/a)^(1/2))*(1+b*cosh(x)^2/
a)^(1/2)/(a+b*cosh(x)^2)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx=-\frac{i\sqrt{\frac{2a+b+b\cosh(2x)}{a+b}}\,\text{EllipticF}\left(ix,\frac{b}{a+b}\right)}{\sqrt{2a+b+b\cosh(2x)}}$$

input
```
Integrate[1/Sqrt[a + b*Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[(2*a + b + b*Cosh[2*x])/(a + b)]*EllipticF[I*x, b/(a + b)])/Sqr
t[2*a + b + b*Cosh[2*x]]
```

3.51.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\frac{b\cosh^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\cosh^2(x)}{a}+1}}dx}{\sqrt{a+b\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{b\cosh^2(x)}{a}+1}\int \frac{1}{\sqrt{\frac{b\sin\left(ix+\frac{\pi}{2}\right)^2}{a}+1}}dx}{\sqrt{a+b\cosh^2(x)}}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{i\sqrt{\frac{b\cosh^2(x)}{a}+1}\,\text{EllipticF}\left(ix+\frac{\pi}{2},-\frac{b}{a}\right)}{\sqrt{a+b\cosh^2(x)}}$$

input
```
Int[1/Sqrt[a + b*Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[1 + (b*Cosh[x]^2)/a]*EllipticF[Pi/2 + I*x, -(b/a)])/Sqrt[a + b*
Cosh[x]^2]
```

3.51.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.51.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\sqrt{\frac{a+b\cosh(x)^2}{a}}\,\sqrt{-\sinh(x)^2}\ \mathrm{EllipticF}\left(\cosh(x)\sqrt{-\frac{b}{a}},\sqrt{-\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\ \sinh(x)\sqrt{a+b\cosh(x)^2}}$	66

input
```
int(1/(a+b*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*((a+b*cosh(x)^2)/a)^(1/2)*(-sinh(x)^2)^(1/2)*EllipticF(cosh
(x)*(-b/a)^(1/2),(-a/b)^(1/2))/sinh(x)/(a+b*cosh(x)^2)^(1/2)
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.69

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx =$$

$$-\frac{2\left(2\,b\sqrt{\frac{a^2+ab}{b^2}}+2\,a+b\right)\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}-2\,a-b}{b}}F(\arcsin\left(\sqrt{\frac{2\,b\sqrt{\frac{a^2+ab}{b^2}}-2\,a-b}{b}}(\cosh\left(x\right)+\sinh\left(x\right))\right) \mid \frac{8\,a^2+8\,ab+}{}}{b^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) -
 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/
b)*(cosh(x) + sinh(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2
+ a*b)/b^2))/b^2)/b^(3/2)
```

3.51.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh^2{(x)}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cosh(x)**2), x)
```

3.51.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cosh(x)^2 + a), x)
```

3.51.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cosh(x)^2 + a), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}^2+a}}\,dx$$

input
```
int(1/(a + b*cosh(x)^2)^(1/2),x)
```

output
```
int(1/(a + b*cosh(x)^2)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{1+\cosh^2(x)}} dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}} dx = -i \operatorname{EllipticF}\left(\frac{\pi}{2}+ix,-1\right)$$

output
```
(-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),I)
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}} dx = -\frac{i \operatorname{EllipticF}\left(ix,\frac{1}{2}\right)}{\sqrt{2}}$$

input
```
Integrate[1/Sqrt[1 + Cosh[x]^2],x]
```

output
```
((-I)*EllipticF[I*x, 1/2])/Sqrt[2]
```

3.52.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cosh^2(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1+\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{3661}$$

$$-i\,\text{EllipticF}\left(ix+\frac{\pi}{2},-1\right)$$

input
```
Int[1/Sqrt[1 + Cosh[x]^2],x]
```

output
```
(-I)*EllipticF[Pi/2 + I*x, -1]
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

3.52.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

method	result	size
default	$-\frac{i\sqrt{\left(1+\cosh(x)^2\right)\sinh(x)^2}\,\sqrt{-\sinh(x)^2}\ \mathrm{EllipticF}(i\cosh(x),i)}{\sqrt{\cosh(x)^4-1}\,\sinh(x)}$	45

input
```
int(1/(1+cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*((1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)/(cosh(x)^4-1)^(1/2)*
EllipticF(I*cosh(x),I)/sinh(x)
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx$$
$$= -2\left(2\sqrt{2}+3\right)\sqrt{2\sqrt{2}-3}F(\arcsin\left(\sqrt{2\sqrt{2}-3}(\cosh{(x)}+\sinh{(x)})\right)|12\sqrt{2}+17)$$

input
```
integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*(2*sqrt(2) + 3)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) -
3)*(cosh(x) + sinh(x))), 12*sqrt(2) + 17)
```

3.52.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{\cosh^2{(x)}+1}}\,dx$$

input
```
integrate(1/(1+cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(cosh(x)**2 + 1), x)
```

3.52.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{\cosh{(x)}^2+1}}\,dx$$

input
```
integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cosh(x)^2 + 1), x)
```

3.52.8 Giac [F]

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{\cosh{(x)}^2+1}}\,dx$$

input
```
integrate(1/(1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cosh(x)^2 + 1), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{\cosh(x)^2+1}}\,dx$$

input
```
int(1/(cosh(x)^2 + 1)^(1/2),x)
```

output
```
int(1/(cosh(x)^2 + 1)^(1/2), x)
```

3.53 $\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx$

3.53.1 Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{\sqrt{-\sinh^2(x)}}$$

output
```
-arctanh(cosh(x))*sinh(x)/(-sinh(x)^2)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = \frac{\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)}{\sqrt{-\sinh^2(x)}}$$

input
```
Integrate[1/Sqrt[1 - Cosh[x]^2],x]
```

output
```
((-Log[Cosh[x/2]] + Log[Sinh[x/2]])*Sinh[x])/Sqrt[-Sinh[x]^2]
```

3.53.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3655, 3042, 3686, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{1-\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{ 3655}$$

$$\int \frac{1}{\sqrt{-\sinh^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin(ix)^2}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\sinh(x)\int \operatorname{csch}(x)dx}{\sqrt{-\sinh^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(x)\int i\csc(ix)dx}{\sqrt{-\sinh^2(x)}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{i\sinh(x)\int \csc(ix)dx}{\sqrt{-\sinh^2(x)}}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{\sinh(x)\text{arctanh}(\cosh(x))}{\sqrt{-\sinh^2(x)}}$$

input
```
Int[1/Sqrt[1 - Cosh[x]^2],x]
```

output
```
-((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[-Sinh[x]^2])
```

3.53.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

method	result	size
default	$-\frac{\sinh(x)\sqrt{-\cosh(x)^2}\arctan\left(\frac{1}{\sqrt{-\cosh(x)^2}}\right)}{\cosh(x)\sqrt{-\sinh(x)^2}}$	34
risch	$-\frac{e^{-x}(e^{2x}-1)\ln(e^x+1)}{\sqrt{-(e^{2x}-1)^2 e^{-2x}}}+\frac{e^{-x}(e^{2x}-1)\ln(e^x-1)}{\sqrt{-(e^{2x}-1)^2 e^{-2x}}}$	67

input
```
int(1/(1-cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sinh(x)*(-cosh(x)^2)^(1/2)*arctan(1/(-cosh(x)^2)^(1/2))/cosh(x)/(-sinh(x)
^2)^(1/2)
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = 2\arctan\left(\frac{\sqrt{-(e^{(4x)}-2e^{(2x)}+1)e^{(-2x)}}e^x}{\cosh(x)e^{(2x)}+(e^{(2x)}-1)\sinh(x)-\cosh(x)}\right)$$

input
```
integrate(1/(1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-(e^(4*x) - 2*e^(2*x) + 1)*e^(-2*x))*e^x/(cosh(x)*e^(2*x) +
(e^(2*x) - 1)*sinh(x) - cosh(x)))
```

3.53.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx$$

input
```
integrate(1/(1-cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - cosh(x)**2), x)
```

3.53.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = -i\log\left(e^{(-x)}+1\right)+i\log\left(e^{(-x)}-1\right)$$

input
```
integrate(1/(1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

3.53.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = -\frac{i\log\left(e^x+1\right)}{\operatorname{sgn}\left(-e^{(3x)}+e^x\right)}+\frac{i\log\left(|e^x-1|\right)}{\operatorname{sgn}\left(-e^{(3x)}+e^x\right)}$$

input
```
integrate(1/(1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-I*log(e^x + 1)/sgn(-e^(3*x) + e^x) + I*log(abs(e^x - 1))/sgn(-e^(3*x) + e
^x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\cosh\left(x\right)^2}}\,dx$$

input
```
int(1/(1 - cosh(x)^2)^(1/2),x)
```

output
```
int(1/(1 - cosh(x)^2)^(1/2), x)
```

3.54 $\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx$

3.54.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{\sqrt{\sinh^2(x)}}$$

output
```
-arctanh(cosh(x))*sinh(x)/(sinh(x)^2)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = \frac{\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)}{\sqrt{\sinh^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 + Cosh[x]^2],x]
```

output
```
((-Log[Cosh[x/2]] + Log[Sinh[x/2]])*Sinh[x])/Sqrt[Sinh[x]^2]
```

3.54.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 3655, 3042, 3686, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cosh^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-1+\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{3655}$$

$$\int \frac{1}{\sqrt{\sinh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sin(ix)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sinh(x)\int \operatorname{csch}(x)dx}{\sqrt{\sinh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\int i\csc(ix)dx}{\sqrt{\sinh^2(x)}}$$

$$\Big\downarrow \text{26}$$

$$\frac{i\sinh(x)\int \csc(ix)dx}{\sqrt{\sinh^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sinh(x)\operatorname{arctanh}(\cosh(x))}{\sqrt{\sinh^2(x)}}$$

input
```
Int[1/Sqrt[-1 + Cosh[x]^2],x]
```

output
```
-((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[Sinh[x]^2])
```

3.54.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.54.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2x)}{2}}\ \operatorname{arctanh}(\cosh(x))}{\sinh(x)}$	16
risch	$-\frac{e^{-x}(e^{2x}-1)\ln(e^x+1)}{\sqrt{(e^{2x}-1)^2 e^{-2x}}}+\frac{e^{-x}(e^{2x}-1)\ln(e^x-1)}{\sqrt{(e^{2x}-1)^2 e^{-2x}}}$	65

input
```
int(1/(cosh(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(sinh(x)^2)^(1/2)*arctanh(cosh(x))/sinh(x)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = -\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

input
```
integrate(1/(-1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

3.54.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{\cosh^2\left(x\right)-1}}\,dx$$

input
```
integrate(1/(-1+cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(cosh(x)**2 - 1), x)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = \log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

input
```
integrate(1/(-1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(-x) + 1) - log(e^(-x) - 1)
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = -\frac{\log(e^x+1)}{\operatorname{sgn}(e^{(3x)}-e^x)} + \frac{\log(|e^x-1|)}{\operatorname{sgn}(e^{(3x)}-e^x)}$$

input
```
integrate(1/(-1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(e^x + 1)/sgn(e^(3*x) - e^x) + log(abs(e^x - 1))/sgn(e^(3*x) - e^x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{\cosh(x)^2-1}}\,dx$$

input
```
int(1/(cosh(x)^2 - 1)^(1/2),x)
```

output
```
int(1/(cosh(x)^2 - 1)^(1/2), x)
```

3.55 $\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx$

3.55.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = -\frac{i\sqrt{1+\cosh^2(x)}\,\text{EllipticF}\left(\frac{\pi}{2}+ix,-1\right)}{\sqrt{-1-\cosh^2(x)}}$$

output
```
(-sinh(x)^2)^(1/2)/sinh(x)*EllipticF(cosh(x),I)*(1+cosh(x)^2)^(1/2)/(-1-co
sh(x)^2)^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = -\frac{i\sqrt{3+\cosh(2x)}\,\text{EllipticF}\left(ix,\frac{1}{2}\right)}{\sqrt{2}\sqrt{-3-\cosh(2x)}}$$

input
```
Integrate[1/Sqrt[-1 - Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[3 + Cosh[2*x]]*EllipticF[I*x, 1/2])/(Sqrt[2]*Sqrt[-3 - Cosh[2*x
]])
```

3.55.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3662, 3042, 3661}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\cosh^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-1-\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{3662}$$

$$\frac{\sqrt{\cosh^2(x)+1}\int \frac{1}{\sqrt{\cosh^2(x)+1}}dx}{\sqrt{-\cosh^2(x)-1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh^2(x)+1}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}}dx}{\sqrt{-\cosh^2(x)-1}}$$

$$\Big\downarrow \text{3661}$$

$$-\frac{i\sqrt{\cosh^2(x)+1}\,\text{EllipticF}\left(ix+\frac{\pi}{2},-1\right)}{\sqrt{-\cosh^2(x)-1}}$$

input
```
Int[1/Sqrt[-1 - Cosh[x]^2],x]
```

output
```
((-I)*Sqrt[1 + Cosh[x]^2]*EllipticF[Pi/2 + I*x, -1])/Sqrt[-1 - Cosh[x]^2]
```

3.55.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3661
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

rule 3662
```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2]   Int[1/Sqrt[1 + (b*Si
n[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{\sqrt{-\left(1+\cosh \left(x\right)^{2}\right) \sinh \left(x\right)^{2}}\, \sqrt{-\sinh \left(x\right)^{2}}\, \sqrt{1+\cosh \left(x\right)^{2}}\, \operatorname{EllipticF}\left(\cosh \left(x\right), i\right)}{\sqrt{1-\cosh \left(x\right)^{4}}\, \sinh \left(x\right) \sqrt{-1-\cosh \left(x\right)^{2}}}$	61

input
```
int(1/(-1-cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(1+cosh(x)^2)*sinh(x)^2)^(1/2)*(-sinh(x)^2)^(1/2)*(1+cosh(x)^2)^(1/2)/(1
-cosh(x)^4)^(1/2)*EllipticF(cosh(x),I)/sinh(x)/(-1-cosh(x)^2)^(1/2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = -2\sqrt{2\sqrt{2}-3}\left(-2i\sqrt{2}-3i\right)F(\arcsin\left(\sqrt{2\sqrt{2}-3}e^{x}\right) \mid 12\sqrt{2} + 17)$$

input
```
integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2*sqrt(2) - 3)*(-2*I*sqrt(2) - 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(
2) - 3)*e^x), 12*sqrt(2) + 17)
```

3.55.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cosh^2(x)-1}}\,dx$$

input
```
integrate(1/(-1-cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-cosh(x)**2 - 1), x)
```

3.55.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cosh(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-cosh(x)^2 - 1), x)
```

3.55.8 Giac [F]

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cosh(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-cosh(x)^2 - 1), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\cosh{(x)}^2-1}}\,dx$$

input
```
int(1/(- cosh(x)^2 - 1)^(1/2),x)
```

output
```
int(1/(- cosh(x)^2 - 1)^(1/2), x)
```

3.56 $\int \frac{1}{a+b\cosh^3(x)}\,dx$

3.56.1 Optimal result

Integrand size = 10, antiderivative size = 288

$$\int \frac{1}{a+b\cosh^3(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
2/3*arctanh((a^(1/3)-b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)+b^(1/3))^(1/2))/a
^(2/3)/(a^(1/3)-b^(1/3))^(1/2)/(a^(1/3)+b^(1/3))^(1/2)+2/3*arctanh((a^(1/3
)+(-1)^(1/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)
)/a^(2/3)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^
(1/2)+2/3*arctanh((a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)+
(-1)^(2/3)*b^(1/3))^(1/2))/a^(2/3)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)/(a^(
1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

3.56.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cosh^3(x)}\,dx=\$\text{Aborted}$$

input
```
Integrate[(a + b*Cosh[x]^3)^(-1),x]
```

output
```
$Aborted
```

3.56.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\cosh(x)\right)}-\frac{1}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}\cosh(x)-\sqrt[3]{a}\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cosh(x)\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a + b*Cosh[x]^3)^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) + b^(1/3)]])/(
3*a^(2/3)*Sqrt[a^(1/3) - b^(1/3)]*Sqrt[a^(1/3) + b^(1/3)]) + (2*ArcTanh[(S
qrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) - (-1)^(1/3)*b^(
1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^
(1/3)*b^(1/3)]) + (2*ArcTanh[(Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]*Tanh[x/2]
)/Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(2/3
)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)])
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.56.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left((a-b)\_Z^6+(-3a-3b)\_Z^4+(3a-3b)\_Z^2-a-b\right)}\frac{\left(-\_R^4+2\_R^2-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a-\_R^5b-2\_R^3a-2\_R^3b+\_Ra-\_Rb}\right)}{3}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(-1+(729a^6-729a^4b^2)\_Z^6-243a^4\_Z^4+27a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^x+\left(\frac{486a^6}{b}-486a^4b\right)\_R^5+\left(-\frac{81a^5}{b}+\right.\right.$

input
```
int(1/(a+b*cosh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((-_R^4+2*_R^2-1)/(_R^5*a-_R^5*b-2*_R^3*a-2*_R^3*b+_R*a-_R*b)*ln(ta
nh(1/2*x)-_R),_R=RootOf((a-b)*_Z^6+(-3*a-3*b)*_Z^4+(3*a-3*b)*_Z^2-a-b))
```

3.56.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 18612, normalized size of antiderivative = 64.62

$$\int \frac{1}{a+b\cosh^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cosh^3(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)**3),x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{1}{a+b\cosh^3(x)}\,dx = \int \frac{1}{b\cosh{(x)}^3+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^3),x, algorithm="maxima")
```

output
```
integrate(1/(b*cosh(x)^3 + a), x)
```

3.56.8 Giac [F]

$$\int \frac{1}{a+b\cosh^3(x)}\,dx = \int \frac{1}{b\cosh{(x)}^3+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^3),x, algorithm="giac")
```

output
```
integrate(1/(b*cosh(x)^3 + a), x)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.20

$$\int \frac{1}{a+b\cosh^3(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{\left(-4\,\mathrm{e}^{x}+\mathrm{root}(729\,a^4\,b^2\,d^6-729\,a^6\,d^6+243\,a^4\,d^4-27\,a^2\,d^2+1,d,k)\;b+\mathrm{root}(729\,a^4\,b^2\,d^6-7\right.}{}\right.$$
$$-729\,a^6\,d^6+243\,a^4\,d^4-27\,a^2\,d^2+1,d,k)$$

input
```
int(1/(a + b*cosh(x)^3),x)
```

output
```
symsum(log(-(24576*(root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*
a^2*d^2 + 1, d, k)*b - 4*exp(x) + 54*root(729*a^4*b^2*d^6 - 729*a^6*d^6 +
243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^3*a^2*b + 108*root(729*a^4*b^2*d^6 - 7
29*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^4*a^3*b + 81*root(729*a^4
*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^4*b + 24*
root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2
*a^2*exp(x) + 216*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^
2*d^2 + 1, d, k)^3*a^3*exp(x) + 108*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 2
43*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^4*a^4*exp(x) - 324*root(729*a^4*b^2*d^6
 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^5*exp(x) + 12*roo
t(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2*a*
b - 20*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1,
d, k)*a*exp(x) + 27*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*
a^2*d^2 + 1, d, k)^4*a^2*b^2*exp(x) + 405*root(729*a^4*b^2*d^6 - 729*a^6*d
^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^3*b^2*exp(x)))/b^5)*root(729*
a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k), k, 1, 6)
```

3.57 $\int \frac{1}{a-b\cosh^3(x)}\,dx$

3.57.1 Optimal result

Integrand size = 11, antiderivative size = 288

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
2/3*arctanh((a^(1/3)+b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)-b^(1/3))^(1/2))/a
^(2/3)/(a^(1/3)-b^(1/3))^(1/2)/(a^(1/3)+b^(1/3))^(1/2)+2/3*arctanh((a^(1/3
)-(-1)^(1/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)
)/a^(2/3)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^
(1/2)+2/3*arctanh((a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)*tanh(1/2*x)/(a^(1/3)-
(-1)^(2/3)*b^(1/3))^(1/2))/a^(2/3)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)/(a^(
1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.36

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = -\frac{2}{3}\text{RootSum}\left[b+3b\#1^2-8a\#1^3+3b\#1^4+b\#1^6\&,\frac{x\#1+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1}{b-4a\#1+2b\#1^2+b\#1^4}\&\right]$$

input
```
Integrate[(a - b*Cosh[x]^3)^(-1),x]
```

output
```
(-2*RootSum[b + 3*b*#1^2 - 8*a*#1^3 + 3*b*#1^4 + b*#1^6 & , (x*#1 + 2*Log[
-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1)/(b - 4*a*#1 + 2*
b*#1^2 + b*#1^4) & ])/3
```

3.57.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cosh^3(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a-b\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \ 3692$$

$$\int \left(\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-\sqrt[3]{b}\cosh(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\cosh(x)\right)}+\frac{1}{3a^{2/3}\left(\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\cosh(x)\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Cosh[x]^3)^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a^(1/3) + b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) - b^(1/3)]])/(
3*a^(2/3)*Sqrt[a^(1/3) - b^(1/3)]*Sqrt[a^(1/3) + b^(1/3)]) + (2*ArcTanh[(S
qrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tanh[x/2])/Sqrt[a^(1/3) + (-1)^(1/3)*b^(
1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^
(1/3)*b^(1/3)]) + (2*ArcTanh[(Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)]*Tanh[x/2]
)/Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]])/(3*a^(2/3)*Sqrt[a^(1/3) - (-1)^(2/3
)*b^(1/3)]*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)])
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.57.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+(-3a+3b)\_Z^4+(3a+3b)\_Z^2-a+b\right)}\frac{\left(-\_R^4+2\_R^2-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a+\_R^5b-2\_R^3a+2\_R^3b+\_Ra+\_Rb}\right)}{3}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+(729a^6-729a^4b^2)\_Z^6-243a^4\_Z^4+27a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^x+\left(-\frac{486a^6}{b}+486a^4b\right)\_R^5+\left(\frac{81a^5}{b}-\right.\right.$

input
```
int(1/(a-b*cosh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((-_R^4+2*_R^2-1)/(_R^5*a+_R^5*b-2*_R^3*a+2*_R^3*b+_R*a+_R*b)*ln(ta
nh(1/2*x)-_R),_R=RootOf((a+b)*_Z^6+(-3*a+3*b)*_Z^4+(3*a+3*b)*_Z^2-a+b))
```

3.57.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 18612, normalized size of antiderivative = 64.62

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cosh(x)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cosh(x)**3),x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = \int -\frac{1}{b\cosh{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^3),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cosh(x)^3 - a), x)
```

3.57.8 Giac [F]

$$\int \frac{1}{a-b\cosh^3(x)}\,dx = \int -\frac{1}{b\cosh{(x)}^3-a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^3),x, algorithm="giac")
```

output
```
integrate(-1/(b*cosh(x)^3 - a), x)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.20

$$\int \frac{1}{a-b\cosh^3(x)}\,dx$$
$$=\sum_{k=1}^{6}\ln\left(-\frac{\left(4\,e^x+\mathrm{root}(729\,a^4\,b^2\,d^6-729\,a^6\,d^6+243\,a^4\,d^4-27\,a^2\,d^2+1,d,k)\;b+\mathrm{root}(729\,a^4\,b^2\,d^6-729\right.}{}\right.$$
$$-729\,a^6\,d^6+243\,a^4\,d^4-27\,a^2\,d^2+1,d,k)$$

input
```
int(1/(a - b*cosh(x)^3),x)
```

output
```
symsum(log(-(24576*(4*exp(x) + root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^
4*d^4 - 27*a^2*d^2 + 1, d, k)*b + 54*root(729*a^4*b^2*d^6 - 729*a^6*d^6 +
243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^3*a^2*b + 108*root(729*a^4*b^2*d^6 - 7
29*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^4*a^3*b + 81*root(729*a^4
*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^4*b - 24*
root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2
*a^2*exp(x) - 216*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^
2*d^2 + 1, d, k)^3*a^3*exp(x) - 108*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 2
43*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^4*a^4*exp(x) + 324*root(729*a^4*b^2*d^6
 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^5*exp(x) + 12*roo
t(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^2*a*
b + 20*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1,
d, k)*a*exp(x) - 27*root(729*a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*
a^2*d^2 + 1, d, k)^4*a^2*b^2*exp(x) - 405*root(729*a^4*b^2*d^6 - 729*a^6*d
^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k)^5*a^3*b^2*exp(x)))/b^5)*root(729*
a^4*b^2*d^6 - 729*a^6*d^6 + 243*a^4*d^4 - 27*a^2*d^2 + 1, d, k), k, 1, 6)
```

3.58 $\int \frac{1}{1+\cosh^3(x)}\,dx$

3.58.1 Optimal result

Integrand size = 8, antiderivative size = 91

$$\int \frac{1}{1+\cosh^3(x)}\,dx = -\frac{2\sqrt[4]{-\frac{1}{3}}\arctan\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)} - \frac{2\sqrt[4]{-\frac{1}{3}}\operatorname{arctanh}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)} + \frac{\sinh(x)}{3(1+\cosh(x))}$$

output
```
-2/9*(-1)^(1/4)*3^(3/4)*arctan((-1)^(3/4)*3^(1/4)*tanh(1/2*x))/(1-(-1)^(1/
3))-2/9*(-1)^(1/4)*3^(3/4)*arctanh((-1)^(3/4)*3^(1/4)*tanh(1/2*x))/(1+(-1)
^(2/3))+1/3*sinh(x)/(1+cosh(x))
```

3.58.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{1}{1+\cosh^3(x)}\,dx = \frac{1}{18}\left(-\sqrt{6+2i\sqrt{3}}\left(3i+\sqrt{3}\right)\arctan\left(\frac{\left(3+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{6-2i\sqrt{3}}}\right) - \sqrt{6-2i\sqrt{3}}\left(-3i+\sqrt{3}\right)\arctan\left(\frac{\left(3-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{6+2i\sqrt{3}}}\right) + 6\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(1 + Cosh[x]^3)^(-1),x]
```

output
```
(-(Sqrt[6 + (2*I)*Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((3 + I*Sqrt[3])*Tanh[x/
2])/Sqrt[6 - (2*I)*Sqrt[3]]]) - Sqrt[6 - (2*I)*Sqrt[3]]*(-3*I + Sqrt[3])*A
rcTan[((3 - I*Sqrt[3])*Tanh[x/2])/Sqrt[6 + (2*I)*Sqrt[3]]] + 6*Tanh[x/2])/
18
```

3.58.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^3(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1+\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{3\left(\sqrt[3]{-1}\cosh(x)-1\right)}-\frac{1}{3\left(-(-1)^{2/3}\cosh(x)-1\right)}-\frac{1}{3(-\cosh(x)-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt[4]{-\frac{1}{3}}\arctan\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1-\sqrt[3]{-1}\right)}-\frac{2\sqrt[4]{-\frac{1}{3}}\text{arctanh}\left((-1)^{3/4}\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)\right)}{3\left(1+(-1)^{2/3}\right)}+\frac{\sinh(x)}{3(\cosh(x)+1)}$$

input
```
Int[(1 + Cosh[x]^3)^(-1),x]
```

output
```
(-2*(-1/3)^(1/4)*ArcTan[(-1)^(3/4)*3^(1/4)*Tanh[x/2]])/(3*(1 - (-1)^(1/3))
) - (2*(-1/3)^(1/4)*ArcTanh[(-1)^(3/4)*3^(1/4)*Tanh[x/2]])/(3*(1 + (-1)^(2
/3))) + Sinh[x]/(3*(1 + Cosh[x]))
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.58.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{2}{3(e^{x}+1)}+\left(\sum_{\_R=\operatorname{RootOf}(243\_Z^{4}-27\_Z^{2}+1)}\_R\ln\left(-162\_R^{3}+27\_R^{2}+9\_R+e^{x}-2\right)\right)$
default	$\frac{\tanh\left(\frac{x}{2}\right)}{3}+\frac{3^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+\frac{\sqrt{3}}{3}}{\tanh\left(\frac{x}{2}\right)^{2}-\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+\frac{\sqrt{3}}{3}}\right)+2\arctan\left(\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+1\right)+2\arctan\left(\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)-1\right)\right)}{36}-\frac{3^{\frac{1}{4}}\cdots}{}$

input
```
int(1/(1+cosh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(exp(x)+1)+sum(_R*ln(-162*_R^3+27*_R^2+9*_R+exp(x)-2),_R=RootOf(243*_
Z^4-27*_Z^2+1))
```

3.58.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.48

$$\int \frac{1}{1+\cosh^3(x)}\,dx = -\frac{\left(\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)+\sqrt{3}\right)\sqrt{2i\sqrt{3}+6}\log\left(i\sqrt{3}+i\sqrt{2i\sqrt{3}+6}+2\cosh(x)+2\sinh(x)-1\right.}{}$$

input
```
integrate(1/(1+cosh(x)^3),x, algorithm="fricas")
```

output
```
-1/18*((sqrt(3)*cosh(x) + sqrt(3)*sinh(x) + sqrt(3))*sqrt(2*I*sqrt(3) + 6)
*log(I*sqrt(3) + I*sqrt(2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) - 1) - (s
qrt(3)*cosh(x) + sqrt(3)*sinh(x) + sqrt(3))*sqrt(2*I*sqrt(3) + 6)*log(I*sq
rt(3) - I*sqrt(2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) - 1) - (sqrt(3)*co
sh(x) + sqrt(3)*sinh(x) + sqrt(3))*sqrt(-2*I*sqrt(3) + 6)*log(-I*sqrt(3) +
 I*sqrt(-2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) - 1) + (sqrt(3)*cosh(x)
+ sqrt(3)*sinh(x) + sqrt(3))*sqrt(-2*I*sqrt(3) + 6)*log(-I*sqrt(3) - I*sqr
t(-2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) - 1) + 12)/(cosh(x) + sinh(x)
+ 1)
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(73) = 146.

Time = 1.51 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.63

$$\int \frac{1}{1+\cosh^3(x)}\,dx = -\frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tanh^2\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{18+18\sqrt{3}} - \frac{3\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tanh^2\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{18+18\sqrt{3}} + \frac{3\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tanh^2\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{18+18\sqrt{3}} + \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tanh^2\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{18+18\sqrt{3}} + \frac{6\tanh\left(\frac{x}{2}\right)}{18+18\sqrt{3}} + \frac{6\sqrt{3}\tanh\left(\frac{x}{2}\right)}{18+18\sqrt{3}} - \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)-1\right)}{18+18\sqrt{3}} - \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+1\right)}{18+18\sqrt{3}}$$

input
```
integrate(1/(1+cosh(x)**3),x)
```

output
```
-2*sqrt(2)*3**(3/4)*log(36*tanh(x/2)**2 - 12*sqrt(2)*3**(3/4)*tanh(x/2) +
12*sqrt(3))/(18 + 18*sqrt(3)) - 3*sqrt(2)*3**(1/4)*log(36*tanh(x/2)**2 - 1
2*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/(18 + 18*sqrt(3)) + 3*sqrt(2)*3
**(1/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/
(18 + 18*sqrt(3)) + 2*sqrt(2)*3**(3/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3*
*(3/4)*tanh(x/2) + 12*sqrt(3))/(18 + 18*sqrt(3)) + 6*tanh(x/2)/(18 + 18*sq
rt(3)) + 6*sqrt(3)*tanh(x/2)/(18 + 18*sqrt(3)) - 2*sqrt(2)*3**(3/4)*atan(s
qrt(2)*3**(1/4)*tanh(x/2) - 1)/(18 + 18*sqrt(3)) - 2*sqrt(2)*3**(3/4)*atan
(sqrt(2)*3**(1/4)*tanh(x/2) + 1)/(18 + 18*sqrt(3))
```

3.58.7 Maxima [F]

$$\int \frac{1}{1+\cosh^3(x)}\,dx = \int \frac{1}{\cosh(x)^3+1}\,dx$$

input
```
integrate(1/(1+cosh(x)^3),x, algorithm="maxima")
```

output
```
-2/3/(e^x + 1) - integrate(2/3*(e^(3*x) - 4*e^(2*x) + e^x)/(e^(4*x) - 2*e^
(3*x) + 6*e^(2*x) - 2*e^x + 1), x)
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.02

$$\int \frac{1}{1+\cosh^3(x)}\,dx = \frac{1}{18}\sqrt{6\sqrt{3}+9}\log\left(4\left(2\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{6\sqrt{3}+9}+6e^x-3\right)^2 + 4\left(\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{3}\right)^2\right) - \frac{1}{18}\sqrt{6\sqrt{3}+9}\log\left(4\left(2\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{6\sqrt{3}+9}-6e^x+3\right)^2 + 4\left(\sqrt{3}\sqrt{6\sqrt{3}+9}+3\sqrt{3}\right)^2\right) - \frac{\sqrt{3}\sqrt{6\sqrt{3}+9}\arctan\left(\frac{3\left(\sqrt{2\sqrt{3}-3}+2e^x-1\right)}{\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{3}}\right)}{9\left(2\sqrt{3}+3\right)} - \frac{\sqrt{3}\sqrt{6\sqrt{3}+9}\arctan\left(-\frac{3\left(\sqrt{2\sqrt{3}-3}-2e^x+1\right)}{\sqrt{3}\sqrt{6\sqrt{3}+9}+3\sqrt{3}}\right)}{9\left(2\sqrt{3}+3\right)} - \frac{2}{3\left(e^x+1\right)}$$

input
```
integrate(1/(1+cosh(x)^3),x, algorithm="giac")
```

output
```
1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(6*s
qrt(3) + 9) + 6*e^x - 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(3))^2
) - 1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt
(6*sqrt(3) + 9) - 6*e^x + 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3
))^2) - 1/9*sqrt(3)*sqrt(6*sqrt(3) + 9)*arctan(3*(sqrt(2*sqrt(3) - 3) + 2*
e^x - 1)/(sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(3)))/(2*sqrt(3) + 3) - 1/9*
sqrt(3)*sqrt(6*sqrt(3) + 9)*arctan(-3*(sqrt(2*sqrt(3) - 3) - 2*e^x + 1)/(s
qrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3)))/(2*sqrt(3) + 3) - 2/3/(e^x + 1)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.20

$$
\begin{aligned}
\int \frac{1}{1+\cosh^3(x)}\,dx = {} & \ln\left(\frac{128}{9}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(\frac{160}{3}\right.\right. \\
& \left.\left.+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(384\,\mathrm{e}^x+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(1152\,\mathrm{e}^x-864\right)-192\right)\right.\right. \\
& \left.\left.-\frac{32\,\mathrm{e}^x}{3}\right)-\frac{32\,\mathrm{e}^x}{3}\right)\,\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}+\ln\left(\frac{128}{9}+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(\frac{160}{3}\right.\right. \\
& \left.\left.+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(384\,\mathrm{e}^x+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(1152\,\mathrm{e}^x-864\right)-192\right)\right.\right. \\
& \left.\left.-\frac{32\,\mathrm{e}^x}{3}\right)-\frac{32\,\mathrm{e}^x}{3}\right)\,\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}-\ln\left(\frac{128}{9}-\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(\frac{160}{3}\right.\right. \\
& \left.\left.+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(192+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(1152\,\mathrm{e}^x-864\right)-384\,\mathrm{e}^x\right)\right.\right. \\
& \left.\left.-\frac{32\,\mathrm{e}^x}{3}\right)-\frac{32\,\mathrm{e}^x}{3}\right)\,\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}-\ln\left(\frac{128}{9}-\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(\frac{160}{3}\right.\right. \\
& \left.\left.+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(192+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,\left(1152\,\mathrm{e}^x-864\right)-384\,\mathrm{e}^x\right)\right.\right. \\
& \left.\left.-\frac{32\,\mathrm{e}^x}{3}\right)-\frac{32\,\mathrm{e}^x}{3}\right)\,\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}-\frac{2}{3\,\left(\mathrm{e}^x+1\right)}
\end{aligned}
$$

input
```
int(1/(cosh(x)^3 + 1),x)
```

output
```
log((1/18 - (3^(1/2)*1i)/54)^(1/2)*((1/18 - (3^(1/2)*1i)/54)^(1/2)*(384*ex
p(x) + (1/18 - (3^(1/2)*1i)/54)^(1/2)*(1152*exp(x) - 864) - 192) - (32*exp
(x))/3 + 160/3) - (32*exp(x))/3 + 128/9)*(1/18 - (3^(1/2)*1i)/54)^(1/2) +
log(((3^(1/2)*1i)/54 + 1/18)^(1/2)*(((3^(1/2)*1i)/54 + 1/18)^(1/2)*(384*ex
p(x) + ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(1152*exp(x) - 864) - 192) - (32*exp
(x))/3 + 160/3) - (32*exp(x))/3 + 128/9)*((3^(1/2)*1i)/54 + 1/18)^(1/2) -
log(128/9 - (1/18 - (3^(1/2)*1i)/54)^(1/2)*((1/18 - (3^(1/2)*1i)/54)^(1/2)
*((1/18 - (3^(1/2)*1i)/54)^(1/2)*(1152*exp(x) - 864) - 384*exp(x) + 192) -
 (32*exp(x))/3 + 160/3) - (32*exp(x))/3)*(1/18 - (3^(1/2)*1i)/54)^(1/2) -
log(128/9 - ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(((3^(1/2)*1i)/54 + 1/18)^(1/2)
*(((3^(1/2)*1i)/54 + 1/18)^(1/2)*(1152*exp(x) - 864) - 384*exp(x) + 192) -
 (32*exp(x))/3 + 160/3) - (32*exp(x))/3)*((3^(1/2)*1i)/54 + 1/18)^(1/2) -
2/(3*(exp(x) + 1))
```

3.58. $\int \frac{1}{1+\cosh^3(x)}\,dx$

3.59 $\int \frac{1}{1-\cosh^3(x)} dx$

3.59.1 Optimal result

Integrand size = 10, antiderivative size = 95

$$\int \frac{1}{1-\cosh^3(x)} dx = -\frac{2\sqrt[4]{-1}\arctan\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)} - \frac{2\sqrt[4]{-1}\text{arctanh}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)} - \frac{\sinh(x)}{3(1-\cosh(x))}$$

output
```
-2/3*(-1)^(1/4)*arctan(1/3*(-1)^(3/4)*tanh(1/2*x)*3^(3/4))*3^(1/4)/(1-(-1)
^(2/3))-2/3*(-1)^(1/4)*arctanh(1/3*(-1)^(3/4)*tanh(1/2*x)*3^(3/4))*3^(1/4)
/(1+(-1)^(1/3))-1/3*sinh(x)/(1-cosh(x))
```

3.59.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{1-\cosh^3(x)} dx = \$\text{Aborted}$$

input
```
Integrate[(1 - Cosh[x]^3)^(-1),x]
```

output
```
$Aborted
```

3.59.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh^3(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$
$$\Big\downarrow \text{3692}$$
$$\int \left(\frac{1}{3\left(\sqrt[3]{-1}\cosh(x)+1\right)}+\frac{1}{3\left(1-(-1)^{2/3}\cosh(x)\right)}+\frac{1}{3(1-\cosh(x))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2\sqrt[4]{-1}\arctan\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1-(-1)^{2/3}\right)}-\frac{2\sqrt[4]{-1}\text{arctanh}\left(\frac{(-1)^{3/4}\tanh\left(\frac{x}{2}\right)}{\sqrt[4]{3}}\right)}{3^{3/4}\left(1+\sqrt[3]{-1}\right)}-\frac{\sinh(x)}{3(1-\cosh(x))}$$

input
```
Int[(1 - Cosh[x]^3)^(-1),x]
```

output
```
(-2*(-1)^(1/4)*ArcTan[((-1)^(3/4)*Tanh[x/2])/3^(1/4)])/(3^(3/4)*(1 - (-1)^
(2/3))) - (2*(-1)^(1/4)*ArcTanh[((-1)^(3/4)*Tanh[x/2])/3^(1/4)])/(3^(3/4)*
(1 + (-1)^(1/3))) - Sinh[x]/(3*(1 - Cosh[x]))
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.59.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

method	result
risch	$\frac{2}{3(\mathrm{e}^{x}-1)}+\left(\sum_{\_R=\mathrm{RootOf}(243\_Z^{4}-27\_Z^{2}+1)}\_R\ln\left(162\_R^{3}-27\_R^{2}-9\_R+\mathrm{e}^{x}+2\right)\right)$
default	$\frac{3^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+\sqrt{3}}{\tanh\left(\frac{x}{2}\right)^{2}-\sqrt{2}\,3^{\frac{1}{4}}\tanh\left(\frac{x}{2}\right)+\sqrt{3}}\right)+2\arctan\left(\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}+1\right)+2\arctan\left(\frac{3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)\sqrt{2}}{3}-1\right)\right)}{12}-\frac{3^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{\tanh}{\tanh}\right.\right.}{}$

input
```
int(1/(1-cosh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2/3/(exp(x)-1)+sum(_R*ln(162*_R^3-27*_R^2-9*_R+exp(x)+2),_R=RootOf(243*_Z^
4-27*_Z^2+1))
```

3.59.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.46

$$\int \frac{1}{1-\cosh^3(x)}\,dx = -\frac{\left(\sqrt{3}\cosh\left(x\right)+\sqrt{3}\sinh\left(x\right)-\sqrt{3}\right)\sqrt{-2i\sqrt{3}+6}\log\left(i\sqrt{3}+i\sqrt{-2i\sqrt{3}+6}+2\cosh\left(x\right)+2\sinh\left(x\right)+\cdots\right.}{}$$

input
```
integrate(1/(1-cosh(x)^3),x, algorithm="fricas")
```

output
```
-1/18*((sqrt(3)*cosh(x) + sqrt(3)*sinh(x) - sqrt(3))*sqrt(-2*I*sqrt(3) + 6
)*log(I*sqrt(3) + I*sqrt(-2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) + 1) -
(sqrt(3)*cosh(x) + sqrt(3)*sinh(x) - sqrt(3))*sqrt(-2*I*sqrt(3) + 6)*log(I
*sqrt(3) - I*sqrt(-2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) + 1) - (sqrt(3
)*cosh(x) + sqrt(3)*sinh(x) - sqrt(3))*sqrt(2*I*sqrt(3) + 6)*log(-I*sqrt(3
) + I*sqrt(2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) + 1) + (sqrt(3)*cosh(x
) + sqrt(3)*sinh(x) - sqrt(3))*sqrt(2*I*sqrt(3) + 6)*log(-I*sqrt(3) - I*sq
rt(2*I*sqrt(3) + 6) + 2*cosh(x) + 2*sinh(x) + 1) - 12)/(cosh(x) + sinh(x)
- 1)
```

3.59.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(78) = 156$.

Time = 1.56 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.37

$$\int \frac{1}{1-\cosh^3(x)}\,dx = -\frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{12} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{36} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{36} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{12} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}-1\right)}{18} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}-1\right)}{6} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}+1\right)}{18} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}+1\right)}{6} + \frac{1}{3\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cosh(x)**3),x)
```

output
```
-sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(x/2) + 4*sq
rt(3))/12 - sqrt(2)*3**(3/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(
x/2) + 4*sqrt(3))/36 + sqrt(2)*3**(3/4)*log(4*tanh(x/2)**2 + 4*sqrt(2)*3**
(1/4)*tanh(x/2) + 4*sqrt(3))/36 + sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 + 4*
sqrt(2)*3**(1/4)*tanh(x/2) + 4*sqrt(3))/12 - sqrt(2)*3**(3/4)*atan(sqrt(2)
*3**(3/4)*tanh(x/2)/3 - 1)/18 + sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*tan
h(x/2)/3 - 1)/6 - sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 + 1)/
18 + sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 + 1)/6 + 1/(3*tanh
(x/2))
```

3.59.7 Maxima [F]

$$\int \frac{1}{1-\cosh^3(x)}\,dx = \int -\frac{1}{\cosh(x)^3-1}\,dx$$

input
```
integrate(1/(1-cosh(x)^3),x, algorithm="maxima")
```

output
```
2/3/(e^x - 1) + integrate(2/3*(e^(3*x) + 4*e^(2*x) + e^x)/(e^(4*x) + 2*e^(
3*x) + 6*e^(2*x) + 2*e^x + 1), x)
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.89

$$\int \frac{1}{1-\cosh^3(x)}\,dx = -\frac{1}{18}\sqrt{6\sqrt{3}+9}\log\left(4\left(2\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{6\sqrt{3}+9}+6e^x+3\right)^2 + 4\left(\sqrt{3}\sqrt{6\sqrt{3}+9}+3\sqrt{3}\right)^2\right) + \frac{1}{18}\sqrt{6\sqrt{3}+9}\log\left(4\left(2\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{6\sqrt{3}+9}-6e^x-3\right)^2 + 4\left(\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{3}\right)^2\right) + \frac{\sqrt{3}\sqrt{6\sqrt{3}+9}\arctan\left(\frac{3\left(\sqrt{2\sqrt{3}-3}+2e^x+1\right)}{\sqrt{3}\sqrt{6\sqrt{3}+9}+3\sqrt{3}}\right)}{9\left(2\sqrt{3}+3\right)} + \frac{\sqrt{3}\sqrt{6\sqrt{3}+9}\arctan\left(-\frac{3\left(\sqrt{2\sqrt{3}-3}-2e^x-1\right)}{\sqrt{3}\sqrt{6\sqrt{3}+9}-3\sqrt{3}}\right)}{9\left(2\sqrt{3}+3\right)} + \frac{2}{3\left(e^x-1\right)}$$

input
```
integrate(1/(1-cosh(x)^3),x, algorithm="giac")
```

output
```
-1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(6*
sqrt(3) + 9) + 6*e^x + 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3))^
2) + 1/18*sqrt(6*sqrt(3) + 9)*log(4*(2*sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqr
t(6*sqrt(3) + 9) - 6*e^x - 3)^2 + 4*(sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(
3))^2) + 1/9*sqrt(3)*sqrt(6*sqrt(3) + 9)*arctan(3*(sqrt(2*sqrt(3) - 3) + 2
*e^x + 1)/(sqrt(3)*sqrt(6*sqrt(3) + 9) + 3*sqrt(3)))/(2*sqrt(3) + 3) + 1/9
*sqrt(3)*sqrt(6*sqrt(3) + 9)*arctan(-3*(sqrt(2*sqrt(3) - 3) - 2*e^x - 1)/(
sqrt(3)*sqrt(6*sqrt(3) + 9) - 3*sqrt(3)))/(2*sqrt(3) + 3) + 2/3/(e^x - 1)
```

3.59. $\int \frac{1}{1-\cosh^3(x)}\,dx$

3.59.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.11

$$\int \frac{1}{1-\cosh^3(x)}\,dx = \ln\left(\frac{32\,e^x}{3}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(\frac{32\,e^x}{3}-\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(384\,e^x+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,(1152\,e^x+864)+192\right)+\frac{160}{3}\right)+\frac{128}{9}\right)\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}+\ln\left(\frac{32\,e^x}{3}+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(\frac{32\,e^x}{3}-\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(384\,e^x+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,(1152\,e^x+864)+192\right)+\frac{160}{3}\right)+\frac{128}{9}\right)\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}-\ln\left(\frac{32\,e^x}{3}-\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(\frac{32\,e^x}{3}+\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(384\,e^x-\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,(1152\,e^x+864)+192\right)+\frac{160}{3}\right)+\frac{128}{9}\right)\sqrt{\frac{1}{18}-\frac{\sqrt{3}\,1\mathrm{i}}{54}}-\ln\left(\frac{32\,e^x}{3}-\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(\frac{32\,e^x}{3}+\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\left(384\,e^x-\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}\,(1152\,e^x+864)+192\right)+\frac{160}{3}\right)+\frac{128}{9}\right)\sqrt{\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{54}}+\frac{2}{3\,(e^x-1)}$$

input
```
int(-1/(cosh(x)^3 - 1),x)
```

output
```
log((32*exp(x))/3 + (1/18 - (3^(1/2)*1i)/54)^(1/2)*((32*exp(x))/3 - (1/18
- (3^(1/2)*1i)/54)^(1/2)*(384*exp(x) + (1/18 - (3^(1/2)*1i)/54)^(1/2)*(115
2*exp(x) + 864) + 192) + 160/3) + 128/9)*(1/18 - (3^(1/2)*1i)/54)^(1/2) +
log((32*exp(x))/3 + ((3^(1/2)*1i)/54 + 1/18)^(1/2)*((32*exp(x))/3 - ((3^(1
/2)*1i)/54 + 1/18)^(1/2)*(384*exp(x) + ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(115
2*exp(x) + 864) + 192) + 160/3) + 128/9)*((3^(1/2)*1i)/54 + 1/18)^(1/2) -
log((32*exp(x))/3 - (1/18 - (3^(1/2)*1i)/54)^(1/2)*((32*exp(x))/3 + (1/18
- (3^(1/2)*1i)/54)^(1/2)*(384*exp(x) - (1/18 - (3^(1/2)*1i)/54)^(1/2)*(115
2*exp(x) + 864) + 192) + 160/3) + 128/9)*(1/18 - (3^(1/2)*1i)/54)^(1/2) -
log((32*exp(x))/3 - ((3^(1/2)*1i)/54 + 1/18)^(1/2)*((32*exp(x))/3 + ((3^(1
/2)*1i)/54 + 1/18)^(1/2)*(384*exp(x) - ((3^(1/2)*1i)/54 + 1/18)^(1/2)*(115
2*exp(x) + 864) + 192) + 160/3) + 128/9)*((3^(1/2)*1i)/54 + 1/18)^(1/2) +
2/(3*(exp(x) - 1))
```

3.59. $\int \frac{1}{1-\cosh^3(x)}\,dx$

3.60 $\int \frac{1}{a+b\cosh^4(x)}\,dx$

3.60.1 Optimal result

Integrand size = 10, antiderivative size = 361

$$\begin{aligned}\int \frac{1}{a+b\cosh^4(x)}\,dx &= \frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{a+b}}\\ &- \frac{\sqrt{\sqrt{a}-\sqrt{a+b}}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{a+b}}\\ &- \frac{\sqrt{\sqrt{a}+\sqrt{a+b}}\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{a+b}}\tanh(x)+\sqrt{a}\tanh^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt{a+b}}\\ &+ \frac{\sqrt{\sqrt{a}+\sqrt{a+b}}\log\left(\sqrt{a+b}+\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{a+b}}\tanh(x)+\sqrt{a}\tanh^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt{a+b}}\end{aligned}$$

output
```
1/4*arctanh(((a^(1/2)+(a+b)^(1/2))^(1/2)-a^(1/4)*2^(1/2)*tanh(x))/(a^(1/2)
-(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1/
2)-1/4*arctanh(((a^(1/2)+(a+b)^(1/2))^(1/2)+a^(1/4)*2^(1/2)*tanh(x))/(a^(1
/2)-(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^
(1/2)-1/8*ln((a+b)^(1/2)-a^(1/4)*2^(1/2)*(a^(1/2)+(a+b)^(1/2))^(1/2)*tanh(
x)+a^(1/2)*tanh(x)^2)*(a^(1/2)+(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1
/2)+1/8*ln((a+b)^(1/2)+a^(1/4)*2^(1/2)*(a^(1/2)+(a+b)^(1/2))^(1/2)*tanh(x)
+a^(1/2)*tanh(x)^2)*(a^(1/2)+(a+b)^(1/2))^(1/2)/a^(3/4)*2^(1/2)/(a+b)^(1/2
)
```

3.60.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.34

$$\int \frac{1}{a+b\cosh^4(x)}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}}$$

input
```
Integrate[(a + b*Cosh[x]^4)^(-1),x]
```

output
```
-1/2*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[-a + I*Sqrt[a]*Sqrt[b]]]/(Sqrt[a]*Sqrt[
-a + I*Sqrt[a]*Sqrt[b]]) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + I*Sqrt[a]*Sq
rt[b]]]/(2*Sqrt[a]*Sqrt[a + I*Sqrt[a]*Sqrt[b]])
```

3.60.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.50, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3688, 1483, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^4}dx$$

$\Big\downarrow$ 3688

$$\int \frac{1-\coth^2(x)}{(a+b)\coth^4(x)-2a\coth^2(x)+a}d\coth(x)$$

$\Big\downarrow$ 1483

$$\frac{\sqrt[4]{a+b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\coth(x)}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\sqrt[4]{a+b}\int \frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\coth(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

$\Big\downarrow$ 1142

$$\frac{\sqrt[4]{a+b}\left(-\frac{\sqrt[4]{a}(\sqrt{a}-\sqrt{a+b})\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{1}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}(a+b)^{5/4}}-\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int -\frac{\sqrt{2}\Big(\cdots}{\coth^2(x)-\cdots}\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+$$

$$\frac{\sqrt[4]{a+b}\left(\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\sqrt{2}\left(\sqrt{2}\coth(x)+\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}\right)}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)-\frac{\sqrt[4]{a}(\sqrt{a}-\sqrt{a+b})\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{\coth^2(x)+\cdots}}{\sqrt{2}(a+b\cdots}\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

$\Big\downarrow$ 25

3.60. $\int \frac{1}{a+b\cosh^4(x)}\,dx$

$$\frac{\sqrt[4]{a+b}\left(\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\sqrt{2}\left(\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\coth(x)\right)}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)-\frac{\sqrt[4]{a}\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{\coth^2(x)-\cdots}}{\sqrt{2}(a+b\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\sqrt{2}\left(\sqrt{2}\coth(x)+\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}\right)}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)-\frac{\sqrt[4]{a}\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{\coth^2(x)+\cdots}}{\sqrt{2}(a+b\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 27

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\coth(x)}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}}-\frac{\sqrt[4]{a}\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\cdots}}{(\cdots}}}{\sqrt{2}(a+b)^{5/4}\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\sqrt{2}\coth(x)+\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}}-\frac{\sqrt[4]{a}\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\cdots}}{(\cdots}}}{\sqrt{2}(a+b)^{5/4}\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 1083

3.60. $\int \frac{1}{a+b\cosh^4(x)}\,dx$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\coth(x)}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}}+\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\coth(x)-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\cdots\right.}}{\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\sqrt{2}\coth(x)+\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}}+\frac{\sqrt{2}\sqrt[4]{a}\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\int \frac{\cdots}{-\left(2\coth(x)+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\cdots\right.}}{\cdots}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\cdots}$$

$\downarrow$ 217

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}-\sqrt{2}\coth(x)}{\coth^2(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}}-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\coth(x)-\cdots\right.}{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}\cdots\right.}{\sqrt{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\int \frac{\sqrt{2}\coth(x)+\frac{\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}}{(a+b)^{3/4}}}{\coth^2(x)+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+\sqrt{a+b}\sqrt{a}+b}\coth(x)}{(a+b)^{3/4}}+\frac{\sqrt{a}}{\sqrt{a+b}}}d\coth(x)}{\sqrt{2}}-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{(\cdots}\cdots\right.}{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}\cdots\right.}{\sqrt{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

$\downarrow$ 1103

3.60. $\int \frac{1}{a+b\cosh^4(x)}\,dx$

$$\frac{\sqrt[4]{a+b}\left(-\frac{(\sqrt{a}-\sqrt{a+b})\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\arctan\left(\frac{(a+b)^{3/4}\left(2\coth(x)-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}{(a+b)^{3/4}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}\right)}{\sqrt{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}-\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\log\left((a+b)^{3/4}\mathrm{co}\right.\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

$$\frac{\sqrt[4]{a+b}\left(\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{a+b}}+1\right)\log\left((a+b)^{3/4}\coth^2(x)+\sqrt{2}\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}\coth(x)+\sqrt{a}\sqrt[4]{a+b}\right)-\frac{(\sqrt{a}-\sqrt{a+b})}{\cdots}\right.}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

input
```
Int[(a + b*Cosh[x]^4)^(-1),x]
```

output
```
((a + b)^(1/4)*(-(((Sqrt[a] - Sqrt[a + b])*Sqrt[a + b + Sqrt[a]*Sqrt[a + b
]]*ArcTan[((a + b)^(3/4)*(-((Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a +
 b]])/(a + b)^(3/4)) + 2*Coth[x]))/(Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*S
qrt[a + b]])])/(Sqrt[a + b]*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])) - ((1 + Sq
rt[a]/Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b
+ Sqrt[a]*Sqrt[a + b]]*Coth[x] + (a + b)^(3/4)*Coth[x]^2])/2))/(2*Sqrt[2]*
a^(3/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + ((a + b)^(1/4)*(-(((Sqrt[a] -
 Sqrt[a + b])*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]*ArcTan[((a + b)^(3/4)*((Sq
rt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])/(a + b)^(3/4) + 2*Coth[x]
))/(Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]])])/(Sqrt[a + b]*Sqrt
[a + b - Sqrt[a]*Sqrt[a + b]])) + ((1 + Sqrt[a]/Sqrt[a + b])*Log[Sqrt[a]*(
a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]*Coth[x] +
 (a + b)^(3/4)*Coth[x]^2])/2))/(2*Sqrt[2]*a^(3/4)*Sqrt[a + b + Sqrt[a]*Sqr
t[a + b]])
```

3.60. $\int \frac{1}{a+b\cosh^4(x)}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.60.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.27

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^4+256a^3b)\_Z^4-32a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2x}+\left(-\frac{128a^4}{b}-128a^3\right)\_R^3+\left(\frac{32a^3}{b}+32a^2\right)\_R^2\right.$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^8+(-4a+4b)\_Z^6+(6a+6b)\_Z^4+(-4a+4b)\_Z^2+a+b\right)} \frac{\left(-\_R^6+3\_R^4-3\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right.}{\_R^7a+\_R^7b-3\_R^5a+3\_R^5b+3\_R^3a+3\_R^3b-\_}\right.}{4}$

input
```
int(1/(a+b*cosh(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*x)+(-128*a^4/b-128*a^3)*_R^3+(32/b*a^3+32*a^2)*_R^2+(8*a^2
/b-8*a)*_R-2*a/b+1),_R=RootOf(1+(256*a^4+256*a^3*b)*_Z^4-32*a^2*_Z^2))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(247) = 494.

Time = 0.29 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.14

$$\int \frac{1}{a+b\cosh^4(x)}\,dx$$

$$= -\frac{1}{4}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+1}{a^2+ab}}\log\Bigg(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2$$

$$+2\left(ab+\left(a^4+a^3b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+1}{a^2+ab}}$$

$$+2\left(a^3+a^2b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+b\Bigg)$$

$$+\frac{1}{4}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+1}{a^2+ab}}\log\Bigg(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2$$

$$-2\left(ab+\left(a^4+a^3b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+1}{a^2+ab}}$$

$$+2\left(a^3+a^2b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+b\Bigg)$$

$$-\frac{1}{4}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}-1}{a^2+ab}}\log\Bigg(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2$$

$$+2\left(ab-\left(a^4+a^3b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}-1}{a^2+ab}}$$

$$-2\left(a^3+a^2b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+b\Bigg)$$

$$+\frac{1}{4}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}-1}{a^2+ab}}\log\Bigg(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2$$

$$-2\left(ab-\left(a^4+a^3b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}-1}{a^2+ab}}$$

input
```
integrate(1/(a+b*cosh(x)^4),x, algorithm="fricas")
```

output
```
-1/4*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)
)*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*(a*b + (a^4 + a^
3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 +
2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) + 2*(a^3 + a^2*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) + b) + 1/4*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3
*b^2)) + 1)/(a^2 + a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)
^2 - 2*(a*b + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))*sqrt(((a^2
 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) + 2*(a^3 + a^
2*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + b) - 1/4*sqrt(-((a^2 + a*b)*sqrt
(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(b*cosh(x)^2 + 2*b*cos
h(x)*sinh(x) + b*sinh(x)^2 + 2*(a*b - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b
 + a^3*b^2)))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(
a^2 + a*b)) - 2*(a^3 + a^2*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + b) + 1/
4*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*
log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - 2*(a*b - (a^4 + a^3*
b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2
*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 2*(a^3 + a^2*b)*sqrt(-b/(a^5 + 2*a^
4*b + a^3*b^2)) + b)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x)**4),x)
```

output
```
Timed out
```

3.60.7 Maxima [F]

$$\int \frac{1}{a+b\cosh^4(x)}\,dx = \int \frac{1}{b\cosh{(x)}^4+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(b*cosh(x)^4 + a), x)
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1781 vs. $2(247) = 494$.

Time = 2.36 (sec) , antiderivative size = 1781, normalized size of antiderivative = 4.93

$$\int \frac{1}{a+b\cosh^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)^4),x, algorithm="giac")
```

output
```
1/4*sqrt((a^2 + sqrt(-a*b)*a)/(a^4 + a^3*b))*log(abs(60*a^4*b*e^(2*x) + 68
*a^3*b^2*e^(2*x) - 16*a^2*b^3*e^(2*x) + 24*sqrt(-a*b)*a^4*e^(2*x) + 48*sqr
t(a^2 - sqrt(-a*b)*a)*a^3*b*e^(2*x) - 16*sqrt(-a*b)*a^3*b*e^(2*x) + 61*sqr
t(a^2 - sqrt(-a*b)*a)*a^2*b^2*e^(2*x) - 64*sqrt(-a*b)*a^2*b^2*e^(2*x) - 4*
sqrt(a^2 - sqrt(-a*b)*a)*a*b^3*e^(2*x) + 6*a^4*b + 2*a^3*b^2 - 8*a^2*b^3 +
 24*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^3*e^(2*x) + 5*sqrt(a^2 - sqrt(-a
*b)*a)*sqrt(-a*b)*a^2*b*e^(2*x) - 36*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a
*b^2*e^(2*x) + 6*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b - 12*sqrt(-a*b)*a^3*b + 5*
sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2 - 16*sqrt(-a*b)*a^2*b^2 - 4*sqrt(a^2 - sq
rt(-a*b)*a)*a*b^3 - 9*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b - 12*sqrt(
a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b^2)) - 1/4*sqrt((a^2 + sqrt(-a*b)*a)/(a^
4 + a^3*b))*log(abs(60*a^4*b*e^(2*x) + 68*a^3*b^2*e^(2*x) - 16*a^2*b^3*e^(
2*x) + 24*sqrt(-a*b)*a^4*e^(2*x) - 48*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b*e^(2*
x) - 16*sqrt(-a*b)*a^3*b*e^(2*x) - 61*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2*e^(
2*x) - 64*sqrt(-a*b)*a^2*b^2*e^(2*x) + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3*e^
(2*x) + 6*a^4*b + 2*a^3*b^2 - 8*a^2*b^3 - 24*sqrt(a^2 - sqrt(-a*b)*a)*sqrt
(-a*b)*a^3*e^(2*x) - 5*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b*e^(2*x) +
 36*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b^2*e^(2*x) - 6*sqrt(a^2 - sqrt(
-a*b)*a)*a^3*b - 12*sqrt(-a*b)*a^3*b - 5*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2
- 16*sqrt(-a*b)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3 + 9*sqrt(a^2...
```

3.60.9 Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 1563, normalized size of antiderivative = 4.33

$$\int \frac{1}{a+b\cosh^4(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*cosh(x)^4),x)
```

output
```
log((524288*(1024*a^3*exp(2*x) - 35*b^3*exp(2*x) + 185*a*b^2 + 464*a^2*b +
 256*a^3 - 24*b^3 + 988*a*b^2*exp(2*x) + 2048*a^2*b*exp(2*x)))/(a*b^6*(a +
 b)^2) - (((((4194304*(253*a*b^3 - b^4*exp(2*x) + 1184*a^3*b + 512*a^4 - b
^4 + 930*a^2*b^2 + 1392*a^2*b^2*exp(2*x) + 627*a*b^3*exp(2*x) + 768*a^3*b*
exp(2*x)))/(b^6*(a + b)^2) + (8388608*a*((a^2 + (-a^3*b)^(1/2))/(a^3*(a +
b)))^(1/2)*(512*a^3*exp(2*x) - 6*b^3*exp(2*x) + 181*a*b^2 + 432*a^2*b + 25
6*a^3 - 5*b^3 + 622*a*b^2*exp(2*x) + 1152*a^2*b*exp(2*x)))/(b^6*(a + b)))*
((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4 - (2097152*(176*a*b + 1536
*a^2*exp(2*x) - 134*b^2*exp(2*x) + 256*a^2 - 75*b^2 + 1408*a*b*exp(2*x)))/
(b^6*(a + b)))*((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2))/4)*((a^2 + (-
a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2) - log((524288*(1024*a^3*exp(2*x) -
 35*b^3*exp(2*x) + 185*a*b^2 + 464*a^2*b + 256*a^3 - 24*b^3 + 988*a*b^2*ex
p(2*x) + 2048*a^2*b*exp(2*x)))/(a*b^6*(a + b)^2) - (((((4194304*(253*a*b^3
 - b^4*exp(2*x) + 1184*a^3*b + 512*a^4 - b^4 + 930*a^2*b^2 + 1392*a^2*b^2*
exp(2*x) + 627*a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a + b)^2) - (83
88608*a*((a^2 + (-a^3*b)^(1/2))/(a^3*(a + b)))^(1/2)*(512*a^3*exp(2*x) - 6
*b^3*exp(2*x) + 181*a*b^2 + 432*a^2*b + 256*a^3 - 5*b^3 + 622*a*b^2*exp(2*
x) + 1152*a^2*b*exp(2*x)))/(b^6*(a + b)))*((a^2 + (-a^3*b)^(1/2))/(a^3*(a
+ b)))^(1/2))/4 + (2097152*(176*a*b + 1536*a^2*exp(2*x) - 134*b^2*exp(2*x)
 + 256*a^2 - 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a + b)))*((a^2 + (-a^3*...
```

3.61 $\int \frac{1}{a-b\cosh^4(x)}\,dx$

3.61.1 Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{1}{a-b\cosh^4(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}\tanh(x)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

output
```
1/2*arctanh(a^(1/4)*tanh(x)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/4)/(a^(1/2)-b^(1
/2))^(1/2)+1/2*arctanh(a^(1/4)*tanh(x)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/4)/(a
^(1/2)+b^(1/2))^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{1}{a-b\cosh^4(x)}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}$$

input
```
Integrate[(a - b*Cosh[x]^4)^(-1),x]
```

output
```
-1/2*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(Sqrt[a]*Sqrt[-a
 + Sqrt[a]*Sqrt[b]]) + ArcTanh[(Sqrt[a]*Tanh[x])/Sqrt[a + Sqrt[a]*Sqrt[b]]
]/(2*Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]])
```

3.61.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3688, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cosh^4(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\int \frac{1-\coth^2(x)}{(a-b)\coth^4(x)-2a\coth^2(x)+a}d\coth(x)$$

$$\Big\downarrow \text{1480}$$

$$-\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\int \frac{1}{(a-b)\coth^2(x)-\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}d\coth(x)-\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\int \frac{1}{(a-b)\coth^2(x)-\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}d\coth(x)$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(\frac{\sqrt{b}}{\sqrt{a}}+1\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\coth(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\left(\sqrt{a}+\sqrt{b}\right)}+\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\coth(x)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[(a - b*Cosh[x]^4)^(-1),x]
```

output
```
((1 + Sqrt[b]/Sqrt[a])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Coth[x])/a^(1/4)])
/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*(Sqrt[a] + Sqrt[b])) + ((1 - Sqrt[b]/S
qrt[a])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Coth[x])/a^(1/4)])/(2*a^(1/4)*(Sq
rt[a] - Sqrt[b])*Sqrt[Sqrt[a] + Sqrt[b]])
```

3.61.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.61.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

method	result
risch	$\sum_{\_R=\operatorname{RootOf}\left(1+\left(256a^4-256a^3b\right)\_Z^4-32a^2\_Z^2\right)} \_R\ln\left(e^{2x}+\left(\frac{128a^4}{b}-128a^3\right)\_R^3+\left(-\frac{32a^3}{b}+32a^2\right)\_R^2$
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left((a-b)\_Z^8+(-4a-4b)\_Z^6+(6a-6b)\_Z^4+(-4a-4b)\_Z^2+a-b\right)} \frac{\left(-\_R^6+3\_R^4-3\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right.}{\_R^7a-\_R^7b-3\_R^5a-3\_R^5b+3\_R^3a-3\_R^3b-\_}\right.}{4}$

input
```
int(1/(a-b*cosh(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*x)+(128*a^4/b-128*a^3)*_R^3+(-32/b*a^3+32*a^2)*_R^2+(-8*a^
2/b-8*a)*_R+2*a/b+1),_R=RootOf(1+(256*a^4-256*a^3*b)*_Z^4-32*a^2*_Z^2))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 779, normalized size of antiderivative = 7.71

$$\int \frac{1}{a-b\cosh^4(x)}\,dx$$

$$= -\frac{1}{4}\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+1}{a^2-ab}}\log\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2 + 2\left(ab-(a^4-a^3b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}\right)\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+1}{a^2-ab}} - 2\,(a^3-a^2b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+b\right)$$

$$+ \frac{1}{4}\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+1}{a^2-ab}}\log\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2 - 2\left(ab-(a^4-a^3b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}\right)\sqrt{\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+1}{a^2-ab}} - 2\,(a^3-a^2b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+b\right)$$

$$- \frac{1}{4}\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}-1}{a^2-ab}}\log\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2 + 2\left(ab+(a^4-a^3b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}\right)\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}-1}{a^2-ab}} + 2\,(a^3-a^2b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+b\right)$$

$$+ \frac{1}{4}\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}-1}{a^2-ab}}\log\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2 - 2\left(ab+(a^4-a^3b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}\right)\sqrt{-\frac{(a^2-ab)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}-1}{a^2-ab}} + 2\,(a^3-a^2b)\sqrt{\frac{b}{a^5-2\,a^4b+a^3b^2}}+b\right)$$

3.61. $\int \frac{1}{a-b\cosh^4(x)}\,dx$

input
```
integrate(1/(a-b*cosh(x)^4),x, algorithm="fricas")
```

output
```
-1/4*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b))
*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + 2*(a*b - (a^4 - a^3
*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a
^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) - 2*(a^3 - a^2*b)*sqrt(b/(a^5 - 2*a^4*b
 + a^3*b^2)) + b) + 1/4*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)
) + 1)/(a^2 - a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 -
2*(a*b - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))*sqrt(((a^2 - a*b
)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) - 2*(a^3 - a^2*b)*sq
rt(b/(a^5 - 2*a^4*b + a^3*b^2)) + b) - 1/4*sqrt(-((a^2 - a*b)*sqrt(b/(a^5
- 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(b*cosh(x)^2 + 2*b*cosh(x)*sinh
(x) + b*sinh(x)^2 + 2*(a*b + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2
)))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))
 + 2*(a^3 - a^2*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + b) + 1/4*sqrt(-((a^
2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - 2*(a*b + (a^4 - a^3*b)*sqrt(b/(a^
5 - 2*a^4*b + a^3*b^2)))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^
2)) - 1)/(a^2 - a*b)) + 2*(a^3 - a^2*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))
+ b)
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a - b\cosh^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a-b*cosh(x)**4),x)
```

output
```
Timed out
```

3.61.7 Maxima [F]

$$\int \frac{1}{a-b\cosh^4(x)}\,dx=\int -\frac{1}{b\cosh(x)^4-a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^4),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cosh(x)^4 - a), x)
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(65)=130$.

Time = 2.46 (sec) , antiderivative size = 1697, normalized size of antiderivative = 16.80

$$\int \frac{1}{a-b\cosh^4(x)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a-b*cosh(x)^4),x, algorithm="giac")
```

output
```
1/4*sqrt((a^2 - sqrt(a*b)*a)/(a^4 - a^3*b))*log(abs(60*a^4*b*e^(2*x) - 68*
a^3*b^2*e^(2*x) - 16*a^2*b^3*e^(2*x) + 24*sqrt(a*b)*a^4*e^(2*x) + 48*sqrt(
a^2 + sqrt(a*b)*a)*a^3*b*e^(2*x) + 16*sqrt(a*b)*a^3*b*e^(2*x) - 61*sqrt(a^
2 + sqrt(a*b)*a)*a^2*b^2*e^(2*x) - 64*sqrt(a*b)*a^2*b^2*e^(2*x) - 4*sqrt(a
^2 + sqrt(a*b)*a)*a*b^3*e^(2*x) + 6*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 + 24*sqr
t(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^3*e^(2*x) - 5*sqrt(a^2 + sqrt(a*b)*a)*sqr
t(a*b)*a^2*b*e^(2*x) - 36*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b^2*e^(2*x)
+ 6*sqrt(a^2 + sqrt(a*b)*a)*a^3*b + 12*sqrt(a*b)*a^3*b - 5*sqrt(a^2 + sqrt
(a*b)*a)*a^2*b^2 - 16*sqrt(a*b)*a^2*b^2 - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3
+ 9*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b - 12*sqrt(a^2 + sqrt(a*b)*a)*s
qrt(a*b)*a*b^2)) - 1/4*sqrt((a^2 - sqrt(a*b)*a)/(a^4 - a^3*b))*log(abs(60*
a^4*b*e^(2*x) - 68*a^3*b^2*e^(2*x) - 16*a^2*b^3*e^(2*x) + 24*sqrt(a*b)*a^4
*e^(2*x) - 48*sqrt(a^2 + sqrt(a*b)*a)*a^3*b*e^(2*x) + 16*sqrt(a*b)*a^3*b*e
^(2*x) + 61*sqrt(a^2 + sqrt(a*b)*a)*a^2*b^2*e^(2*x) - 64*sqrt(a*b)*a^2*b^2
*e^(2*x) + 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3*e^(2*x) + 6*a^4*b - 2*a^3*b^2 -
 8*a^2*b^3 - 24*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^3*e^(2*x) + 5*sqrt(a^2
 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b*e^(2*x) + 36*sqrt(a^2 + sqrt(a*b)*a)*sqrt(
a*b)*a*b^2*e^(2*x) - 6*sqrt(a^2 + sqrt(a*b)*a)*a^3*b + 12*sqrt(a*b)*a^3*b
+ 5*sqrt(a^2 + sqrt(a*b)*a)*a^2*b^2 - 16*sqrt(a*b)*a^2*b^2 + 4*sqrt(a^2 +
sqrt(a*b)*a)*a*b^3 - 9*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b + 12*sqr...
```

3.61.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 1487, normalized size of antiderivative = 14.72

$$\int \frac{1}{a - b\cosh^4(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(a - b*cosh(x)^4),x)
```

output
```
log((((1/(a^2 - (a^3*b)^(1/2)))^(1/2)*((4194304*(b^4*exp(2*x) + 253*a*b^3
 + 1184*a^3*b - 512*a^4 + b^4 - 930*a^2*b^2 - 1392*a^2*b^2*exp(2*x) + 627*
a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a - b)^2) - (8388608*a*(1/(a^2
 - (a^3*b)^(1/2)))^(1/2)*(512*a^3*exp(2*x) + 6*b^3*exp(2*x) + 181*a*b^2 -
432*a^2*b + 256*a^3 + 5*b^3 + 622*a*b^2*exp(2*x) - 1152*a^2*b*exp(2*x)))/(
b^6*(a - b))))/4 - (2097152*(176*a*b - 1536*a^2*exp(2*x) + 134*b^2*exp(2*x
) - 256*a^2 + 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a - b)))*(1/(a^2 - (a^3*b
)^(1/2)))^(1/2))/4 + (524288*(1024*a^3*exp(2*x) + 35*b^3*exp(2*x) + 185*a*
b^2 - 464*a^2*b + 256*a^3 + 24*b^3 + 988*a*b^2*exp(2*x) - 2048*a^2*b*exp(2
*x)))/(a*b^6*(a - b)^2))*(-(a^2 + (a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2)
 - log((((1/(a^2 - (a^3*b)^(1/2)))^(1/2)*((4194304*(b^4*exp(2*x) + 253*a*
b^3 + 1184*a^3*b - 512*a^4 + b^4 - 930*a^2*b^2 - 1392*a^2*b^2*exp(2*x) + 6
27*a*b^3*exp(2*x) + 768*a^3*b*exp(2*x)))/(b^6*(a - b)^2) + (8388608*a*(1/(
a^2 - (a^3*b)^(1/2)))^(1/2)*(512*a^3*exp(2*x) + 6*b^3*exp(2*x) + 181*a*b^2
 - 432*a^2*b + 256*a^3 + 5*b^3 + 622*a*b^2*exp(2*x) - 1152*a^2*b*exp(2*x))
)/(b^6*(a - b))))/4 + (2097152*(176*a*b - 1536*a^2*exp(2*x) + 134*b^2*exp(
2*x) - 256*a^2 + 75*b^2 + 1408*a*b*exp(2*x)))/(b^6*(a - b)))*(1/(a^2 - (a^
3*b)^(1/2)))^(1/2))/4 + (524288*(1024*a^3*exp(2*x) + 35*b^3*exp(2*x) + 185
*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 + 988*a*b^2*exp(2*x) - 2048*a^2*b*ex
p(2*x)))/(a*b^6*(a - b)^2))*(-(a^2 + (a^3*b)^(1/2))/(16*(a^3*b - a^4)))...
```

3.62 $\int \frac{1}{1+\cosh^4(x)}\,dx$

3.62.1 Optimal result

Integrand size = 8, antiderivative size = 176

$$\int \frac{1}{1+\cosh^4(x)}\,dx = -\frac{\arctan\left(\frac{\sqrt{1+\sqrt{2}}-2\coth(x)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\arctan\left(\frac{\sqrt{1+\sqrt{2}}+2\coth(x)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}} - \frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}-2\sqrt{1+\sqrt{2}}\coth(x)+2\coth^2(x)\right) + \frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(1+\sqrt{2\left(1+\sqrt{2}\right)}\coth(x)+\sqrt{2}\coth^2(x)\right)$$

output
```
-1/4*arctan((-2*coth(x)+(1+2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2))/(1+2^(1/2))^
(1/2)+1/4*arctan((2*coth(x)+(1+2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2))/(1+2^(1/
2))^(1/2)-1/8*ln(2*coth(x)^2+2^(1/2)-2*coth(x)*(1+2^(1/2))^(1/2))*(1+2^(1/
2))^(1/2)+1/8*ln(1+coth(x)^2*2^(1/2)+coth(x)*(2+2*2^(1/2))^(1/2))*(1+2^(1/
2))^(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.26

$$\int \frac{1}{1+\cosh^4(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{2\sqrt{1-i}} + \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{2\sqrt{1+i}}$$

input
```
Integrate[(1 + Cosh[x]^4)^(-1),x]
```

output
```
ArcTanh[Tanh[x]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTanh[Tanh[x]/Sqrt[1 + I]
]/(2*Sqrt[1 + I])
```

3.62.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3688, 1483, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^4(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1+\sin\left(\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3688}$$

$$\int \frac{1-\coth^2(x)}{2\coth^4(x)-2\coth^2(x)+1}d\coth(x)$$

$$\Big\downarrow \text{1483}$$

$$\frac{\int \frac{2\sqrt{1+\sqrt{2}}-\left(2+\sqrt{2}\right)\coth(x)}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)}{2\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\int \frac{\left(2+\sqrt{2}\right)\coth(x)+2\sqrt{1+\sqrt{2}}}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)-\frac{1}{4}\left(2+\sqrt{2}\right)\int -\frac{2\left(\sqrt{1+\sqrt{2}}-2\coth(x)\right)}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)}{2\sqrt{2\left(1+\sqrt{2}\right)}}+$$
$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{2\left(2\coth(x)+\sqrt{1+\sqrt{2}}\right)}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$\downarrow$ 27

$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)+\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{\sqrt{1+\sqrt{2}}-2\coth(x)}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)}{2\sqrt{2\left(1+\sqrt{2}\right)}}+$$
$$\frac{\sqrt{\frac{1}{2}\left(\sqrt{2}-1\right)}\int \frac{1}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)+\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{2\coth(x)+\sqrt{1+\sqrt{2}}}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$\downarrow$ 1083

$$\frac{\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{\sqrt{1+\sqrt{2}}-2\coth(x)}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)-\sqrt{2\left(\sqrt{2}-1\right)}\int \frac{1}{4\left(1-\sqrt{2}\right)-\left(4\coth(x)-2\sqrt{1+\sqrt{2}}\right)^2}d(4\coth(x)-2}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$
$$\frac{\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{2\coth(x)+\sqrt{1+\sqrt{2}}}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)-\sqrt{2\left(\sqrt{2}-1\right)}\int \frac{1}{4\left(1-\sqrt{2}\right)-\left(4\coth(x)+2\sqrt{1+\sqrt{2}}\right)^2}d(4\coth(x)+2}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$\downarrow$ 217

$$\frac{\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{\sqrt{1+\sqrt{2}}-2\coth(x)}{2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)+\frac{\arctan\left(\frac{4\coth(x)-2\sqrt{1+\sqrt{2}}}{2\sqrt{\sqrt{2}-1}}\right)}{\sqrt{2}}}{2\sqrt{2\left(1+\sqrt{2}\right)}}+$$
$$\frac{\frac{1}{2}\left(2+\sqrt{2}\right)\int \frac{2\coth(x)+\sqrt{1+\sqrt{2}}}{2\coth^2(x)+2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}}d\coth(x)+\frac{\arctan\left(\frac{4\coth(x)+2\sqrt{1+\sqrt{2}}}{2\sqrt{\sqrt{2}-1}}\right)}{\sqrt{2}}}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$\downarrow$ 1103

3.62. $\int \frac{1}{1+\cosh^4(x)}\,dx$

$$\frac{\frac{\arctan\left(\frac{4\coth(x)-2\sqrt{1+\sqrt{2}}}{2\sqrt{\sqrt{2}-1}}\right)}{\sqrt{2}}-\frac{1}{4}(2+\sqrt{2})\log\left(2\coth^2(x)-2\sqrt{1+\sqrt{2}}\coth(x)+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\frac{\arctan\left(\frac{4\coth(x)+2\sqrt{1+\sqrt{2}}}{2\sqrt{\sqrt{2}-1}}\right)}{\sqrt{2}}+\frac{1}{4}(2+\sqrt{2})\log\left(\sqrt{2}\coth^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\coth(x)+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

input
```
Int[(1 + Cosh[x]^4)^(-1),x]
```

output
```
(ArcTan[(-2*Sqrt[1 + Sqrt[2]] + 4*Coth[x])/(2*Sqrt[-1 + Sqrt[2]])]/Sqrt[2]
 - ((2 + Sqrt[2])*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Coth[x] + 2*Coth[x]^2]
)/4)/(2*Sqrt[2*(1 + Sqrt[2])]) + (ArcTan[(2*Sqrt[1 + Sqrt[2]] + 4*Coth[x])
/(2*Sqrt[-1 + Sqrt[2]])]/Sqrt[2] + ((2 + Sqrt[2])*Log[1 + Sqrt[2*(1 + Sqrt
[2])]*Coth[x] + Sqrt[2]*Coth[x]^2])/4)/(2*Sqrt[2*(1 + Sqrt[2])])
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.62.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.20

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}\left(512\_Z^4-32\_Z^2+1\right)} \_R\ln\left(-256\_R^3+64\_R^2+\mathrm{e}^{2x}-1\right)$	36
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-2\_Z^2+1\right)} \_R\ln\left(2\tanh\left(\frac{x}{2}\right)\_R+\tanh\left(\frac{x}{2}\right)^2+1\right)\right)}{4}$	37

input
```
int(1/(1+cosh(x)^4),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(-256*_R^3+64*_R^2+exp(2*x)-1),_R=RootOf(512*_Z^4-32*_Z^2+1))
```

3.62.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+\cosh^4(x)}\,dx = -\frac{1}{8}\sqrt{2}\sqrt{i+1}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+(i+1)\sqrt{2}\sqrt{i+1}+2i+1\right)+\frac{1}{8}\sqrt{2}\sqrt{i+1}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-(i+1)\sqrt{2}\sqrt{i+1}+2i+1\right)-\frac{1}{8}\sqrt{2}\sqrt{-i+1}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-(i-1)\sqrt{2}\sqrt{-i+1}-2i+1\right)+\frac{1}{8}\sqrt{2}\sqrt{-i+1}\log\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+(i-1)\sqrt{2}\sqrt{-i+1}-2i+1\right)$$

input
```
integrate(1/(1+cosh(x)^4),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*sqrt(I + 1)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + (
I + 1)*sqrt(2)*sqrt(I + 1) + 2*I + 1) + 1/8*sqrt(2)*sqrt(I + 1)*log(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - (I + 1)*sqrt(2)*sqrt(I + 1) + 2*I +
1) - 1/8*sqrt(2)*sqrt(-I + 1)*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 - (I - 1)*sqrt(2)*sqrt(-I + 1) - 2*I + 1) + 1/8*sqrt(2)*sqrt(-I + 1)*log
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + (I - 1)*sqrt(2)*sqrt(-I + 1)
- 2*I + 1)
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cosh^4(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cosh(x)**4),x)
```

output
```
Timed out
```

3.62.7 Maxima [F]

$$\int \frac{1}{1+\cosh^4(x)}\,dx=\int \frac{1}{\cosh(x)^4+1}\,dx$$

input
```
integrate(1/(1+cosh(x)^4),x, algorithm="maxima")
```

output
```
integrate(1/(cosh(x)^4 + 1), x)
```

3.62.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{1}{1+\cosh^4(x)}\,dx =& -\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)\log\left((20i+10)\sqrt{2}e^{(2x)}\right.\\ &+10\sqrt{2}\sqrt{10\sqrt{2}+14}+50\sqrt{2}-(2i-14)\sqrt{10\sqrt{2}+14}\\ &\left.+(28i+14)e^{(2x)}+70\right)\\ &+\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)\log\left((20i+10)\sqrt{2}e^{(2x)}\right.\\ &-10\sqrt{2}\sqrt{10\sqrt{2}+14}+50\sqrt{2}+(2i-14)\sqrt{10\sqrt{2}+14}\\ &\left.+(28i+14)e^{(2x)}+70\right)\\ &-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}+2}\left(-\frac{i}{\sqrt{2}+1}+1\right)\log\left(2\sqrt{2}e^{(2x)}\right.\\ &\left.+2\sqrt{2}\sqrt{2\sqrt{2}-2}+(4i+2)\sqrt{2}+(2i-2)\sqrt{2\sqrt{2}-2}-2e^{(2x)}-4i-2\right)\\ &+\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}+2}\left(-\frac{i}{\sqrt{2}+1}+1\right)\log\left(2\sqrt{2}e^{(2x)}\right.\\ &\left.-2\sqrt{2}\sqrt{2\sqrt{2}-2}+(4i+2)\sqrt{2}-(2i-2)\sqrt{2\sqrt{2}-2}-2e^{(2x)}-4i-2\right)\end{aligned}$$

input
```
integrate(1/(1+cosh(x)^4),x, algorithm="giac")
```

output
```
-(1/16*I + 1/16)*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10
)*sqrt(2)*e^(2*x) + 10*sqrt(2)*sqrt(10*sqrt(2) + 14) + 50*sqrt(2) - (2*I -
 14)*sqrt(10*sqrt(2) + 14) + (28*I + 14)*e^(2*x) + 70) + (1/16*I + 1/16)*s
qrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)*sqrt(2)*e^(2*x)
- 10*sqrt(2)*sqrt(10*sqrt(2) + 14) + 50*sqrt(2) + (2*I - 14)*sqrt(10*sqrt(
2) + 14) + (28*I + 14)*e^(2*x) + 70) - (1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)
*(-I/(sqrt(2) + 1) + 1)*log(2*sqrt(2)*e^(2*x) + 2*sqrt(2)*sqrt(2*sqrt(2) -
 2) + (4*I + 2)*sqrt(2) + (2*I - 2)*sqrt(2*sqrt(2) - 2) - 2*e^(2*x) - 4*I
- 2) + (1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(sqrt(2) + 1) + 1)*log(2*sq
rt(2)*e^(2*x) - 2*sqrt(2)*sqrt(2*sqrt(2) - 2) + (4*I + 2)*sqrt(2) - (2*I -
 2)*sqrt(2*sqrt(2) - 2) - 2*e^(2*x) - 4*I - 2)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{1}{1+\cosh^4(x)}\,dx$$

$$= \frac{\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152+91291648\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,(-9830400+56623104\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\mathrm{e}^{2x}\,(2188\right.}{8}$$

$$-\frac{\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152+91291648\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,(9830400-56623104\mathrm{i})+\sqrt{2}\,\sqrt{1-\mathrm{i}}\,\mathrm{e}^{2x}\,(-21\right.}{8}$$

$$+\frac{\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152-91291648\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,(-9830400-56623104\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\mathrm{e}^{2x}\right.}{8}$$

$$-\frac{\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\ln\left(\mathrm{e}^{2x}\,(436273152-91291648\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,(9830400+56623104\mathrm{i})+\sqrt{2}\,\sqrt{1+1\mathrm{i}}\,\mathrm{e}^{2x}\,(\right.}{8}$$

input
```
int(1/(cosh(x)^4 + 1),x)
```

output
```
(2^(1/2)*(1 - 1i)^(1/2)*log(exp(2*x)*(436273152 + 91291648i) - 2^(1/2)*(1
- 1i)^(1/2)*(9830400 - 56623104i) + 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(21889
0240 + 149422080i) + (21168128 + 94306304i)))/8 - (2^(1/2)*(1 - 1i)^(1/2)*
log(exp(2*x)*(436273152 + 91291648i) + 2^(1/2)*(1 - 1i)^(1/2)*(9830400 - 5
6623104i) - 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(218890240 + 149422080i) + (21
168128 + 94306304i)))/8 + (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152
- 91291648i) - 2^(1/2)*(1 + 1i)^(1/2)*(9830400 + 56623104i) + 2^(1/2)*(1 +
 1i)^(1/2)*exp(2*x)*(218890240 - 149422080i) + (21168128 - 94306304i)))/8
- (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 91291648i) + 2^(1/2)*(
1 + 1i)^(1/2)*(9830400 + 56623104i) - 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(218
890240 - 149422080i) + (21168128 - 94306304i)))/8
```

3.63 $\int \frac{1}{1-\cosh^4(x)}\,dx$

3.63.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{1-\cosh^4(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{\coth(x)}{2}$$

output
```
1/2*coth(x)+1/4*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{1-\cosh^4(x)}\,dx = \frac{1}{4}\left(\sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right) + 2\coth(x)\right)$$

input
```
Integrate[(1 - Cosh[x]^4)^(-1),x]
```

output
```
(Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]] + 2*Coth[x])/4
```

3.63.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3688, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh^4(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{1-\sin\left(\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{ 3688}$$

$$\int \frac{1-\coth^2(x)}{1-2\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{ 299}$$

$$\frac{1}{2}\int \frac{1}{1-2\coth^2(x)}d\coth(x)+\frac{\coth(x)}{2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\coth(x)\right)}{2\sqrt{2}}+\frac{\coth(x)}{2}$$

input
```
Int[(1 - Cosh[x]^4)^(-1),x]
```

output
```
ArcTanh[Sqrt[2]*Coth[x]]/(2*Sqrt[2]) + Coth[x]/2
```

3.63.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3688
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + 2*a*ff^2*x^2 + (
a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /
; FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result
risch	$\frac{1}{e^{2x}-1}+\frac{\sqrt{2}\,\ln\left(e^{2x}+3-2\sqrt{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2x}+3+2\sqrt{2}\right)}{8}$
default	$\frac{\tanh\left(\frac{x}{2}\right)}{4}+\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^{2}-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}-1\right)\right)}{16}-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^{2}-}{\tanh\left(\frac{x}{2}\right)^{2}+}\right.\right.}{}$

input
```
int(1/(1-cosh(x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/(exp(2*x)-1)+1/8*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-1/8*2^(1/2)*ln(exp(2*x
)+3+2*2^(1/2))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.60

$$\int \frac{1}{1-\cosh^4(x)}\,dx$$
$$= \frac{\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{2}\cosh(x)\sinh(x) + \sqrt{2}\sinh(x)^2 - \sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)}{\cosh(x)^2+\sinh(x)^2+}\right.}{8\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)}$$

input
```
integrate(1/(1-cosh(x)^4),x, algorithm="fricas")
```

output
```
1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 -
sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sin
h(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^2
 + 3)) + 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(22) = 44$.

Time = 0.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\int \frac{1}{1-\cosh^4(x)}\,dx = -\frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{8} + \frac{\sqrt{2}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\tanh\left(\frac{x}{2}\right)+4\right)}{8} + \frac{\tanh\left(\frac{x}{2}\right)}{4} + \frac{1}{4\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate(1/(1-cosh(x)**4),x)
```

output
```
-sqrt(2)*log(4*tanh(x/2)**2 - 4*sqrt(2)*tanh(x/2) + 4)/8 + sqrt(2)*log(4*t
anh(x/2)**2 + 4*sqrt(2)*tanh(x/2) + 4)/8 + tanh(x/2)/4 + 1/(4*tanh(x/2))
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{1}{1-\cosh^4(x)}\,dx = -\frac{1}{8}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right) - \frac{1}{e^{(-2x)}-1}$$

input
```
integrate(1/(1-cosh(x)^4),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 1/(e^(-2*x) - 1)
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{1}{1-\cosh^4(x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) + \frac{1}{e^{(2x)}-1}$$

input
```
integrate(1/(1-cosh(x)^4),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/
(e^(2*x) - 1)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{1}{1-\cosh^4(x)}\,dx = \frac{\sqrt{2}\,\ln\left(-2\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{8}\right)}{8} - \frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(12\,e^{2\,x}+4\right)}{8}-2\,e^{2\,x}\right)}{8} + \frac{1}{e^{2\,x}-1}$$

input
```
int(-1/(cosh(x)^4 - 1),x)
```

output
```
(2^(1/2)*log(- 2*exp(2*x) - (2^(1/2)*(12*exp(2*x) + 4))/8))/8 - (2^(1/2)*l
og((2^(1/2)*(12*exp(2*x) + 4))/8 - 2*exp(2*x)))/8 + 1/(exp(2*x) - 1)
```

3.64 $\int \frac{1}{a+b\cosh^5(x)}\,dx$

3.64.1 Optimal result

Integrand size = 10, antiderivative size = 494

$$\begin{aligned}\int \frac{1}{a+b\cosh^5(x)}\,dx &= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\end{aligned}$$

output
```
2/5*arctanh((a^(1/5)-b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+b^(1/5))^(1/2))/a
^(4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctanh((a^(1/5
)+(-1)^(1/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)
)/a^(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^
(1/2)+2/5*arctanh((a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+
(-1)^(2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(
1/5)+(-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)+(-1)^(3/5)*b^(1/5))^(1
/2)*tanh(1/2*x)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^
(3/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/
5)-(-1)^(4/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2
))/a^(4/5)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))
^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.28

$$\int \frac{1}{a+b\cosh^5(x)}\,dx = \frac{8}{5}\text{RootSum}\left[b+5b\#1^2+10b\#1^4+32a\#1^5+10b\#1^6+5b\#1^8 + b\#1^{10}\&, \frac{x\#1^3+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^3}{b+4b\#1^2+16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\right]$$

input
```
Integrate[(a + b*Cosh[x]^5)^(-1),x]
```

output
```
(8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b
*#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/
2]*#1]*#1^3)/(b + 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ]
)/5
```

3.64.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^5(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\cosh(x)\right)}-\frac{1}{5a^{4/5}\left(\sqrt[5]{-1}\sqrt[5]{b}\cosh(x)-\sqrt[5]{a}\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cosh(x)\right)}-\frac{}{5a\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

input
```
Int[(a + b*Cosh[x]^5)^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a^(1/5) - b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + b^(1/5)]])/(
5*a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTanh[(S
qrt[a^(1/5) + (-1)^(1/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - (-1)^(1/5)*b^(
1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^
(1/5)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*Tanh[x/2]
)/Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5
)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5)
+ (-1)^(3/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]])/(5*a
^(4/5)*Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5
)]) + (2*ArcTanh[(Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/
5) + (-1)^(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*S
qrt[a^(1/5) + (-1)^(4/5)*b^(1/5)])
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.64.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.32

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left((a-b)\_Z^{10}+(-5a-5b)\_Z^{8}+(10a-10b)\_Z^{6}+(-10a-10b)\_Z^{4}+(5a-5b)\_Z^{2}-a-b\right)} \frac{\left(-\_R^{8}+4\_R^{6}-6\_R\right.}{\_R^{9}a-\_R^{9}b-4\_R^{7}a-4\_R^{7}b+6\_}\right.}{5}$
risch	$\sum_{\_R=\operatorname{RootOf}\left(-1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}-1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}-6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)} \_R\ln\left(e^{x}+\left(\right.\right.$

input
```
int(1/(a+b*cosh(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a-_R^9*b-4*_R^7*a-4*_R^7*b+6*
_R^5*a-6*_R^5*b-4*_R^3*a-4*_R^3*b+_R*a-_R*b)*ln(tanh(1/2*x)-_R),_R=RootOf(
(a-b)*_Z^10+(-5*a-5*b)*_Z^8+(10*a-10*b)*_Z^6+(-10*a-10*b)*_Z^4+(5*a-5*b)*_
Z^2-a-b))
```

3.64.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cosh^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*cosh(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.64.6 Sympy [F]

$$\int \frac{1}{a+b\cosh^5(x)}\,dx = \int \frac{1}{a+b\cosh^5{(x)}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)**5),x)
```

output
```
Integral(1/(a + b*cosh(x)**5), x)
```

3.64.7 Maxima [F]

$$\int \frac{1}{a+b\cosh^5(x)}\,dx = \int \frac{1}{b\cosh{(x)}^5+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^5),x, algorithm="maxima")
```

output
```
integrate(1/(b*cosh(x)^5 + a), x)
```

3.64.8 Giac [F]

$$\int \frac{1}{a + b\cosh^5(x)}\,dx = \int \frac{1}{b\cosh{(x)}^5 + a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^5),x, algorithm="giac")
```

output
```
integrate(1/(b*cosh(x)^5 + a), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh^5(x)}\,dx = \text{Hanged}$$

input
```
int(1/(a + b*cosh(x)^5),x)
```

output
```
\text{Hanged}
```

3.65 $\int \frac{1}{a+b\cosh^6(x)}\,dx$

3.65.1 Optimal result

Integrand size = 10, antiderivative size = 171

$$\int \frac{1}{a+b\cosh^6(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)+b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)+b^(1
/3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2))
/a^(5/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a
^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/
2)
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+b\cosh^6(x)}\,dx = \frac{16}{3}\text{RootSum}\Big[b+6b\#1+15b\#1^2+64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5 + b\#1^6\&, \frac{x\#1^2+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^2}{b+5b\#1+32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\Big]$$

input
```
Integrate[(a + b*Cosh[x]^6)^(-1),x]
```

output
```
(16*RootSum[b + 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6
*b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[
x]*#1]*#1^2)/(b + 5*b*#1 + 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 + 5*b*#1^4 +
b*#1^5) & ])/3
```

3.65.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3690, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\frac{\sqrt[3]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{1-\left(\frac{\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\coth^2(x)}d\coth(x)}{3a}+\frac{\int \frac{1}{1-\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\coth^2(x)}d\coth(x)}{3a}+\frac{\int \frac{1}{1-\left(\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\coth^2(x)}d\coth(x)}{3a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\coth(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\coth(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\coth(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a + b*Cosh[x]^6)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^(1/3) + b^(1/3)]*Coth[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3)
 + b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Coth[x])/a^(1/6
)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3)
+ (-1)^(2/3)*b^(1/3)]*Coth[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/
3)*b^(1/3)])
```

3.65.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.65.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+\left(46656a^6+46656a^5b\right)\_Z^6-3888a^4\_Z^4+108a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2x}+\left(-\frac{15552a^6}{b}-15552a^5\right)\_R^5+\right.$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{12}+(-6a+6b)\_Z^{10}+(15a+15b)\_Z^8+(-20a+20b)\_Z^6+(15a+15b)\_Z^4+(-6a+6b)\_Z^2+a+b\right)} \frac{\_R^{11}a+\_R^{11}b-5\_}{}\right.}{6}$

input
```
int(1/(a+b*cosh(x)^6),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*x)+(-15552*a^6/b-15552*a^5)*_R^5+(2592*a^5/b+2592*a^4)*_R^
4+(864*a^4/b-432*a^3)*_R^3+(-144/b*a^3+72*a^2)*_R^2+(-12*a^2/b-12*a)*_R+2*
a/b+1),_R=RootOf(-1+(46656*a^6+46656*a^5*b)*_Z^6-3888*a^4*_Z^4+108*a^2*_Z^
2))
```

3.65.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 15201, normalized size of antiderivative = 88.89

$$\int \frac{1}{a+b\cosh^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

3.65.6 Sympy [F]

$$\int \frac{1}{a+b\cosh^6(x)}\,dx = \int \frac{1}{a+b\cosh^6{(x)}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)**6),x)
```

output
```
Integral(1/(a + b*cosh(x)**6), x)
```

3.65.7 Maxima [F]

$$\int \frac{1}{a+b\cosh^6(x)}\,dx = \int \frac{1}{b\cosh{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^6),x, algorithm="maxima")
```

output
```
integrate(1/(b*cosh(x)^6 + a), x)
```

3.65.8 Giac [F]

$$\int \frac{1}{a+b\cosh^6(x)}\,dx=\int \frac{1}{b\cosh{(x)}^6+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^6),x, algorithm="giac")
```

output
```
sage0*x
```

3.65.9 Mupad [B] (verification not implemented)

Time = 61.87 (sec) , antiderivative size = 844, normalized size of antiderivative = 4.94

$$\int \frac{1}{a+b\cosh^6(x)}\,dx=\sum_{k=1}^{6}\ln\Bigg(\mathrm{root}\big(46656\,a^5\,b\,d^6+46656\,a^6\,d^6-3888\,a^4\,d^4+108\,a^2\,d^2$$
$$-1,d,k\big)\Bigg(\mathrm{root}\big(46656\,a^5\,b\,d^6+46656\,a^6\,d^6-3888\,a^4\,d^4+108\,a^2\,d^2-1,d,k\big)\Bigg(\mathrm{root}\big(46656\,a^5\,b\,d^6+46656\,a$$
$$+\frac{13510798882111488\,(655360\,a^3\,e^{2\,x}+11382\,b^3\,e^{2\,x}+144416\,a\,b^2+269056\,a^2\,b+131072\,a^3+6459\,b^3+}{b^{10}\,(a+b)^2}$$
$$+\frac{1125899906842624\,(851968\,a^4\,e^{2\,x}+6006\,b^4\,e^{2\,x}+211497\,a\,b^3+597504\,a^3\,b+196608\,a^4+3840\,b^4+60}{b^{10}\,(a+b)^2\,\left(a^2+b\,a\right)}$$
$$+46656\,a^6\,d^6-3888\,a^4\,d^4+108\,a^2\,d^2-1,d,k\big)$$

input
```
int(1/(a + b*cosh(x)^6),x)
```

output

```
symsum(log(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 108*a^2*d
^2 - 1, d, k)*(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 108*a
^2*d^2 - 1, d, k)*(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4 + 1
08*a^2*d^2 - 1, d, k)*(root(46656*a^5*b*d^6 + 46656*a^6*d^6 - 3888*a^4*d^4
 + 108*a^2*d^2 - 1, d, k)*((1459166279268040704*(327680*a^7*exp(2*x) + 298
496*a^6*b + 65536*a^7 + 158*a^2*b^5 + 91315*a^3*b^4 + 348176*a^4*b^3 + 489
952*a^5*b^2 + 196*a^2*b^5*exp(2*x) + 274019*a^3*b^4*exp(2*x) + 1132876*a^4
*b^3*exp(2*x) + 1770440*a^5*b^2*exp(2*x) + 1239040*a^6*b*exp(2*x)))/(b^10*
(a + b)^3) + (17509995351216488448*root(46656*a^5*b*d^6 + 46656*a^6*d^6 -
3888*a^4*d^4 + 108*a^2*d^2 - 1, d, k)*(262144*a^7*exp(2*x) + 203520*a^6*b
+ 65536*a^7 + 453*a^3*b^4 + 72022*a^4*b^3 + 209472*a^5*b^2 + 630*a^3*b^4*e
xp(2*x) + 254512*a^4*b^3*exp(2*x) + 767508*a^5*b^2*exp(2*x) + 775680*a^6*b
*exp(2*x)))/(b^10*(a + b)^2)) - (486388759756013568*(655360*a^5*exp(2*x) -
 9*a*b^4 + 370176*a^4*b + 196608*a^5 - 24408*a^2*b^3 + 149088*a^3*b^2 - 63
676*a^2*b^3*exp(2*x) + 526248*a^3*b^2*exp(2*x) - 10*a*b^4*exp(2*x) + 12451
84*a^4*b*exp(2*x)))/(b^10*(a + b)^2)) - (40532396646334464*(655360*a^5*exp
(2*x) - b^5*exp(2*x) - 24677*a*b^4 + 773120*a^4*b + 262144*a^5 - b^5 + 198
071*a^2*b^3 + 733696*a^3*b^2 + 477713*a^2*b^3*exp(2*x) + 1770640*a^3*b^2*e
xp(2*x) - 53861*a*b^4*exp(2*x) + 1894400*a^4*b*exp(2*x)))/(b^10*(a + b)^3)
) + (13510798882111488*(655360*a^3*exp(2*x) + 11382*b^3*exp(2*x) + 1444...
```

3.65. $\int \frac{1}{a+b\cosh^6(x)}\,dx$

3.66 $\int \frac{1}{a+b\cosh^8(x)}\,dx$

3.66.1 Optimal result

Integrand size = 10, antiderivative size = 245

$$\int \frac{1}{a+b\cosh^8(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{-a}\tanh(x)}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

output
```
-1/4*arctanh((-a)^(1/8)*tanh(x)/((-a)^(1/4)-b^(1/4))^(1/2))/(-a)^(7/8)/((-
a)^(1/4)-b^(1/4))^(1/2)-1/4*arctanh((-a)^(1/8)*tanh(x)/((-a)^(1/4)-I*b^(1/
4))^(1/2))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctanh((-a)^(1/8)*
tanh(x)/((-a)^(1/4)+I*b^(1/4))^(1/2))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1
/2)-1/4*arctanh((-a)^(1/8)*tanh(x)/((-a)^(1/4)+b^(1/4))^(1/2))/(-a)^(7/8)/
((-a)^(1/4)+b^(1/4))^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.64

$$\int \frac{1}{a+b\cosh^8(x)}\,dx = 16\text{RootSum}\Big[b+8b\#1+28b\#1^2+56b\#1^3+256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7 +b\#1^8\&, \frac{x\#1^3+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^3}{b+7b\#1+21b\#1^2+128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\Big]$$

input
```
Integrate[(a + b*Cosh[x]^8)^(-1),x]
```

output
```
16*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 + 5
6*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh
[x] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 + 128*a*#1^3
+ 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

3.66.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3690, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh^8(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(\frac{\pi}{2}+ix\right)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{-a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)\coth^2(x)}d\coth(x)}{4a}+\frac{\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)\coth^2(x)}d\coth(x)}{4a}+\frac{\int \frac{1}{1-\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}+1\right)\coth^2(x)}d\coth(x)}{4a}+\frac{\int \frac{1}{1-\left(1-\frac{a\sqrt[4]{b}}{(-a)^{5/4}}\right)\coth^2(x)}d\coth(x)}{4a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt[8]{-a}\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\coth(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\sqrt[8]{-a}\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\coth(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\sqrt[8]{-a}\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\coth(x)}{\sqrt[8]{-a}}\right)}{4a\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{(-a)^{5/8}\text{arctanh}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\coth(x)}{(-a)^{5/8}}\right)}{4a\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

input
```
Int[(a + b*Cosh[x]^8)^(-1),x]
```

output
```
((-a)^(1/8)*ArcTanh[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Coth[x])/(-a)^(1/8)])/(4
*a*Sqrt[(-a)^(1/4) - I*b^(1/4)]) + ((-a)^(1/8)*ArcTanh[(Sqrt[(-a)^(1/4) +
I*b^(1/4)]*Coth[x])/(-a)^(1/8)])/(4*a*Sqrt[(-a)^(1/4) + I*b^(1/4)]) + ((-a
)^(1/8)*ArcTanh[(Sqrt[(-a)^(1/4) + b^(1/4)]*Coth[x])/(-a)^(1/8)])/(4*a*Sqr
t[(-a)^(1/4) + b^(1/4)]) + ((-a)^(5/8)*ArcTanh[(Sqrt[(-a)^(5/4) + a*b^(1/4
)]*Coth[x])/(-a)^(5/8)])/(4*a*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

3.66. $\int \frac{1}{a+b\cosh^8(x)}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.66.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.75

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8+16777216a^7b)\_Z^8-1048576a^6\_Z^6+24576a^4\_Z^4-256a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2x}+\left(-\frac{4194304a^8}{b}-\right.\right.$
default	$\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{16}+(-8a+8b)\_Z^{14}+(28a+28b)\_Z^{12}+(-56a+56b)\_Z^{10}+(70a+70b)\_Z^8+(-56a+56b)\_Z^6+(28a+28b)\_Z^4+(-8a+8b\right.}\right.$

input
```
int(1/(a+b*cosh(x)^8),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*x)+(-4194304*a^8/b-4194304*a^7)*_R^7+(524288*a^7/b+524288*
a^6)*_R^6+(196608*a^6/b-65536*a^5)*_R^5+(-24576*a^5/b+8192*a^4)*_R^4+(-307
2*a^4/b-1024*a^3)*_R^3+(384/b*a^3+128*a^2)*_R^2+(16*a^2/b-16*a)*_R-2*a/b+1
),_R=RootOf(1+(16777216*a^8+16777216*a^7*b)*_Z^8-1048576*a^6*_Z^6+24576*a^
4*_Z^4-256*a^2*_Z^2))
```

3.66.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 661324 vs. 2(165) = 330.

Time = 3.14 (sec) , antiderivative size = 661324, normalized size of antiderivative = 2699.28

$$\int \frac{1}{a+b\cosh^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

3.66.6 Sympy [F]

$$\int \frac{1}{a+b\cosh^8(x)}\,dx = \int \frac{1}{a+b\cosh^8{(x)}}\,dx$$

input
```
integrate(1/(a+b*cosh(x)**8),x)
```

output
```
Integral(1/(a + b*cosh(x)**8), x)
```

3.66.7 Maxima [F]

$$\int \frac{1}{a+b\cosh^8(x)}\,dx = \int \frac{1}{b\cosh{(x)}^8+a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(b*cosh(x)^8 + a), x)
```

3.66.8 Giac [F]

$$\int \frac{1}{a + b\cosh^8(x)}\,dx = \int \frac{1}{b\cosh(x)^8 + a}\,dx$$

input
```
integrate(1/(a+b*cosh(x)^8),x, algorithm="giac")
```

output
```
sage0*x
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\cosh^8(x)}\,dx = \text{Hanged}$$

input
```
int(1/(a + b*cosh(x)^8),x)
```

output
```
\text{Hanged}
```

3.67 $\int \frac{1}{a-b\cosh^5(x)}\,dx$

3.67.1 Optimal result

Integrand size = 11, antiderivative size = 494

$$\begin{aligned}\int \frac{1}{a-b\cosh^5(x)}\,dx &= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}} \\
&+ \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\end{aligned}$$

output
```
2/5*arctanh((a^(1/5)+b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-b^(1/5))^(1/2))/a
^(4/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctanh((a^(1/5
)-(-1)^(1/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2)
)/a^(4/5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^
(1/2)+2/5*arctanh((a^(1/5)+(-1)^(2/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-
(-1)^(2/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(
1/5)+(-1)^(2/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/5)-(-1)^(3/5)*b^(1/5))^(1
/2)*tanh(1/2*x)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^
(3/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctanh((a^(1/
5)+(-1)^(4/5)*b^(1/5))^(1/2)*tanh(1/2*x)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2
))/a^(4/5)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))
^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.28

$$\int \frac{1}{a-b\cosh^5(x)}\,dx = -\frac{8}{5}\text{RootSum}\Big[b+5b\#1^2+10b\#1^4-32a\#1^5+10b\#1^6+5b\#1^8 + b\#1^{10}\&, \frac{x\#1^3+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)\#1^3}{b+4b\#1^2-16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\Big]$$

input
```
Integrate[(a - b*Cosh[x]^5)^(-1),x]
```

output
```
(-8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 - 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 +
b*#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x
/2]*#1]*#1^3)/(b + 4*b*#1^2 - 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) &
])/5
```

3.67.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cosh^5(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\sin\left(\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\cosh(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cosh(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cosh(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{\right.}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\text{arctanh}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tanh\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

input
```
Int[(a - b*Cosh[x]^5)^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a^(1/5) + b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) - b^(1/5)]])/(
5*a^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTanh[(S
qrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + (-1)^(1/5)*b^(
1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^
(1/5)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]*Tanh[x/2]
)/Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5
)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTanh[(Sqrt[a^(1/5)
- (-1)^(3/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]])/(5*a
^(4/5)*Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5
)]) + (2*ArcTanh[(Sqrt[a^(1/5) + (-1)^(4/5)*b^(1/5)]*Tanh[x/2])/Sqrt[a^(1/
5) - (-1)^(4/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*S
qrt[a^(1/5) + (-1)^(4/5)*b^(1/5)])
```

3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.67.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{10}+(-5a+5b)\_Z^{8}+(10a+10b)\_Z^{6}+(-10a+10b)\_Z^{4}+(5a+5b)\_Z^{2}-a+b\right)}\frac{\left(-\_R^{8}+4\_R^{6}-6\_R\right.}{\_R^{9}a+\_R^{9}b-4\_R^{7}a+4\_R^{7}b+6\_}\right.}{5}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}-1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}-6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{x}+\left(\right.\right.$

input
```
int(1/(a-b*cosh(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a+_R^9*b-4*_R^7*a+4*_R^7*b+6*
_R^5*a+6*_R^5*b-4*_R^3*a+4*_R^3*b+_R*a+_R*b)*ln(tanh(1/2*x)-_R),_R=RootOf(
(a+b)*_Z^10+(-5*a+5*b)*_Z^8+(10*a+10*b)*_Z^6+(-10*a+10*b)*_Z^4+(5*a+5*b)*_
Z^2-a+b))
```

3.67.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{a-b\cosh^5(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a-b*cosh(x)^5),x, algorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

3.67.6 Sympy [F]

$$\int \frac{1}{a-b\cosh^5(x)}\,dx = \int \frac{1}{a-b\cosh^5{(x)}}\,dx$$

input
```
integrate(1/(a-b*cosh(x)**5),x)
```

output
```
Integral(1/(a - b*cosh(x)**5), x)
```

3.67.7 Maxima [F]

$$\int \frac{1}{a-b\cosh^5(x)}\,dx = \int -\frac{1}{b\cosh{(x)}^5-a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^5),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cosh(x)^5 - a), x)
```

3.67.8 Giac [F]

$$\int \frac{1}{a - b\cosh^5(x)}\,dx = \int -\frac{1}{b\cosh{(x)}^5 - a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^5),x, algorithm="giac")
```

output
```
integrate(-1/(b*cosh(x)^5 - a), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a - b\cosh^5(x)}\,dx = \text{Hanged}$$

input
```
int(1/(a - b*cosh(x)^5),x)
```

output
```
\text{Hanged}
```

3.68 $\int \frac{1}{a-b\cosh^6(x)}\,dx$

3.68.1 Optimal result

Integrand size = 11, antiderivative size = 175

$$\int \frac{1}{a-b\cosh^6(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\tanh(x)}{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

output
```
1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)-b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)-b^(1
/3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2))
/a^(5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh(a^(1/6)*tanh(x)/(a
^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/
2)
```

3.68.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.75

$$\int \frac{1}{a-b\cosh^6(x)}\,dx = -\frac{16}{3}\text{RootSum}\Big[b+6b\#1+15b\#1^2-64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5 + b\#1^6\&, \frac{x\#1^2+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^2}{b+5b\#1-32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\Big]$$

input
```
Integrate[(a - b*Cosh[x]^6)^(-1),x]
```

output
```
(-16*RootSum[b + 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 +
6*b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh
[x]*#1]*#1^2)/(b + 5*b*#1 - 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 + 5*b*#1^4 +
 b*#1^5) & ])/3
```

3.68.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cosh^6(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a-b\sin\left(\frac{\pi}{2}+ix\right)^6}dx$$

$$\Big\downarrow \text{ 3690}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\cosh^2(x)}{\sqrt[3]{a}}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}+\frac{\int \frac{1}{\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}+1}dx}{3a}+\frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[3]{a}}}dx}{3a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{1-\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)\coth^2(x)}d\coth(x)}{3a}+\frac{\int \frac{1}{1-\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}+1\right)\coth^2(x)}d\coth(x)}{3a}+\frac{\int \frac{1}{1-\left(1-\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt[3]{a}}\right)\coth^2(x)}d\coth(x)}{3a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\coth(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\coth(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\coth(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

input
```
Int[(a - b*Cosh[x]^6)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Coth[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3)
 - b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Coth[x])/a^(1/6
)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3)
- (-1)^(2/3)*b^(1/3)]*Coth[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/
3)*b^(1/3)])
```

3.68.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.68.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(-1+\left(46656a^6-46656a^5b\right)\_Z^6-3888a^4\_Z^4+108a^2\_Z^2\right)} \_R\ln\left(e^{2x}+\left(\frac{15552a^6}{b}-15552a^5\right)\_R^5+\left(-\right.\right.$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{12}+(-6a-6b)\_Z^{10}+(15a-15b)\_Z^8+(-20a-20b)\_Z^6+(15a-15b)\_Z^4+(-6a-6b)\_Z^2+a-b\right)} \overline{\_R^{11}a-\_R^{11}b-5\_}\right.}{6}$

input
```
int(1/(a-b*cosh(x)^6),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*x)+(15552*a^6/b-15552*a^5)*_R^5+(-2592*a^5/b+2592*a^4)*_R^
4+(-864*a^4/b-432*a^3)*_R^3+(144/b*a^3+72*a^2)*_R^2+(12*a^2/b-12*a)*_R-2*a
/b+1),_R=RootOf(-1+(46656*a^6-46656*a^5*b)*_Z^6-3888*a^4*_Z^4+108*a^2*_Z^2
))
```

3.68.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 16379, normalized size of antiderivative = 93.59

$$\int \frac{1}{a - b\cosh^6(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cosh(x)^6),x, algorithm="fricas")
```

output
```
Too large to include
```

3.68.6 Sympy [F]

$$\int \frac{1}{a - b\cosh^6(x)}\,dx = \int \frac{1}{a - b\cosh^6(x)}\,dx$$

input
```
integrate(1/(a-b*cosh(x)**6),x)
```

output
```
Integral(1/(a - b*cosh(x)**6), x)
```

3.68.7 Maxima [F]

$$\int \frac{1}{a - b\cosh^6(x)}\,dx = \int -\frac{1}{b\cosh(x)^6 - a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^6),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cosh(x)^6 - a), x)
```

3.68.8 Giac [F]

$$\int \frac{1}{a-b\cosh^6(x)}\,dx = \int -\frac{1}{b\cosh{(x)}^6 - a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^6),x, algorithm="giac")
```

output
```
sage0*x
```

3.68.9 Mupad [B] (verification not implemented)

Time = 62.08 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.89

$$\int \frac{1}{a-b\cosh^6(x)}\,dx = \sum_{k=1}^{6} \ln\Bigg(\mathrm{root}\big(46656\,a^5\,b\,d^6 - 46656\,a^6\,d^6 + 3888\,a^4\,d^4 - 108\,a^2\,d^2$$
$$+1, d, k\big)\,\Bigg(\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a^6\,d^6+3888\,a^4\,d^4-108\,a^2\,d^2+1, d, k\big)\,\Bigg(\mathrm{root}\big(46656\,a^5\,b\,d^6-46656\,a$$
$$+\frac{13510798882111488\,(655360\,a^3\,e^{2\,x} - 11382\,b^3\,e^{2\,x} + 144416\,a\,b^2 - 269056\,a^2\,b + 131072\,a^3 - 6459\,b^3 +}{b^{10}\,(a-b)^2}$$
$$-\frac{1125899906842624\,(851968\,a^4\,e^{2\,x} + 6006\,b^4\,e^{2\,x} - 211497\,a\,b^3 - 597504\,a^3\,b + 196608\,a^4 + 3840\,b^4 + 6}{b^{10}\,(a-b)^2\,(a\,b-a^2)}$$
$$- 46656\,a^6\,d^6 + 3888\,a^4\,d^4 - 108\,a^2\,d^2 + 1, d, k\big)$$

input
```
int(1/(a - b*cosh(x)^6),x)
```

output
```
symsum(log(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2*d
^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a
^2*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 1
08*a^2*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4
 - 108*a^2*d^2 + 1, d, k)*((14591662792680407O4*(327680*a^7*exp(2*x) - 298
496*a^6*b + 65536*a^7 - 158*a^2*b^5 + 91315*a^3*b^4 - 348176*a^4*b^3 + 489
952*a^5*b^2 - 196*a^2*b^5*exp(2*x) + 274019*a^3*b^4*exp(2*x) - 1132876*a^4
*b^3*exp(2*x) + 1770440*a^5*b^2*exp(2*x) - 1239040*a^6*b*exp(2*x)))/(b^10*
(a - b)^3) + (17509995351216488448*root(46656*a^5*b*d^6 - 46656*a^6*d^6 +
3888*a^4*d^4 - 108*a^2*d^2 + 1, d, k)*(262144*a^7*exp(2*x) - 203520*a^6*b
+ 65536*a^7 + 453*a^3*b^4 - 72022*a^4*b^3 + 209472*a^5*b^2 + 630*a^3*b^4*e
xp(2*x) - 254512*a^4*b^3*exp(2*x) + 767508*a^5*b^2*exp(2*x) - 775680*a^6*b
*exp(2*x)))/(b^10*(a - b)^2)) - (486388759756013568*(655360*a^5*exp(2*x) -
 9*a*b^4 - 370176*a^4*b + 196608*a^5 + 24408*a^2*b^3 + 149088*a^3*b^2 + 63
676*a^2*b^3*exp(2*x) + 526248*a^3*b^2*exp(2*x) - 10*a*b^4*exp(2*x) - 12451
84*a^4*b*exp(2*x)))/(b^10*(a - b)^2)) - (40532396646334464*(655360*a^5*exp
(2*x) + b^5*exp(2*x) - 24677*a*b^4 - 773120*a^4*b + 262144*a^5 + b^5 - 198
071*a^2*b^3 + 733696*a^3*b^2 - 477713*a^2*b^3*exp(2*x) + 1770640*a^3*b^2*e
xp(2*x) - 53861*a*b^4*exp(2*x) - 1894400*a^4*b*exp(2*x)))/(b^10*(a - b)^3)
) + (13510798882111488*(655360*a^3*exp(2*x) - 11382*b^3*exp(2*x) + 1444...
```

3.68. $\int \frac{1}{a-b\cosh^6(x)}\,dx$

3.69 $\int \frac{1}{a-b\cosh^8(x)}\,dx$

3.69.1 Optimal result

Integrand size = 11, antiderivative size = 213

$$\int \frac{1}{a-b\cosh^8(x)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{a}\tanh(x)}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

output
```
1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)-b^(1/4))^(1/2))/a^(7/8)/(a^(1/4)-b^(1
/4))^(1/2)+1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)-I*b^(1/4))^(1/2))/a^(7/8)/
(a^(1/4)-I*b^(1/4))^(1/2)+1/4*arctanh(a^(1/8)*tanh(x)/(a^(1/4)+I*b^(1/4))^
(1/2))/a^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)+1/4*arctanh(a^(1/8)*tanh(x)/(a^(1
/4)+b^(1/4))^(1/2))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.63 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.74

$$\int \frac{1}{a-b\cosh^8(x)}\,dx = -16\text{RootSum}\Big[b+8b\#1+28b\#1^2+56b\#1^3-256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7+b\#1^8\&,\frac{x\#1^3+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^3}{b+7b\#1+21b\#1^2-128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\Big]$$

input
```
Integrate[(a - b*Cosh[x]^8)^(-1),x]
```

output
```
-16*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 +
56*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sin
h[x] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 - 128*a*#1^3
 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

3.69.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3690, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-b\cosh^8(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a-b\sin\left(\frac{\pi}{2}+ix\right)^8}dx$$

$$\Big\downarrow \ 3690$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\cosh^2(x)}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}}dx}{4a}+\frac{\int \frac{1}{\frac{i\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}+1}dx}{4a}+\frac{\int \frac{1}{\frac{\sqrt[4]{b}\sin\left(ix+\frac{\pi}{2}\right)^2}{\sqrt[4]{a}}+1}dx}{4a}$$

$$\Big\downarrow \text{3660}$$

$$\frac{\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)\coth^2(x)}d\coth(x)}{4a}+\frac{\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)\coth^2(x)}d\coth(x)}{4a}+\frac{\int \frac{1}{1-\left(\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\coth^2(x)}d\coth(x)}{4a}+\frac{\int \frac{1}{1-\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}+1\right)\coth^2(x)}d\coth(x)}{4a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\coth(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\coth(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\coth(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\text{arctanh}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\coth(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

input
```
Int[(a - b*Cosh[x]^8)^(-1),x]
```

output
```
ArcTanh[(Sqrt[a^(1/4) - b^(1/4)]*Coth[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4)
 - b^(1/4)]) + ArcTanh[(Sqrt[a^(1/4) - I*b^(1/4)]*Coth[x])/a^(1/8)]/(4*a^(
7/8)*Sqrt[a^(1/4) - I*b^(1/4)]) + ArcTanh[(Sqrt[a^(1/4) + I*b^(1/4)]*Coth[
x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTanh[(Sqrt[a^(1/4)
 + b^(1/4)]*Coth[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

3.69.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.69.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.86

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8-16777216a^7b)\_Z^8-1048576a^6\_Z^6+24576a^4\_Z^4-256a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2x}+\left(\frac{4194304a^8}{b}-4\right.\right.$
default	$\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{16}+(-8a-8b)\_Z^{14}+(28a-28b)\_Z^{12}+(-56a-56b)\_Z^{10}+(70a-70b)\_Z^8+(-56a-56b)\_Z^6+(28a-28b)\_Z^4+(-8a-8b\right.}\right.$

input
```
int(1/(a-b*cosh(x)^8),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*x)+(4194304*a^8/b-4194304*a^7)*_R^7+(-524288*a^7/b+524288*
a^6)*_R^6+(-196608*a^6/b-65536*a^5)*_R^5+(24576*a^5/b+8192*a^4)*_R^4+(3072
*a^4/b-1024*a^3)*_R^3+(-384/b*a^3+128*a^2)*_R^2+(-16*a^2/b-16*a)*_R+2*a/b+
1),_R=RootOf(1+(16777216*a^8-16777216*a^7*b)*_Z^8-1048576*a^6*_Z^6+24576*a
^4*_Z^4-256*a^2*_Z^2))
```

3.69.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631813 vs. 2(133) = 266.

Time = 3.11 (sec) , antiderivative size = 631813, normalized size of antiderivative = 2966.26

$$\int \frac{1}{a-b\cosh^8(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-b*cosh(x)^8),x, algorithm="fricas")
```

output
```
Too large to include
```

3.69.6 Sympy [F]

$$\int \frac{1}{a-b\cosh^8(x)}\,dx=\int \frac{1}{a-b\cosh^8{(x)}}\,dx$$

input
```
integrate(1/(a-b*cosh(x)**8),x)
```

output
```
Integral(1/(a - b*cosh(x)**8), x)
```

3.69.7 Maxima [F]

$$\int \frac{1}{a-b\cosh^8(x)}\,dx=\int -\frac{1}{b\cosh{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^8),x, algorithm="maxima")
```

output
```
-integrate(1/(b*cosh(x)^8 - a), x)
```

3.69.8 Giac [F]

$$\int \frac{1}{a-b\cosh^8(x)}\,dx=\int -\frac{1}{b\cosh{(x)}^8-a}\,dx$$

input
```
integrate(1/(a-b*cosh(x)^8),x, algorithm="giac")
```

output
```
sage0*x
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a - b\cosh^8(x)}\,dx = \text{Hanged}$$

input
```
int(1/(a - b*cosh(x)^8),x)
```

output
```
\text{Hanged}
```

3.70 $\int \frac{1}{1+\cosh^5(x)}\,dx$

3.70.1 Optimal result

Integrand size = 8, antiderivative size = 223

$$\int \frac{1}{1+\cosh^5(x)}\,dx = -\frac{2\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}} - \frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\arctan\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)} + \frac{2\text{arctanh}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\text{arctanh}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}} + \frac{\sinh(x)}{5(1+\cosh(x))}$$

output
```
1/5*sinh(x)/(1+cosh(x))-2/5*arctan(tanh(1/2*x)/((-1+(-1)^(1/5))/(1+(-1)^(1
/5)))^(1/2))/(-1+(-1)^(2/5))^(1/2)+2/5*arctanh(((1-(-1)^(4/5))/(1+(-1)^(4/
5)))^(1/2)*tanh(1/2*x))/(1+(-1)^(3/5))^(1/2)-2/5*arctan(((-1-(-1)^(3/5))/(
1-(-1)^(3/5)))^(1/2)*tanh(1/2*x))*((-1-(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)/(
1+(-1)^(3/5))+2/5*arctanh(((1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2)*tanh(1/2*x
))/(1-(-1)^(4/5))^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.03 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.00

$$\int \frac{1}{1+\cosh^5(x)}\,dx$$
$$= -\frac{1}{10}\text{RootSum}\Big[1-2\#1+8\#1^2-14\#1^3+30\#1^4-14\#1^5+8\#1^6-2\#1^7$$
$$+\#1^8\&, \frac{x+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)-4x\#1-8\log\left(-\cosh\left(\frac{x}{2}\right)-\text{s}\right.}{}$$
$$+\frac{1}{5}\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(1 + Cosh[x]^5)^(-1),x]
```

output
```
-1/10*RootSum[1 - 2*#1 + 8*#1^2 - 14*#1^3 + 30*#1^4 - 14*#1^5 + 8*#1^6 - 2
*#1^7 + #1^8 & , (x + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x
/2]*#1] - 4*x*#1 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]
*#1]*#1 + 15*x*#1^2 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[
x/2]*#1]*#1^2 - 40*x*#1^3 - 80*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 -
 Sinh[x/2]*#1]*#1^3 + 15*x*#1^4 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2
]*#1 - Sinh[x/2]*#1]*#1^4 - 4*x*#1^5 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh
[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-1 + 8*#1 - 21*#1^2 + 60*#1^3 - 35*#1^4
+ 24*#1^5 - 7*#1^6 + 4*#1^7) & ] + Tanh[x/2]/5
```

3.70.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^5(x)+1}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{1+\sin \left(\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(-\frac{1}{5\left(\sqrt[5]{-1}\cosh(x)-1\right)}-\frac{1}{5\left(-(-1)^{2/5}\cosh(x)-1\right)}-\frac{1}{5\left((-1)^{3/5}\cosh(x)-1\right)}-\frac{1}{5\left(-(-1)^{4/5}\cosh(x)-1\right)}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\arctan\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)}+$$
$$\frac{2\text{arctanh}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\text{arctanh}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{\sinh(x)}{5(\cosh(x)+1)}$$

input
```
Int[(1 + Cosh[x]^5)^(-1),x]
```

output
```
(-2*ArcTan[Tanh[x/2]/Sqrt[-((1 - (-1)^(1/5))/(1 + (-1)^(1/5)))]])/(5*Sqrt[
-1 + (-1)^(2/5)]) - (2*Sqrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*ArcTan[S
qrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*Tanh[x/2]])/(5*(1 + (-1)^(3/5)))
 + (2*ArcTanh[Sqrt[(1 - (-1)^(2/5))/(1 + (-1)^(2/5))]*Tanh[x/2]])/(5*Sqrt[
1 - (-1)^(4/5)]) + (2*ArcTanh[Sqrt[(1 - (-1)^(4/5))/(1 + (-1)^(4/5))]*Tanh
[x/2]])/(5*Sqrt[1 + (-1)^(3/5)]) + Sinh[x]/(5*(1 + Cosh[x]))
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.70.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.28

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^{8}+10\_Z^{4}+1\right)}\frac{\left(-5\_R^{6}+5\_R^{4}-5\_R^{2}+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}+\_R^{3}}\right)}{50}$
risch	$-\frac{2}{5(\mathrm{e}^{x}+1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^{8}-156250\_Z^{6}+6250\_Z^{4}-125\_Z^{2}+1\right)}\_R\ln\left(2343750\_R^{7}-234375\_R^{6}\right.\right.$

input
```
int(1/(1+cosh(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/5*tanh(1/2*x)+1/50*sum((-5*_R^6+5*_R^4-5*_R^2+1)/(_R^7+_R^3)*ln(tanh(1/2
*x)-_R),_R=RootOf(5*_Z^8+10*_Z^4+1))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(150) = 300.

Time = 0.30 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.75

$$\int \frac{1}{1+\cosh^5(x)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(1+cosh(x)^5),x, algorithm="fricas")
```

output
```
1/50*((sqrt(5)*cosh(x) + sqrt(5)*sinh(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(2*
sqrt(5) - 5) + 10)*log(sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*(3*sqrt(5)
 + 5)*sqrt(2*sqrt(5) - 5) - 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3) + 5*sqrt(5
) + 20*cosh(x) + 20*sinh(x) - 5) - (sqrt(5)*cosh(x) + sqrt(5)*sinh(x) + sq
rt(5))*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*log(-sqrt(2*sqrt(5)*sqrt(2
*sqrt(5) - 5) + 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5) - 5*sqrt(2*sqrt(5)
 - 5)*(sqrt(5) + 3) + 5*sqrt(5) + 20*cosh(x) + 20*sinh(x) - 5) - (sqrt(5)*
cosh(x) + sqrt(5)*sinh(x) + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) +
 10)*log(sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*(3*sqrt(5) + 5)*sqrt(2*
sqrt(5) - 5) + 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3) + 5*sqrt(5) + 20*cosh(x
) + 20*sinh(x) - 5) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x) + sqrt(5))*sqrt(-
2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) -
5) + 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5) + 5*sqrt(2*sqrt(5) - 5)*(sqrt
(5) + 3) + 5*sqrt(5) + 20*cosh(x) + 20*sinh(x) - 5) - (sqrt(5)*cosh(x) + s
qrt(5)*sinh(x) + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) + 10)*log(sq
rt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) + 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) -
5) - 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5) - 5*sqrt(5) + 20*cosh(x) + 20*si
nh(x) - 5) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x) + sqrt(5))*sqrt(2*sqrt(5)*
sqrt(-2*sqrt(5) - 5) + 10)*log(-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) + 10)*
(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) - 5) - 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) ...
```

3.70.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{1+\cosh^5(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(1+cosh(x)**5),x)
```

output
```
Exception raised: ValueError >> Exceeds the limit (4300 digits) for intege
r string conversion; use sys.set_int_max_str_digits() to increase the limi
t
```

3.70.7 Maxima [F]

$$\int \frac{1}{1+\cosh^5(x)}\,dx = \int \frac{1}{\cosh(x)^5+1}\,dx$$

input
```
integrate(1/(1+cosh(x)^5),x, algorithm="maxima")
```

output
```
-2/5/(e^x + 1) - integrate(2/5*(e^(7*x) - 4*e^(6*x) + 15*e^(5*x) - 40*e^(4
*x) + 15*e^(3*x) - 4*e^(2*x) + e^x)/(e^(8*x) - 2*e^(7*x) + 8*e^(6*x) - 14*
e^(5*x) + 30*e^(4*x) - 14*e^(3*x) + 8*e^(2*x) - 2*e^x + 1), x)
```

3.70.8 Giac [F]

$$\int \frac{1}{1+\cosh^5(x)}\,dx = \int \frac{1}{\cosh(x)^5+1}\,dx$$

input
```
integrate(1/(1+cosh(x)^5),x, algorithm="giac")
```

output
```
sage0*x
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+\cosh^5(x)}\,dx = \text{Hanged}$$

input
```
int(1/(cosh(x)^5 + 1),x)
```

output
```
\text{Hanged}
```

3.71 $\int \frac{1}{1+\cosh^6(x)} dx$

3.71.1 Optimal result

Integrand size = 8, antiderivative size = 83

$$\int \frac{1}{1+\cosh^6(x)} dx = \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{3\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}} + \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

output
```
1/6*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)+1/3*arctanh(tanh(x)/(1-(-1)^(1/3)
)^(1/2))/(1-(-1)^(1/3))^(1/2)+1/3*arctanh(tanh(x)/(1+(-1)^(2/3))^(1/2))/(1
+(-1)^(2/3))^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+\cosh^6(x)} dx = \frac{1}{6}\Bigg(\arctan(\operatorname{csch}(x)\operatorname{sech}(x)) + i\sqrt{3}\left(\arctan\left(\frac{1-2i\tanh(x)}{\sqrt{3}}\right) - \arctan\left(\frac{1+2i\tanh(x)}{\sqrt{3}}\right)\right) + \sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)\Bigg)$$

input
```
Integrate[(1 + Cosh[x]^6)^(-1),x]
```

output
```
(ArcTan[Csch[x]*Sech[x]] + I*Sqrt[3]*(ArcTan[(1 - (2*I)*Tanh[x])/Sqrt[3]]
- ArcTan[(1 + (2*I)*Tanh[x])/Sqrt[3]]) + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]])
/6
```

3.71.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3690, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^6(x)+1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1+\sin\left(\frac{\pi}{2}+ix\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{3}\int \frac{1}{\cosh^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\cosh^2(x)}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\cosh^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \frac{1}{(-1)^{2/3}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{3}\int \frac{1}{1-2\coth^2(x)}d\coth(x)+\frac{1}{3}\int \frac{1}{1-\left(1-\sqrt[3]{-1}\right)\coth^2(x)}d\coth(x)+\frac{1}{3}\int \frac{1}{1-\left(1+(-1)^{2/3}\right)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{3\sqrt{2}}+\frac{\text{arctanh}\left(\sqrt{1-\sqrt[3]{-1}}\coth(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\text{arctanh}\left(\sqrt{1+(-1)^{2/3}}\coth(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

input
```
Int[(1 + Cosh[x]^6)^(-1),x]
```

output
```
ArcTanh[Sqrt[2]*Coth[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Coth[x
]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Coth[x]]/(3*Sqr
t[1 + (-1)^(2/3)])
```

3.71.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.71.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.86 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e^{2x}+3-2\sqrt{2}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2x}+3+2\sqrt{2}\right)}{12}+\left(\sum_{\_R=\mathrm{RootOf}\left(1296\_Z^4-36\_Z^2+1\right)}\_R\ln\left(-432\_R^3+72\_R^2+\right.\right.$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1\right)}\frac{\left(-\_R^2+4\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3-3\_R^2+2\_R+1}\right)}{6}+\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\,\mathrm{ar}\right.}{}$

input
```
int(1/(1+cosh(x)^6),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-1/12*2^(1/2)*ln(exp(2*x)+3+2*2^(1/2)
)+sum(_R*ln(-432*_R^3+72*_R^2+exp(2*x)-1),_R=RootOf(1296*_Z^4-36*_Z^2+1))
```

3.71.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.80

$$\int \frac{1}{1+\cosh^6(x)}\,dx = -\frac{1}{12}\sqrt{2}\sqrt{i\sqrt{3}+1}\log\Big(2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2 + \left(i\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{i\sqrt{3}+1}+2i\sqrt{3}\Big) + \frac{1}{12}\sqrt{2}\sqrt{i\sqrt{3}+1}\log\Big(2\cosh(x)^2 + 4\cosh(x)\sinh(x)+2\sinh(x)^2+\left(-i\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{i\sqrt{3}+1}+2i\sqrt{3}\Big) + \frac{1}{12}\sqrt{2}\sqrt{-i\sqrt{3}+1}\log\Big(2\cosh(x)^2+4\cosh(x)\sinh(x)+2\sinh(x)^2 + \left(i\sqrt{3}\sqrt{2}-\sqrt{2}\right)\sqrt{-i\sqrt{3}+1}-2i\sqrt{3}\Big) - \frac{1}{12}\sqrt{2}\sqrt{-i\sqrt{3}+1}\log\Big(2\cosh(x)^2 + 4\cosh(x)\sinh(x)+2\sinh(x)^2+\left(-i\sqrt{3}\sqrt{2}+\sqrt{2}\right)\sqrt{-i\sqrt{3}+1}-2i\sqrt{3}\Big) + \frac{1}{12}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2+2\sqrt{2}-}{\cosh(x)^2+\sinh(x)^2+3}\right.$$

input
```
integrate(1/(1+cosh(x)^6),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*sqrt(I*sqrt(3) + 1)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*
sinh(x)^2 + (I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(I*sqrt(3) + 1) + 2*I*sqrt(3
)) + 1/12*sqrt(2)*sqrt(I*sqrt(3) + 1)*log(2*cosh(x)^2 + 4*cosh(x)*sinh(x)
+ 2*sinh(x)^2 + (-I*sqrt(3)*sqrt(2) - sqrt(2))*sqrt(I*sqrt(3) + 1) + 2*I*s
qrt(3)) + 1/12*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log(2*cosh(x)^2 + 4*cosh(x)*si
nh(x) + 2*sinh(x)^2 + (I*sqrt(3)*sqrt(2) - sqrt(2))*sqrt(-I*sqrt(3) + 1) -
 2*I*sqrt(3)) - 1/12*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log(2*cosh(x)^2 + 4*cosh
(x)*sinh(x) + 2*sinh(x)^2 + (-I*sqrt(3)*sqrt(2) + sqrt(2))*sqrt(-I*sqrt(3)
 + 1) - 2*I*sqrt(3)) + 1/12*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*
(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2)
- 3)/(cosh(x)^2 + sinh(x)^2 + 3))
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cosh^6(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cosh(x)**6),x)
```

output
```
Timed out
```

3.71.7 Maxima [F]

$$\int \frac{1}{1+\cosh^6(x)}\,dx = \int \frac{1}{\cosh(x)^6+1}\,dx$$

input
```
integrate(1/(1+cosh(x)^6),x, algorithm="maxima")
```

output
```
-1/12*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3))
- 4/3*integrate(-(6*e^(-2*x) - e^(-4*x) - 1)*e^(-2*x)/(14*e^(-4*x) + e^(-8
*x) + 1), x)
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{1}{1+\cosh^6(x)}\,dx &= \frac{1}{36}\left(\left(2\sqrt{3}-3\right)e^{(4x)}+2\sqrt{3}-3\right)\arctan\left(\frac{e^{(2x)}}{\sqrt{3}+2}\right) \\ &\quad -\frac{1}{36}\left(\left(2\sqrt{3}+3\right)e^{(4x)}+2\sqrt{3}+3\right)\arctan\left(-\frac{e^{(2x)}}{\sqrt{3}-2}\right) \\ &\quad -\frac{1}{12}\sqrt{3}\log\left(\left(\sqrt{3}+2\right)^2+e^{(4x)}\right)+\frac{1}{12}\sqrt{3}\log\left(\left(\sqrt{3}-2\right)^2+e^{(4x)}\right) \\ &\quad +\frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)\end{aligned}$$

input
```
integrate(1/(1+cosh(x)^6),x, algorithm="giac")
```

output
```
1/36*((2*sqrt(3) - 3)*e^(4*x) + 2*sqrt(3) - 3)*arctan(e^(2*x)/(sqrt(3) + 2
)) - 1/36*((2*sqrt(3) + 3)*e^(4*x) + 2*sqrt(3) + 3)*arctan(-e^(2*x)/(sqrt(
3) - 2)) - 1/12*sqrt(3)*log((sqrt(3) + 2)^2 + e^(4*x)) + 1/12*sqrt(3)*log(
(sqrt(3) - 2)^2 + e^(4*x)) + 1/12*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(
2*sqrt(2) + e^(2*x) + 3))
```

3.71.9 Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.06

$$\int \frac{1}{1+\cosh^6(x)}\,dx$$
$$= \frac{\sqrt{3}\,\ln\Big(\big(6177144285775790080\,\mathrm{e}^{2x} - 2167269359741829120\,\sqrt{3} + 3566375915854233600\,\sqrt{3}\,\mathrm{e}^{2x} - 37538}{}$$
$$- \frac{\mathrm{atan}\left(\frac{6177144285775790080\,\mathrm{e}^{2x}-2167269359741829120\,\sqrt{3}+3566375915854233600\,\sqrt{3}\,\mathrm{e}^{2x}-3753820658157486080}{1400944939554045952 0\,\mathrm{e}^{2x}+955607545932677120\,\sqrt{3}+8088359377641144320\,\sqrt{3}\,\mathrm{e}^{2x}+1655160823988879360}\right)}{6}$$
$$- \frac{\sqrt{3}\,\ln\Big(\big(6177144285775790080\,\mathrm{e}^{2x} + 2167269359741829120\,\sqrt{3} - 3566375915854233600\,\sqrt{3}\,\mathrm{e}^{2x} - 375}{}$$
$$- \frac{\pi\,\mathrm{sign}\left(x - \frac{\ln\left(\frac{24639\,\sqrt{3}+42676}{40545\,\sqrt{3}+70226}\right)}{2}\right)}{6}$$
$$+ \frac{\pi\,\mathrm{sign}\big(6177144285775790080\,\mathrm{e}^{2x} - 2167269359741829120\,\sqrt{3} + 3566375915854233600\,\sqrt{3}\,\mathrm{e}^{2x} - 37538}{6}$$
$$- \frac{\sqrt{2}\,\ln\big(2144322552070144000\,\sqrt{2} - 17674880313941032960\,\mathrm{e}^{2x} + 12498027726650736640\,\sqrt{2}\,\mathrm{e}^{2x} - 30}{12}$$
$$+ \frac{\sqrt{2}\,\ln\big(17674880313941032960\,\mathrm{e}^{2x} + 2144322552070144000\,\sqrt{2} + 12498027726650736640\,\sqrt{2}\,\mathrm{e}^{2x} + 30}{12}$$
$$+ \frac{\ln\big(\mathrm{e}^{2x}\,(-14009449395540459520 - 6177144285775790080\mathrm{i}) + \sqrt{3}\,(955607545932677120 - 216726935}{}$$
$$- \frac{\ln\big(\mathrm{e}^{2x}\,(-14009449395540459520 + 6177144285775790080\mathrm{i}) + \sqrt{3}\,(955607545932677120 + 216726935}{}$$

input
```
int(1/(cosh(x)^6 + 1),x)
```

output
```
(log(3^(1/2)*(955607545932677120 - 2167269359741829120i) - exp(2*x)*(14009
449395540459520 + 6177144285775790080i) + 3^(1/2)*exp(2*x)*(80883593776411
44320 + 3566375915854233600i) - (1655160823988879360 - 3753820658157486080
i))*1i)/12 - (log(3^(1/2)*(955607545932677120 + 2167269359741829120i) - ex
p(2*x)*(14009449395540459520 - 6177144285775790080i) + 3^(1/2)*exp(2*x)*(8
088359377641144320 - 3566375915854233600i) - (1655160823988879360 + 375382
0658157486080i))*1i)/12 - atan((6177144285775790080*exp(2*x) - 21672693597
41829120*3^(1/2) + 3566375915854233600*3^(1/2)*exp(2*x) - 3753820658157486
080)/(14009449395540459520*exp(2*x) + 955607545932677120*3^(1/2) + 8088359
377641144320*3^(1/2)*exp(2*x) + 1655160823988879360))/6 + (3^(1/2)*log((61
77144285775790080*exp(2*x) - 2167269359741829120*3^(1/2) + 356637591585423
3600*3^(1/2)*exp(2*x) - 3753820658157486080)^2 + (14009449395540459520*exp
(2*x) + 955607545932677120*3^(1/2) + 8088359377641144320*3^(1/2)*exp(2*x)
+ 1655160823988879360)^2))/12 - (3^(1/2)*log((6177144285775790080*exp(2*x)
 + 2167269359741829120*3^(1/2) - 3566375915854233600*3^(1/2)*exp(2*x) - 37
53820658157486080)^2 + (14009449395540459520*exp(2*x) - 955607545932677120
*3^(1/2) - 8088359377641144320*3^(1/2)*exp(2*x) + 1655160823988879360)^2))
/12 - (pi*sign(x - log((24639*3^(1/2) + 42676)/(40545*3^(1/2) + 70226))/2)
)/6 + (pi*sign(6177144285775790080*exp(2*x) - 2167269359741829120*3^(1/2)
+ 3566375915854233600*3^(1/2)*exp(2*x) - 3753820658157486080))/6 - (2^(...
```

3.71. $\int \frac{1}{1+\cosh^6(x)}\, dx$

3.72 $\int \frac{1}{1+\cosh^8(x)}\,dx$

3.72.1 Optimal result

Integrand size = 8, antiderivative size = 129

$$\int \frac{1}{1+\cosh^8(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\sqrt[4]{-1}}}\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[4]{-1}}}\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{3/4}}}\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+(-1)^{3/4}}}\right)}{4\sqrt{1+(-1)^{3/4}}}$$

output
```
1/4*arctanh(tanh(x)/(1-(-1)^(1/4))^(1/2))/(1-(-1)^(1/4))^(1/2)+1/4*arctanh
(tanh(x)/(1+(-1)^(1/4))^(1/2))/(1+(-1)^(1/4))^(1/2)+1/4*arctanh(tanh(x)/(1
-(-1)^(3/4))^(1/2))/(1-(-1)^(3/4))^(1/2)+1/4*arctanh(tanh(x)/(1+(-1)^(3/4)
)^(1/2))/(1+(-1)^(3/4))^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{1}{1+\cosh^8(x)}\,dx = 16\text{RootSum}\left[1+8\#1+28\#1^2+56\#1^3+326\#1^4+56\#1^5+28\#1^6+8\#1^7+\#1^8\&,\frac{x\#1^3+\log(-\cosh(x)-\sinh(x)+\cosh(x)\#1-\sinh(x)\#1)\#1^3}{1+7\#1+21\#1^2+163\#1^3+35\#1^4+21\#1^5+7\#1^6+\#1^7}\&\right]$$

input
```
Integrate[(1 + Cosh[x]^8)^(-1),x]
```

output
```
16*RootSum[1 + 8*#1 + 28*#1^2 + 56*#1^3 + 326*#1^4 + 56*#1^5 + 28*#1^6 + 8
*#1^7 + #1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#
1]*#1^3)/(1 + 7*#1 + 21*#1^2 + 163*#1^3 + 35*#1^4 + 21*#1^5 + 7*#1^6 + #1^
7) & ]
```

3.72.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3690, 3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^8(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1+\sin\left(\frac{\pi}{2}+ix\right)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\cosh^2(x)}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\cosh^2(x)+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\cosh^2(x)}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\cosh^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{\sqrt[4]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{(-1)^{3/4}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{4}\int \frac{1}{1-\left(1-\sqrt[4]{-1}\right)\coth^2(x)}d\coth(x)+\frac{1}{4}\int \frac{1}{1-\left(1+\sqrt[4]{-1}\right)\coth^2(x)}d\coth(x)+\frac{1}{4}\int \frac{1}{1-\left(1-(-1)^{3/4}\right)\coth^2(x)}d\coth(x)+\frac{1}{4}\int \frac{1}{1-\left(1+(-1)^{3/4}\right)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{1-\sqrt[4]{-1}}\coth(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\operatorname{arctanh}\left(\sqrt{1+\sqrt[4]{-1}}\coth(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\operatorname{arctanh}\left(\sqrt{1-(-1)^{3/4}}\coth(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\operatorname{arctanh}\left(\sqrt{1+(-1)^{3/4}}\coth(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

input
```
Int[(1 + Cosh[x]^8)^(-1),x]
```

output
```
ArcTanh[Sqrt[1 - (-1)^(1/4)]*Coth[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[S
qrt[1 + (-1)^(1/4)]*Coth[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (
-1)^(3/4)]*Coth[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)
]*Coth[x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

3.72.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

3.72.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.83 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.36

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^{8}-4\_Z^{6}+6\_Z^{4}-4\_Z^{2}+1\right)}\_R\ln\left(2\tanh\left(\frac{x}{2}\right)\_R+\tanh\left(\frac{x}{2}\right)^{2}+1\right)\right)}{8}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(33554432\_Z^{8}-1048576\_Z^{6}+24576\_Z^{4}-256\_Z^{2}+1\right)}\_R\ln\left(-8388608\_R^{7}+1048576\_R^{6}+13107\right.$

input
```
int(1/(1+cosh(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/8*sum(_R*ln(2*tanh(1/2*x)*_R+tanh(1/2*x)^2+1),_R=RootOf(2*_Z^8-4*_Z^6+6*
_Z^4-4*_Z^2+1))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(89) = 178.

Time = 0.28 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.71

$$
\begin{aligned}
\int \frac{1}{1+\cosh^8(x)}\,dx = {} & -\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2+2\cosh(x)\sinh(x) \\
& +\sinh(x)^2+\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right) \\
& +\left(\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+1\right)+\sqrt{2}+1\right)\sqrt{\sqrt{2\sqrt{2}-3}+1}+\sqrt{2}+1\Bigg) \\
& +\frac{1}{16}\sqrt{2}\sqrt{\sqrt{2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2+2\cosh(x)\sinh(x) \\
& +\sinh(x)^2+\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right) \\
& -\left(\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+1\right)+\sqrt{2}+1\right)\sqrt{\sqrt{2\sqrt{2}-3}+1}+\sqrt{2}+1\Bigg) \\
& +\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2+2\cosh(x)\sinh(x) \\
& +\sinh(x)^2-\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right) \\
& +\left(\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+1\right)-\sqrt{2}-1\right)\sqrt{-\sqrt{2\sqrt{2}-3}+1}+\sqrt{2}+1\Bigg) \\
& -\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2+2\cosh(x)\sinh(x) \\
& +\sinh(x)^2-\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+2\right) \\
& -\left(\sqrt{2\sqrt{2}-3}\left(\sqrt{2}+1\right)-\sqrt{2}-1\right)\sqrt{-\sqrt{2\sqrt{2}-3}+1}+\sqrt{2}+1\Bigg) \\
& +\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{-2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2+2\cosh(x)\sinh(x) \\
& +\sinh(x)^2+\left(\sqrt{2}-2\right)\sqrt{-2\sqrt{2}-3} \\
& +\left(\left(\sqrt{2}-1\right)\sqrt{-2\sqrt{2}-3}-\sqrt{2}+1\right)\sqrt{-\sqrt{-2\sqrt{2}-3}+1}-\sqrt{2} \\
& +1\Bigg)-\frac{1}{16}\sqrt{2}\sqrt{-\sqrt{-2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2 \\
& +2\cosh(x)\sinh(x)+\sinh(x)^2+\left(\sqrt{2}-2\right)\sqrt{-2\sqrt{2}-3} \\
& -\left(\left(\sqrt{2}-1\right)\sqrt{-2\sqrt{2}-3}-\sqrt{2}+1\right)\sqrt{-\sqrt{-2\sqrt{2}-3}+1}-\sqrt{2} \\
& +1\Bigg)-\frac{1}{16}\sqrt{2}\sqrt{\sqrt{-2\sqrt{2}-3}+1}\log\Bigg(\cosh(x)^2+2\cosh(x)\sinh(x)
\end{aligned}
$$

input
```
integrate(1/(1+cosh(x)^8),x, algorithm="fricas")
```

output
```
-1/16*sqrt(2)*sqrt(sqrt(2*sqrt(2) - 3) + 1)*log(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2 + sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2) + (sqrt(2*sqrt(2) - 3)
*(sqrt(2) + 1) + sqrt(2) + 1)*sqrt(sqrt(2*sqrt(2) - 3) + 1) + sqrt(2) + 1)
 + 1/16*sqrt(2)*sqrt(sqrt(2*sqrt(2) - 3) + 1)*log(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 + sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2) - (sqrt(2*sqrt(2) -
3)*(sqrt(2) + 1) + sqrt(2) + 1)*sqrt(sqrt(2*sqrt(2) - 3) + 1) + sqrt(2) +
1) + 1/16*sqrt(2)*sqrt(-sqrt(2*sqrt(2) - 3) + 1)*log(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 - sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2) + (sqrt(2*sqrt(2)
 - 3)*(sqrt(2) + 1) - sqrt(2) - 1)*sqrt(-sqrt(2*sqrt(2) - 3) + 1) + sqrt(2
) + 1) - 1/16*sqrt(2)*sqrt(-sqrt(2*sqrt(2) - 3) + 1)*log(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - sqrt(2*sqrt(2) - 3)*(sqrt(2) + 2) - (sqrt(2*sqr
t(2) - 3)*(sqrt(2) + 1) - sqrt(2) - 1)*sqrt(-sqrt(2*sqrt(2) - 3) + 1) + sq
rt(2) + 1) + 1/16*sqrt(2)*sqrt(-sqrt(-2*sqrt(2) - 3) + 1)*log(cosh(x)^2 +
2*cosh(x)*sinh(x) + sinh(x)^2 + (sqrt(2) - 2)*sqrt(-2*sqrt(2) - 3) + ((sqr
t(2) - 1)*sqrt(-2*sqrt(2) - 3) - sqrt(2) + 1)*sqrt(-sqrt(-2*sqrt(2) - 3) +
 1) - sqrt(2) + 1) - 1/16*sqrt(2)*sqrt(-sqrt(-2*sqrt(2) - 3) + 1)*log(cosh
(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + (sqrt(2) - 2)*sqrt(-2*sqrt(2) - 3)
 - ((sqrt(2) - 1)*sqrt(-2*sqrt(2) - 3) - sqrt(2) + 1)*sqrt(-sqrt(-2*sqrt(2
) - 3) + 1) - sqrt(2) + 1) - 1/16*sqrt(2)*sqrt(sqrt(-2*sqrt(2) - 3) + 1)*l
og(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - (sqrt(2) - 2)*sqrt(-2*sq...
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1+\cosh^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+cosh(x)**8),x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \frac{1}{1+\cosh^8(x)}\,dx = \int \frac{1}{\cosh(x)^8+1}\,dx$$

input
```
integrate(1/(1+cosh(x)^8),x, algorithm="maxima")
```

output
```
integrate(1/(cosh(x)^8 + 1), x)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{1}{1+\cosh^8(x)}\,dx = 0$$

input
```
integrate(1/(1+cosh(x)^8),x, algorithm="giac")
```

output
```
0
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1+\cosh^8(x)}\,dx = \text{Hanged}$$

input
```
int(1/(cosh(x)^8 + 1),x)
```

output
```
\text{Hanged}
```

3.73 $\int \frac{1}{1-\cosh^5(x)}\,dx$

3.73.1 Optimal result

Integrand size = 10, antiderivative size = 205

$$\int \frac{1}{1-\cosh^5(x)}\,dx = -\frac{2\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{-1+(-1)^{4/5}}}+\frac{2\arctan\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}}+\frac{2\text{arctanh}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\text{arctanh}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{\sinh(x)}{5(1-\cosh(x))}$$

output
```
-1/5*sinh(x)/(1-cosh(x))+2/5*arctanh(((1-(-1)^(3/5))/(1+(-1)^(3/5)))^(1/2)
*tanh(1/2*x))/(1+(-1)^(1/5))^(1/2)+2/5*arctanh(((1-(-1)^(1/5))/(1+(-1)^(1/
5)))^(1/2)*tanh(1/2*x))/(1-(-1)^(2/5))^(1/2)+2/5*arctan(((-1-(-1)^(4/5))/(
1-(-1)^(4/5)))^(1/2)*tanh(1/2*x))/(-1-(-1)^(3/5))^(1/2)-2/5*arctan(tanh(1/
2*x)/((-1+(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2))/(-1+(-1)^(4/5))^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.03 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.17

$$\int \frac{1}{1-\cosh^5(x)}\,dx$$
$$= \frac{1}{5}\coth\left(\frac{x}{2}\right)+\frac{1}{10}\text{RootSum}\Big[1+2\#1+8\#1^2+14\#1^3+30\#1^4+14\#1^5+8\#1^6+2\#1^7$$
$$+\#1^8\&, \frac{x+2\log\left(-\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\#1-\sinh\left(\frac{x}{2}\right)\#1\right)+4x\#1+8\log\left(-\cosh\left(\frac{x}{2}\right)-s\right.}{}$$

input
```
Integrate[(1 - Cosh[x]^5)^(-1),x]
```

output
```
Coth[x/2]/5 + RootSum[1 + 2*#1 + 8*#1^2 + 14*#1^3 + 30*#1^4 + 14*#1^5 + 8*
#1^6 + 2*#1^7 + #1^8 & , (x + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1
- Sinh[x/2]*#1] + 4*x*#1 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - S
inh[x/2]*#1]*#1 + 15*x*#1^2 + 30*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1
 - Sinh[x/2]*#1]*#1^2 + 40*x*#1^3 + 80*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1]*#1^3 + 15*x*#1^4 + 30*Log[-Cosh[x/2] - Sinh[x/2] +
Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 4*x*#1^5 + 8*Log[-Cosh[x/2] - Sinh[x/2
] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x
/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(1 + 8*#1 + 21*#1^2 + 60*#1^3 + 3
5*#1^4 + 24*#1^5 + 7*#1^6 + 4*#1^7) & ]/10
```

3.73.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3692, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh^5(x)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(\frac{\pi}{2}+ix\right)^5}dx$$

$$\Big\downarrow \text{3692}$$

$$\int \left(\frac{1}{5\left(\sqrt[5]{-1}\cosh(x)+1\right)}+\frac{1}{5\left(1-(-1)^{2/5}\cosh(x)\right)}+\frac{1}{5\left((-1)^{3/5}\cosh(x)+1\right)}+\frac{1}{5\left(1-(-1)^{4/5}\cosh(x)\right)}+\frac{1}{5(1$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\arctan\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}}+\frac{2\text{arctanh}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\text{arctanh}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{\sinh(x)}{5(1-\cosh(x))}$$

input
```
Int[(1 - Cosh[x]^5)^(-1),x]
```

output
```
(-2*ArcTan[Tanh[x/2]/Sqrt[-((1 - (-1)^(2/5))/(1 + (-1)^(2/5)))]])/(5*Sqrt[
-1 + (-1)^(4/5)]) + (2*ArcTan[Sqrt[-((1 + (-1)^(4/5))/(1 - (-1)^(4/5)))]*T
anh[x/2]])/(5*Sqrt[-1 - (-1)^(3/5)]) + (2*ArcTanh[Sqrt[(1 - (-1)^(1/5))/(1
 + (-1)^(1/5))]*Tanh[x/2]])/(5*Sqrt[1 - (-1)^(2/5)]) + (2*ArcTanh[Sqrt[(1
- (-1)^(3/5))/(1 + (-1)^(3/5))]*Tanh[x/2]])/(5*Sqrt[1 + (-1)^(1/5)]) - Sin
h[x]/(5*(1 - Cosh[x]))
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3692
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

3.73.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.31

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+10\_Z^4+5\right)}\frac{\left(-\_R^6+5\_R^4-5\_R^2+5\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^7+5\_R^3}\right)}{10}+\frac{1}{5\tanh\left(\frac{x}{2}\right)}$
risch	$\frac{2}{5(e^x-1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^8-156250\_Z^6+6250\_Z^4-125\_Z^2+1\right)}\_R\ln\left(-2343750\_R^7+234375\_R^6\right.\right.$

input
```
int(1/(1-cosh(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/10*sum((-_R^6+5*_R^4-5*_R^2+5)/(_R^7+5*_R^3)*ln(tanh(1/2*x)-_R),_R=RootO
f(_Z^8+10*_Z^4+5))+1/5/tanh(1/2*x)
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(137) = 274.

Time = 0.30 (sec) , antiderivative size = 852, normalized size of antiderivative = 4.16

$$\int \frac{1}{1-\cosh^5(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-cosh(x)^5),x, algorithm="fricas")
```

output
```
-1/50*((sqrt(5)*cosh(x) + sqrt(5)*sinh(x) - sqrt(5))*sqrt(2*sqrt(5)*sqrt(2
*sqrt(5) - 5) + 10)*log(sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*(3*sqrt(5
) + 5)*sqrt(2*sqrt(5) - 5) + 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3) - 5*sqrt(
5) + 20*cosh(x) + 20*sinh(x) + 5) - (sqrt(5)*cosh(x) + sqrt(5)*sinh(x) - s
qrt(5))*sqrt(2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*log(-sqrt(2*sqrt(5)*sqrt(
2*sqrt(5) - 5) + 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5) + 5*sqrt(2*sqrt(5
) - 5)*(sqrt(5) + 3) - 5*sqrt(5) + 20*cosh(x) + 20*sinh(x) + 5) - (sqrt(5)
*cosh(x) + sqrt(5)*sinh(x) - sqrt(5))*sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5)
+ 10)*log(sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*(3*sqrt(5) + 5)*sqrt(2
*sqrt(5) - 5) - 5*sqrt(2*sqrt(5) - 5)*(sqrt(5) + 3) - 5*sqrt(5) + 20*cosh(
x) + 20*sinh(x) + 5) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x) - sqrt(5))*sqrt(
-2*sqrt(5)*sqrt(2*sqrt(5) - 5) + 10)*log(-sqrt(-2*sqrt(5)*sqrt(2*sqrt(5) -
 5) + 10)*(3*sqrt(5) + 5)*sqrt(2*sqrt(5) - 5) - 5*sqrt(2*sqrt(5) - 5)*(sqr
t(5) + 3) - 5*sqrt(5) + 20*cosh(x) + 20*sinh(x) + 5) - (sqrt(5)*cosh(x) +
sqrt(5)*sinh(x) - sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) + 10)*log(s
qrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) + 10)*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) -
 5) + 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5) - 5) + 5*sqrt(5) + 20*cosh(x) + 20*s
inh(x) + 5) + (sqrt(5)*cosh(x) + sqrt(5)*sinh(x) - sqrt(5))*sqrt(2*sqrt(5)
*sqrt(-2*sqrt(5) - 5) + 10)*log(-sqrt(2*sqrt(5)*sqrt(-2*sqrt(5) - 5) + 10)
*(3*sqrt(5) - 5)*sqrt(-2*sqrt(5) - 5) + 5*(sqrt(5) - 3)*sqrt(-2*sqrt(5)...
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\cosh^5(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-cosh(x)**5),x)
```

output
```
Timed out
```

3.73.7 Maxima [F]

$$\int \frac{1}{1-\cosh^5(x)}\,dx = \int -\frac{1}{\cosh{(x)}^5-1}\,dx$$

input
```
integrate(1/(1-cosh(x)^5),x, algorithm="maxima")
```

output
```
2/5/(e^x - 1) + integrate(2/5*(e^(7*x) + 4*e^(6*x) + 15*e^(5*x) + 40*e^(4*
x) + 15*e^(3*x) + 4*e^(2*x) + e^x)/(e^(8*x) + 2*e^(7*x) + 8*e^(6*x) + 14*e
^(5*x) + 30*e^(4*x) + 14*e^(3*x) + 8*e^(2*x) + 2*e^x + 1), x)
```

3.73.8 Giac [F]

$$\int \frac{1}{1-\cosh^5(x)}\,dx = \int -\frac{1}{\cosh{(x)}^5-1}\,dx$$

input
```
integrate(1/(1-cosh(x)^5),x, algorithm="giac")
```

output
```
sage0*x
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{1-\cosh^5(x)}\,dx = \text{Hanged}$$

input
```
int(-1/(cosh(x)^5 - 1),x)
```

output
```
\text{Hanged}
```

3.74 $\int \frac{1}{1-\cosh^6(x)}\,dx$

3.74.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int \frac{1}{1-\cosh^6(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+\sqrt[3]{-1}}}\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-(-1)^{2/3}}}\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\coth(x)}{3}$$

output
```
1/3*coth(x)+1/3*arctanh(tanh(x)/(1+(-1)^(1/3))^(1/2))/(1+(-1)^(1/3))^(1/2)
+1/3*arctanh(tanh(x)/(1-(-1)^(2/3))^(1/2))/(1-(-1)^(2/3))^(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int \frac{1}{1-\cosh^6(x)}\,dx = -\frac{(15+8\cosh(2x)+\cosh(4x))\sinh(x)\left(-6\cosh(x)+\sqrt[4]{-3}\left(\left(3i+\sqrt{3}\right)\arctan\left(\frac{(-1)^{3/4}\left(-i+\sqrt{3}\right)\tanh(x)}{2\sqrt[4]{3}}\right)+\right.\right.}{144\left(-1+\cosh^6(x)\right)}$$

input
```
Integrate[(1 - Cosh[x]^6)^(-1),x]
```

output
```
-1/144*((15 + 8*Cosh[2*x] + Cosh[4*x])*Sinh[x]*(-6*Cosh[x] + (-3)^(1/4)*((
3*I + Sqrt[3])*ArcTan[((-1)^(3/4)*(-I + Sqrt[3])*Tanh[x])/(2*3^(1/4))] + (
3 + I*Sqrt[3])*ArcTan[((-1/3)^(1/4)*(I + Sqrt[3])*Tanh[x])/2])*Sinh[x]))/(
-1 + Cosh[x]^6)
```

3.74.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3042, 3690, 3042, 3654, 25, 3042, 25, 3660, 219, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh^6(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(\frac{\pi}{2}+ix\right)^6}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{3}\int \frac{1}{1-\cosh^2(x)}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\cosh^2(x)+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cosh^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int -\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(ix+\frac{\pi}{2}\right)^2}dx-\frac{1}{3}\int \text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(ix+\frac{\pi}{2}\right)^2}dx-\frac{1}{3}\int -\csc(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[3]{-1}\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\int \csc(ix)^2dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{3}\int \csc(ix)^2dx+\frac{1}{3}\int \frac{1}{1-\left(1+\sqrt[3]{-1}\right)\coth^2(x)}d\coth(x)+\frac{1}{3}\int \frac{1}{1-\left(1-(-1)^{2/3}\right)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\int \csc(ix)^2dx+\frac{\text{arctanh}\left(\sqrt{1+\sqrt[3]{-1}}\coth(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\text{arctanh}\left(\sqrt{1-(-1)^{2/3}}\coth(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{3}i\int 1d(-i\coth(x))+\frac{\text{arctanh}\left(\sqrt{1+\sqrt[3]{-1}}\coth(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\text{arctanh}\left(\sqrt{1-(-1)^{2/3}}\coth(x)\right)}{3\sqrt{1-(-1)^{2/3}}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\text{arctanh}\left(\sqrt{1+\sqrt[3]{-1}}\coth(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\text{arctanh}\left(\sqrt{1-(-1)^{2/3}}\coth(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\coth(x)}{3}$$

input
```
Int[(1 - Cosh[x]^6)^(-1),x]
```

output
```
ArcTanh[Sqrt[1 + (-1)^(1/3)]*Coth[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[S
qrt[1 - (-1)^(2/3)]*Coth[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Coth[x]/3
```

3.74.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.74.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result
risch	$\frac{2}{3(e^{2x}-1)}+\left(\sum_{\_R=\mathrm{RootOf}(3888\_Z^4-108\_Z^2+1)}\_R\ln\left(-1296\_R^3+216\_R^2+e^{2x}-1\right)\right)$
default	$\frac{\tanh(\frac{x}{2})}{6}+\frac{3^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{\tanh(\frac{x}{2})^2+\frac{3^{\frac{3}{4}}\tanh(\frac{x}{2})\sqrt{2}}{3}+\frac{\sqrt{3}}{3}}{\tanh(\frac{x}{2})^2-\frac{3^{\frac{3}{4}}\tanh(\frac{x}{2})\sqrt{2}}{3}+\frac{\sqrt{3}}{3}}\right)+2\arctan\left(\sqrt{2}\,3^{\frac{1}{4}}\tanh(\frac{x}{2})+1\right)+2\arctan\left(\sqrt{2}\,3^{\frac{1}{4}}\tanh(\frac{x}{2})-1\right)\right)}{72}-\frac{3^{\frac{1}{4}}\cdots}{}$

input
```
int(1/(1-cosh(x)^6),x,method=_RETURNVERBOSE)
```

output
```
2/3/(exp(2*x)-1)+sum(_R*ln(-1296*_R^3+216*_R^2+exp(2*x)-1),_R=RootOf(3888*
_Z^4-108*_Z^2+1))
```

3.74.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 358, normalized size of antiderivative = 5.04

$$\int \frac{1}{1-\cosh^6(x)}\,dx = -\frac{\left(\sqrt{6}\cosh(x)^2 + 2\sqrt{6}\cosh(x)\sinh(x) + \sqrt{6}\sinh(x)^2 - \sqrt{6}\right)\sqrt{i\sqrt{3}+3}\log\left(\sqrt{6}(i\sqrt{3}+3)^{\frac{3}{2}} + 6\cosh(\right.}{}$$

input
```
integrate(1/(1-cosh(x)^6),x, algorithm="fricas")
```

output
```
-1/36*((sqrt(6)*cosh(x)^2 + 2*sqrt(6)*cosh(x)*sinh(x) + sqrt(6)*sinh(x)^2
- sqrt(6))*sqrt(I*sqrt(3) + 3)*log(sqrt(6)*(I*sqrt(3) + 3)^(3/2) + 6*cosh(
x)^2 + 12*cosh(x)*sinh(x) + 6*sinh(x)^2 + 6*I*sqrt(3) + 12) - (sqrt(6)*cos
h(x)^2 + 2*sqrt(6)*cosh(x)*sinh(x) + sqrt(6)*sinh(x)^2 - sqrt(6))*sqrt(-I*
sqrt(3) + 3)*log(sqrt(6)*(I*sqrt(3) - 3)*sqrt(-I*sqrt(3) + 3) + 6*cosh(x)^
2 + 12*cosh(x)*sinh(x) + 6*sinh(x)^2 - 6*I*sqrt(3) + 12) + (sqrt(6)*cosh(x
)^2 + 2*sqrt(6)*cosh(x)*sinh(x) + sqrt(6)*sinh(x)^2 - sqrt(6))*sqrt(-I*sqr
t(3) + 3)*log(sqrt(6)*(-I*sqrt(3) + 3)^(3/2) + 6*cosh(x)^2 + 12*cosh(x)*si
nh(x) + 6*sinh(x)^2 - 6*I*sqrt(3) + 12) - (sqrt(6)*cosh(x)^2 + 2*sqrt(6)*c
osh(x)*sinh(x) + sqrt(6)*sinh(x)^2 - sqrt(6))*sqrt(I*sqrt(3) + 3)*log(sqrt
(6)*sqrt(I*sqrt(3) + 3)*(-I*sqrt(3) - 3) + 6*cosh(x)^2 + 12*cosh(x)*sinh(x
) + 6*sinh(x)^2 + 6*I*sqrt(3) + 12) - 24)/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)
```

3.74.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(65) = 130$.

Time = 9.15 (sec) , antiderivative size = 632, normalized size of antiderivative = 8.90

$$\int \frac{1}{1-\cosh^6(x)}\,dx = -\frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{24} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tanh^2\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{72} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{72} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tanh^2\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{24} - \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tanh^2\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{24} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tanh^2\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{72} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tanh^2\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{72} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tanh^2\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{24} + \frac{\tanh\left(\frac{x}{2}\right)}{6} - \frac{\sqrt{2}\cdot\sqrt[4]{3}\,\mathrm{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)-1\right)}{12} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\,\mathrm{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)-1\right)}{36} - \frac{\sqrt{2}\cdot\sqrt[4]{3}\,\mathrm{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+1\right)}{12} + \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\,\mathrm{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tanh\left(\frac{x}{2}\right)+1\right)}{36} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}-1\right)}{36} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}-1\right)}{12} - \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}+1\right)}{36} + \frac{\sqrt{2}\cdot\sqrt[4]{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tanh\left(\frac{x}{2}\right)}{3}+1\right)}{12} + \frac{1}{6\tanh\left(\frac{x}{2}\right)}$$

3.74. $\int \frac{1}{1-\cosh^6(x)}\,dx$

input
```
integrate(1/(1-cosh(x)**6),x)
```

output
```
-sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(x/2) + 4*sq
rt(3))/24 - sqrt(2)*3**(3/4)*log(4*tanh(x/2)**2 - 4*sqrt(2)*3**(1/4)*tanh(
x/2) + 4*sqrt(3))/72 + sqrt(2)*3**(3/4)*log(4*tanh(x/2)**2 + 4*sqrt(2)*3**
(1/4)*tanh(x/2) + 4*sqrt(3))/72 + sqrt(2)*3**(1/4)*log(4*tanh(x/2)**2 + 4*
sqrt(2)*3**(1/4)*tanh(x/2) + 4*sqrt(3))/24 - sqrt(2)*3**(1/4)*log(36*tanh(
x/2)**2 - 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/24 - sqrt(2)*3**(3/4
)*log(36*tanh(x/2)**2 - 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*sqrt(3))/72 + s
qrt(2)*3**(3/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4)*tanh(x/2) + 12*s
qrt(3))/72 + sqrt(2)*3**(1/4)*log(36*tanh(x/2)**2 + 12*sqrt(2)*3**(3/4)*ta
nh(x/2) + 12*sqrt(3))/24 + tanh(x/2)/6 - sqrt(2)*3**(1/4)*atan(sqrt(2)*3**
(1/4)*tanh(x/2) - 1)/12 + sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(1/4)*tanh(x/2)
 - 1)/36 - sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(1/4)*tanh(x/2) + 1)/12 + sqrt
(2)*3**(3/4)*atan(sqrt(2)*3**(1/4)*tanh(x/2) + 1)/36 - sqrt(2)*3**(3/4)*at
an(sqrt(2)*3**(3/4)*tanh(x/2)/3 - 1)/36 + sqrt(2)*3**(1/4)*atan(sqrt(2)*3*
*(3/4)*tanh(x/2)/3 - 1)/12 - sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*tanh(x
/2)/3 + 1)/36 + sqrt(2)*3**(1/4)*atan(sqrt(2)*3**(3/4)*tanh(x/2)/3 + 1)/12
 + 1/(6*tanh(x/2))
```

3.74.7 Maxima [F]

$$\int \frac{1}{1-\cosh^6(x)}\,dx = \int -\frac{1}{\cosh(x)^6-1}\,dx$$

input
```
integrate(1/(1-cosh(x)^6),x, algorithm="maxima")
```

output
```
2/3/(e^(2*x) - 1) + integrate(1/3*(e^(3*x) + 4*e^(2*x) + e^x)/(e^(4*x) + 2
*e^(3*x) + 6*e^(2*x) + 2*e^x + 1), x) - integrate(1/3*(e^(3*x) - 4*e^(2*x)
 + e^x)/(e^(4*x) - 2*e^(3*x) + 6*e^(2*x) - 2*e^x + 1), x)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \frac{1}{1-\cosh^6(x)}\,dx = \frac{2}{3\left(e^{(2x)}-1\right)}$$

input
```
integrate(1/(1-cosh(x)^6),x, algorithm="giac")
```

output
```
2/3/(e^(2*x) - 1)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 329, normalized size of antiderivative = 4.63

$$\int \frac{1}{1-\cosh^6(x)}\,dx = \ln\left(\frac{1061158912\,e^{2x}}{27} + \sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\left(\frac{2539651072\,e^{2x}}{9} - \sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\left(\frac{21515730944\,e^{2x}}{9} + \sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,(19788726272\,e^{2x}+2864709632) + \frac{3870294016}{9}\right) + \frac{548405248}{27}\right) + \frac{351797248}{81}\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}$$
$$+ \ln\left(\frac{1061158912\,e^{2x}}{27} + \sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\left(\frac{2539651072\,e^{2x}}{9} - \sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\left(\frac{21515730944\,e^{2x}}{9} + \sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,(19788726272\,e^{2x}+2864709632) + \frac{3870294016}{9}\right) + \frac{548405248}{27}\right) + \frac{351797248}{81}\right)\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}$$
$$- \ln\left(\frac{1061158912\,e^{2x}}{27} - \sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\left(\frac{2539651072\,e^{2x}}{9} + \sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\left(\frac{21515730944\,e^{2x}}{9} - \sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}\,(19788726272\,e^{2x}+2864709632) + \frac{3870294016}{9}\right) + \frac{548405248}{27}\right) + \frac{351797248}{81}\right)\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1i}{216}}$$
$$- \ln\left(\frac{1061158912\,e^{2x}}{27} - \sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\left(\frac{2539651072\,e^{2x}}{9} + \sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\left(\frac{21515730944\,e^{2x}}{9} - \sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}}\,(19788726272\,e^{2x}+2864709632) + \frac{3870294016}{9}\right) + \frac{548405248}{27}\right) + \frac{351797248}{81}\right)\sqrt{\frac{1}{72}+\frac{\sqrt{3}\,1i}{216}} + \frac{2}{3\,(e^{2x}-1)}$$

input
```
int(-1/(cosh(x)^6 - 1),x)
```

output
```
log((1061158912*exp(2*x))/27 + (1/72 - (3^(1/2)*1i)/216)^(1/2)*((253965107
2*exp(2*x))/9 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*((21515730944*exp(2*x))/9
+ (1/72 - (3^(1/2)*1i)/216)^(1/2)*(19788726272*exp(2*x) + 2864709632) + 38
70294016/9) + 548405248/27) + 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2
) + log((1061158912*exp(2*x))/27 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((25396
51072*exp(2*x))/9 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((21515730944*exp(2*x)
)/9 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*(19788726272*exp(2*x) + 2864709632)
+ 3870294016/9) + 548405248/27) + 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^
(1/2) - log((1061158912*exp(2*x))/27 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*((2
539651072*exp(2*x))/9 + (1/72 - (3^(1/2)*1i)/216)^(1/2)*((21515730944*exp(
2*x))/9 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*(19788726272*exp(2*x) + 28647096
32) + 3870294016/9) + 548405248/27) + 351797248/81)*(1/72 - (3^(1/2)*1i)/2
16)^(1/2) - log((1061158912*exp(2*x))/27 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)
*((2539651072*exp(2*x))/9 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*((21515730944*
exp(2*x))/9 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)*(19788726272*exp(2*x) + 2864
709632) + 3870294016/9) + 548405248/27) + 351797248/81)*((3^(1/2)*1i)/216
+ 1/72)^(1/2) + 2/(3*(exp(2*x) - 1))
```

3.74. $\int \frac{1}{1-\cosh^6(x)}\,dx$

3.75 $\int \frac{1}{1-\cosh^8(x)}\,dx$

3.75.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \frac{1}{1-\cosh^8(x)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{4\sqrt{1-i}}+\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{4\sqrt{1+i}}+\frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{\coth(x)}{4}$$

output
```
1/4*coth(x)+1/4*arctanh(tanh(x)/(1-I)^(1/2))/(1-I)^(1/2)+1/4*arctanh(tanh(
x)/(1+I)^(1/2))/(1+I)^(1/2)+1/8*arctanh(1/2*2^(1/2)*tanh(x))*2^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{1}{1-\cosh^8(x)}\,dx=\frac{1}{8}\left(\frac{2\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{2\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{\sqrt{1+i}}+\sqrt{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{2}}\right)+2\coth(x)\right)$$

input
```
Integrate[(1 - Cosh[x]^8)^(-1),x]
```

output
```
((2*ArcTanh[Tanh[x]/Sqrt[1 - I]])/Sqrt[1 - I] + (2*ArcTanh[Tanh[x]/Sqrt[1
+ I]])/Sqrt[1 + I] + Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]] + 2*Coth[x])/8
```

3.75.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {3042, 3690, 3042, 3654, 25, 3042, 25, 3660, 219, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh^8(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(\frac{\pi}{2}+ix\right)^8}dx$$

$$\Big\downarrow \text{3690}$$

$$\frac{1}{4}\int \frac{1}{1-\cosh^2(x)}dx+\frac{1}{4}\int \frac{1}{1-i\cosh^2(x)}dx+\frac{1}{4}\int \frac{1}{i\cosh^2(x)+1}dx+\frac{1}{4}\int \frac{1}{\cosh^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{1-i\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int -\text{csch}^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx-\frac{1}{4}\int \text{csch}^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx-\frac{1}{4}\int -\csc(ix)^2dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{1}{1-i\sin\left(ix+\frac{\pi}{2}\right)^2}dx+\frac{1}{4}\int \frac{1}{i\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^2+1}dx+\frac{1}{4}\int \csc(ix)^2dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{4}\int \csc(ix)^2dx+\frac{1}{4}\int \frac{1}{1-2\coth^2(x)}d\coth(x)+\frac{1}{4}\int \frac{1}{1-(1+i)\coth^2(x)}d\coth(x)+\frac{1}{4}\int \frac{1}{1-(1-i)\coth^2(x)}d\coth(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\int \csc(ix)^2dx+\frac{\text{arctanh}\left(\sqrt{1-i}\coth(x)\right)}{4\sqrt{1-i}}+\frac{\text{arctanh}\left(\sqrt{1+i}\coth(x)\right)}{4\sqrt{1+i}}+\frac{\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{4\sqrt{2}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}i\int 1d(-i\coth(x))+\frac{\text{arctanh}\left(\sqrt{1-i}\coth(x)\right)}{4\sqrt{1-i}}+\frac{\text{arctanh}\left(\sqrt{1+i}\coth(x)\right)}{4\sqrt{1+i}}+\frac{\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{4\sqrt{2}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\text{arctanh}\left(\sqrt{1-i}\coth(x)\right)}{4\sqrt{1-i}}+\frac{\text{arctanh}\left(\sqrt{1+i}\coth(x)\right)}{4\sqrt{1+i}}+\frac{\text{arctanh}\left(\sqrt{2}\coth(x)\right)}{4\sqrt{2}}+\frac{\coth(x)}{4}$$

input
```
Int[(1 - Cosh[x]^8)^(-1),x]
```

output
```
ArcTanh[Sqrt[1 - I]*Coth[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Coth[x]
]/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Coth[x]]/(4*Sqrt[2]) + Coth[x]/4
```

3.75.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 3690
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Simp[2/(a*n)   Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n
/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.75.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

method	result
risch	$\frac{1}{2\,\mathrm{e}^{2x}-2}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3-2\sqrt{2}\right)}{16}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2x}+3+2\sqrt{2}\right)}{16}+\left(\sum_{\_R=\mathrm{RootOf}\left(8192\_Z^4-128\_Z^2+1\right)}\_R\ln\left(-2048\_R^3+\right.\right.$
default	$\frac{\tanh\left(\frac{x}{2}\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2+\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}{\tanh\left(\frac{x}{2}\right)^2-\tanh\left(\frac{x}{2}\right)\sqrt{2}+1}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}+1\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\sqrt{2}-1\right)\right)}{32}-\frac{\sqrt{2}\left(\ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2-}{\tanh\left(\frac{x}{2}\right)^2+}\right.\right.}{}$

input
```
int(1/(1-cosh(x)^8),x,method=_RETURNVERBOSE)
```

output
```
1/2/(exp(2*x)-1)+1/16*2^(1/2)*ln(exp(2*x)+3-2*2^(1/2))-1/16*2^(1/2)*ln(exp
(2*x)+3+2*2^(1/2))+sum(_R*ln(-2048*_R^3+256*_R^2+exp(2*x)-1),_R=RootOf(819
2*_Z^4-128*_Z^2+1))
```

3.75.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 358, normalized size of antiderivative = 5.19

$$\int \frac{1}{1-\cosh^8(x)}\,dx =$$

$$-\frac{\sqrt{i+1}\left(\sqrt{2}\cosh\left(x\right)^2+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^2-\sqrt{2}\right)\log\left(\cosh\left(x\right)^2+2\cosh\left(x\right)\sinh\left(x\right)\right.}{}$$

input
```
integrate(1/(1-cosh(x)^8),x, algorithm="fricas")
```

output
```
-1/16*(sqrt(I + 1)*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2
)*sinh(x)^2 - sqrt(2))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + (I
+ 1)*sqrt(2)*sqrt(I + 1) + 2*I + 1) - sqrt(I + 1)*(sqrt(2)*cosh(x)^2 + 2*s
qrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(cosh(x)^2 + 2*co
sh(x)*sinh(x) + sinh(x)^2 - (I + 1)*sqrt(2)*sqrt(I + 1) + 2*I + 1) + sqrt(
-I + 1)*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 - sqrt(2))*log(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - (I - 1)*sqrt(2
)*sqrt(-I + 1) - 2*I + 1) - sqrt(-I + 1)*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*co
sh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(cosh(x)^2 + 2*cosh(x)*sin
h(x) + sinh(x)^2 + (I - 1)*sqrt(2)*sqrt(-I + 1) - 2*I + 1) - (sqrt(2)*cosh
(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(-(3*(
2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2
) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2 + sinh(x)^2 + 3)) - 8)/(cosh(
x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{1-\cosh^8(x)}\,dx = \text{Timed out}$$

input
```
integrate(1/(1-cosh(x)**8),x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{1}{1-\cosh^8(x)}\,dx = \int -\frac{1}{\cosh(x)^8-1}\,dx$$

input
```
integrate(1/(1-cosh(x)^8),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1
/2/(e^(2*x) - 1) + 8*integrate(e^(4*x)/(e^(8*x) + 4*e^(6*x) + 22*e^(4*x) +
 4*e^(2*x) + 1), x)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{1}{1-\cosh^8(x)}\,dx = \frac{1}{16}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right) + \frac{1}{2\left(e^{(2x)}-1\right)}$$

input
```
integrate(1/(1-cosh(x)^8),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1
/2/(e^(2*x) - 1)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.93

$$\int \frac{1}{1-\cosh^8(x)}\,dx$$
$$= \frac{\sqrt{2}\ln\left(582732658686033920\,e^{2x} + 70697326355677184\sqrt{2} + 412054214575915008\sqrt{2}\,e^{2x} + 9998111775\right.}{16}$$
$$-\frac{\sqrt{2}\ln\left(70697326355677184\sqrt{2} - 582732658686033920\,e^{2x} + 412054214575915008\sqrt{2}\,e^{2x} - 99981117\right.}{16}$$
$$+\frac{1}{2\,(e^{2x}-1)}$$
$$-\frac{\sqrt{2}\sqrt{1-i}\ln\left(70836483296067584 + \sqrt{2}\sqrt{1-i}\,(-54684829282729984 + 21956972328779776i) + \sqrt{2}\right.}{}$$
$$+\frac{\sqrt{2}\sqrt{1-i}\ln\left(70836483296067584 + \sqrt{2}\sqrt{1-i}\,(54684829282729984 - 21956972328779776i) + \sqrt{2}\sqrt{}\right.}{}$$
$$-\frac{\sqrt{2}\sqrt{1+1i}\ln\left(70836483296067584 + \sqrt{2}\sqrt{1+1i}\,(-54684829282729984 - 21956972328779776i) + \sqrt{}\right.}{}$$
$$+\frac{\sqrt{2}\sqrt{1+1i}\ln\left(70836483296067584 + \sqrt{2}\sqrt{1+1i}\,(54684829282729984 + 21956972328779776i) + \sqrt{2}\right.}{}$$

input
```
int(-1/(cosh(x)^8 - 1),x)
```

output
```
(2^(1/2)*log(582732658686033920*exp(2*x) + 706973263556771 84*2^(1/2) + 412
054214575915008*2^(1/2)*exp(2*x) + 999811177544417 28))/16 - (2^(1/2)*log(7
0697326355677184*2^(1/2) - 582732658686033920*exp(2*x) + 41205421457591500
8*2^(1/2)*exp(2*x) - 99981117754441728))/16 + 1/(2*(exp(2*x) - 1)) - (2^(1
/2)*(1 - 1i)^(1/2)*log((70836483296067584 - 69311013991743488i) - 2^(1/2)*
(1 - 1i)^(1/2)*(54684829282729984 - 21956972328779776i) - 2^(1/2)*(1 - 1i)
^(1/2)*exp(2*x)*(12296353929494528 - 271474128182050816i) - exp(2*x)*(1556
13434002538496 + 429723297714798592i)))/16 + (2^(1/2)*(1 - 1i)^(1/2)*log(2
^(1/2)*(1 - 1i)^(1/2)*(54684829282729984 - 21956972328779776i) - exp(2*x)*
(155613434002538496 + 429723297714798592i) + 2^(1/2)*(1 - 1i)^(1/2)*exp(2*
x)*(12296353929494528 - 271474128182050816i) + (70836483296067584 - 693110
13991743488i)))/16 - (2^(1/2)*(1 + 1i)^(1/2)*log((70836483296067584 + 6931
1013991743488i) - 2^(1/2)*(1 + 1i)^(1/2)*(54684829282729984 + 219569723287
79776i) - 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(12296353929494528 + 27147412818
2050816i) - exp(2*x)*(155613434002538496 - 429723297714798592i)))/16 + (2^
(1/2)*(1 + 1i)^(1/2)*log(2^(1/2)*(1 + 1i)^(1/2)*(54684829282729984 + 21956
972328779776i) - exp(2*x)*(155613434002538496 - 429723297714798592i) + 2^(
1/2)*(1 + 1i)^(1/2)*exp(2*x)*(12296353929494528 + 271474128182050816i) + (
70836483296067584 + 69311013991743488i)))/16
```

3.75. $\int \frac{1}{1-\cosh^8(x)}\,dx$

3.76 $\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx$

3.76.1 Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = \log(\cosh(x)) - \frac{1}{2}\log\left(1+\cosh^2(x)\right)$$

output
```
ln(cosh(x))-1/2*ln(1+cosh(x)^2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = \log(\cosh(x)) - \frac{1}{2}\log\left(1+\cosh^2(x)\right)$$

input
```
Integrate[Tanh[x]/(1 + Cosh[x]^2),x]
```

output
```
Log[Cosh[x]] - Log[1 + Cosh[x]^2]/2
```

3.76.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 26, 3673, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\cosh^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)^2\right)\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)^2+1\right)\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3673}$$

$$\frac{1}{2}\int \frac{\text{sech}^2(x)}{\cosh^2(x)+1}d\cosh^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int \text{sech}^2(x)d\cosh^2(x)-\int \frac{1}{\cosh^2(x)+1}d\cosh^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log\left(\cosh^2(x)\right)-\int \frac{1}{\cosh^2(x)+1}d\cosh^2(x)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(\cosh^2(x)\right)-\log\left(\cosh^2(x)+1\right)\right)$$

input
```
Int[Tanh[x]/(1 + Cosh[x]^2),x]
```

output
```
(Log[Cosh[x]^2] - Log[1 + Cosh[x]^2])/2
```

3.76.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.76.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{4}+1\right)}{2}+\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)$	22
risch	$\ln\left(1+e^{2x}\right)-\frac{\ln\left(e^{4x}+6\,e^{2x}+1\right)}{2}$	24

input
```
int(tanh(x)/(1+cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(tanh(1/2*x)^4+1)+ln(1+tanh(1/2*x)^2)
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = -\frac{1}{2}\log\left(\frac{2\left(\cosh(x)^2+\sinh(x)^2+3\right)}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) + \log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(tanh(x)/(1+cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*log(2*(cosh(x)^2 + sinh(x)^2 + 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2)) + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

3.76.6 Sympy [F]

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = \int \frac{\tanh(x)}{\cosh^2(x)+1}\,dx$$

input
```
integrate(tanh(x)/(1+cosh(x)**2),x)
```

output
```
Integral(tanh(x)/(cosh(x)**2 + 1), x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = -\frac{1}{2}\log\left(6e^{(-2x)}+e^{(-4x)}+1\right)+\log\left(e^{(-2x)}+1\right)$$

input
```
integrate(tanh(x)/(1+cosh(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log(6*e^(-2*x) + e^(-4*x) + 1) + log(e^(-2*x) + 1)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = -\frac{1}{2}\,\log\left(e^{(4\,x)} + 6\,e^{(2\,x)} + 1\right) + \log\left(e^{(2\,x)} + 1\right)$$

input
```
integrate(tanh(x)/(1+cosh(x)^2),x, algorithm="giac")
```

output
```
-1/2*log(e^(4*x) + 6*e^(2*x) + 1) + log(e^(2*x) + 1)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{\tanh(x)}{1+\cosh^2(x)}\,dx = \ln\left(-5184\,\mathrm{e}^{2\,x} - 5184\right) - \frac{\ln\left(54\,\mathrm{e}^{2\,x} + 9\,\mathrm{e}^{4\,x} + 9\right)}{2}$$

input
```
int(tanh(x)/(cosh(x)^2 + 1),x)
```

output
```
log(- 5184*exp(2*x) - 5184) - log(54*exp(2*x) + 9*exp(4*x) + 9)/2
```

3.77 $\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx$

3.77.1 Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx = -\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)+\sqrt{a+b\cosh^2(x)}$$

output
```
-arctanh((a+b*cosh(x)^2)^(1/2)/a^(1/2))*a^(1/2)+(a+b*cosh(x)^2)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx = -\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)+\sqrt{a+b\cosh^2(x)}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]^2]*Tanh[x],x]
```

output
```
-(Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]) + Sqrt[a + b*Cosh[x]^2]
```

3.77.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3673, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\cosh^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}}{\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}}{\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{1}{2}\int \sqrt{b\cosh^2(x)+a}\text{sech}^2(x)d\cosh^2(x)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(a\int \frac{\text{sech}^2(x)}{\sqrt{b\cosh^2(x)+a}}d\cosh^2(x)+2\sqrt{a+b\cosh^2(x)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{2a\int \frac{1}{\frac{\cosh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cosh^2(x)+a}}{b}+2\sqrt{a+b\cosh^2(x)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(2\sqrt{a+b\cosh^2(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*Cosh[x]^2]*Tanh[x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]^
2])/2
```

3.77.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.77.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
default	$\sqrt{a+b\cosh\left(x\right)^{2}}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\cosh(x)^{2}}}{\cosh(x)}\right)$	42

input
```
int((a+b*cosh(x)^2)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
(a+b*cosh(x)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cosh(x)^2)^(1/2))/cos
h(x))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(31) = 62$.

Time = 0.41 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.15

$$\int\sqrt{a+b\cosh^2(x)}\tanh(x)\,dx$$

$$= \left[\frac{\sqrt{a}(\cosh\left(x\right)+\sinh\left(x\right))\log\left(\frac{b\cosh(x)^{4}+4\,b\cosh(x)\sinh(x)^{3}+b\sinh(x)^{4}+2\,(4\,a+b)\cosh(x)^{2}+2\left(3\,b\cosh(x)^{2}+4\,a+b\right)\sinh(x)^{2}-4 \cdots}{\cosh(x)^{4}+4\,\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\,\cosh(x)^{2} \cdots\right.}\right.}{2\,(\cos \cdots}\right.$$

input
```
integrate((a+b*cosh(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*(cosh(x) + sinh(x))*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3
 + b*sinh(x)^4 + 2*(4*a + b)*cosh(x)^2 + 2*(3*b*cosh(x)^2 + 4*a + b)*sinh(
x)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))*(cosh(x) + sinh(x)) + 4*(b*cosh(x)^
3 + (4*a + b)*cosh(x))*sinh(x) + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh
(x))*sinh(x) + 1)) + sqrt(2)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(c
osh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/2*(2*sq
rt(-a)*(cosh(x) + sinh(x))*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a + b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*co
sh(x) + a*sinh(x))) + sqrt(2)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(
cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x) + sinh(x))]
```

3.77.6 Sympy [F]

$$\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx = \int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx$$

input
```
integrate((a+b*cosh(x)**2)**(1/2)*tanh(x),x)
```

output
```
Integral(sqrt(a + b*cosh(x)**2)*tanh(x), x)
```

3.77.7 Maxima [F]

$$\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx = \int \sqrt{b\cosh(x)^2+a}\tanh(x)\,dx$$

input
```
integrate((a+b*cosh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x)^2 + a)*tanh(x), x)
```

3.77.8 Giac [F]

$$\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx=\int \sqrt{b\cosh{(x)}^2+a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x)^2 + a)*tanh(x), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh^2(x)}\tanh(x)\,dx=\int \tanh(x)\,\sqrt{b\cosh{(x)}^2+a}\,dx$$

input
```
int(tanh(x)*(a + b*cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)*(a + b*cosh(x)^2)^(1/2), x)
```

3.78 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx$

3.78.1 Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-arctanh((a+b*cosh(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]/Sqrt[a])
```

3.78.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3673, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i}{\tan\left(\frac{\pi}{2}+ix\right)\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^2+a}\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3673$$

$$\frac{1}{2}\int \frac{\operatorname{sech}^2(x)}{\sqrt{b\cosh^2(x)+a}}d\cosh^2(x)$$

$$\Big\downarrow \ 73$$

$$\frac{\int \frac{1}{\frac{\cosh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\cosh^2(x)+a}}{b}$$

$$\Big\downarrow \ 221$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Cosh[x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Cosh[x]^2]/Sqrt[a]]/Sqrt[a])
```

3.78.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.78.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\cosh(x)^2}}{\cosh(x)}\right)}{\sqrt{a}}$	31

input
```
int(tanh(x)/(a+b*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cosh(x)^2)^(1/2))/cosh(x))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 9.54

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx$$

$$= \left[\frac{\log\left(\frac{b\cosh(x)^4+4\,b\cosh(x)\sinh(x)^3+b\sinh(x)^4+2\,(4\,a+b)\cosh(x)^2+2\left(3\,b\cosh(x)^2+4\,a+b\right)\sinh(x)^2-4\,\sqrt{2}\sqrt{a}\sqrt{\frac{b\cosh(x)^2+b\sinh(x)^2+}{\cosh(x)^2-2\,\cosh(x)\sinh(x)}}}{\cosh(x)^4+4\,\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\,\cosh(x)^2+1\right)\sinh(x)^2+2\,\cosh(x)^2+4\,(\cos}\right.}{2\,\sqrt{a}}\right.$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*(4*a + b)*
cosh(x)^2 + 2*(3*b*cosh(x)^2 + 4*a + b)*sinh(x)^2 - 4*sqrt(2)*sqrt(a)*sqrt
((b*cosh(x)^2 + b*sinh(x)^2 + 2*a + b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + si
nh(x)^2))*(cosh(x) + sinh(x)) + 4*(b*cosh(x)^3 + (4*a + b)*cosh(x))*sinh(x
) + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*
sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(a), s
qrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(x)^2 + b*sinh(x)^2 + 2*a
+ b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x) + a*sinh(x)))
/a]
```

3.78.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*cosh(x)**2), x)
```

3.78.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x)^2 + a), x)
```

3.78.8 Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^2(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{b\cosh(x)^2+a}}\,dx$$

input
```
int(tanh(x)/(a + b*cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)/(a + b*cosh(x)^2)^(1/2), x)
```

3.79 $\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx$

3.79.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = -\text{arctanh}\left(\sqrt{1+\cosh^2(x)}\right)$$

output
```
-arctanh((1+cosh(x)^2)^(1/2))
```

3.79.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = -\text{arctanh}\left(\sqrt{1+\cosh^2(x)}\right)$$

input
```
Integrate[Tanh[x]/Sqrt[1 + Cosh[x]^2],x]
```

output
```
-ArcTanh[Sqrt[1 + Cosh[x]^2]]
```

3.79.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3673, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{\cosh^2(x)+1}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i}{\sqrt{1+\sin\left(\frac{\pi}{2}+ix\right)^2}\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)^2+1}\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3673$$

$$\frac{1}{2}\int \frac{\text{sech}^2(x)}{\sqrt{\cosh^2(x)+1}}d\cosh^2(x)$$

$$\Big\downarrow \; 73$$

$$\int \frac{1}{\cosh^4(x)-1}d\sqrt{\cosh^2(x)+1}$$

$$\Big\downarrow \; 220$$

$$-\text{arctanh}\left(\sqrt{\cosh^2(x)+1}\right)$$

input
```
Int[Tanh[x]/Sqrt[1 + Cosh[x]^2],x]
```

output
```
-ArcTanh[Sqrt[1 + Cosh[x]^2]]
```

3.79.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3673
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m
 + 1)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1
)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Integ
erQ[(m - 1)/2]
```

3.79.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\cosh\left(x\right)^{2}}}\right)$	12

input
```
int(tanh(x)/(1+cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/(1+cosh(x)^2)^(1/2))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.85

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = \log\left(\frac{\sqrt{2}\sqrt{\frac{\cosh(x)^2+\sinh(x)^2+3}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}}-2\cosh(x)-2\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}\right)$$

input
```
integrate(tanh(x)/(1+cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log((sqrt(2)*sqrt((cosh(x)^2 + sinh(x)^2 + 3)/(cosh(x)^2 - 2*cosh(x)*sinh(
x) + sinh(x)^2)) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1))
```

3.79.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{\cosh^2(x)+1}}\,dx$$

input
```
integrate(tanh(x)/(1+cosh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(cosh(x)**2 + 1), x)
```

3.79.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{\cosh(x)^2+1}}\,dx$$

input
```
integrate(tanh(x)/(1+cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(cosh(x)^2 + 1), x)
```

3.79.8 Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{\cosh(x)^2+1}}\,dx$$

input
```
integrate(tanh(x)/(1+cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(cosh(x)^2 + 1), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{1+\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{\cosh(x)^2+1}}\,dx$$

input
```
int(tanh(x)/(cosh(x)^2 + 1)^(1/2),x)
```

output
```
int(tanh(x)/(cosh(x)^2 + 1)^(1/2), x)
```

3.80 $\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx$

3.80.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = -\operatorname{arctanh}\left(\sqrt{-\sinh^2(x)}\right)$$

output
```
-arctanh((-sinh(x)^2)^(1/2))
```

3.80.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = \frac{\arctan(\sinh(x))\sinh(x)}{\sqrt{-\sinh^2(x)}}$$

input
```
Integrate[Tanh[x]/Sqrt[1 - Cosh[x]^2],x]
```

output
```
(ArcTan[Sinh[x]]*Sinh[x])/Sqrt[-Sinh[x]^2]
```

3.80.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 26, 3655, 26, 3042, 26, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i}{\sqrt{1-\sin\left(\frac{\pi}{2}+ix\right)^2}\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\sqrt{1-\sin\left(ix+\frac{\pi}{2}\right)^2}\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3655$$

$$i\int -\frac{i\tanh(x)}{\sqrt{-\sinh^2(x)}}dx$$

$$\Big\downarrow \; 26$$

$$\int \frac{\tanh(x)}{\sqrt{-\sinh^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\tan(ix)}{\sqrt{\sin(ix)^2}}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\tan(ix)}{\sqrt{\sin(ix)^2}}dx$$

$$\Big\downarrow \; 3684$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-\sinh^2(x)}\left(\sinh^2(x)+1\right)}d\sinh^2(x)$$

$$\Big\downarrow \; 73$$

$$-\int \frac{1}{1-\sinh^4(x)}d\sqrt{-\sinh^2(x)}$$
$$\Big\downarrow \text{219}$$
$$-\text{arctanh}\left(\sqrt{-\sinh^2(x)}\right)$$

input
```
Int[Tanh[x]/Sqrt[1 - Cosh[x]^2],x]
```

output
```
-ArcTanh[Sqrt[-Sinh[x]^2]]
```

3.80.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3655
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.80.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

method	result	size
default	$\text{‘int/indef0‘}\left(\frac{\sinh(x)}{\cosh(x)^{2}\sqrt{-\sinh(x)^{2}}}, \sinh\left(x\right)\right)$	19
risch	$\frac{i\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}+i\right)}{\sqrt{-\left(\mathrm{e}^{2x}-1\right)^{2}\mathrm{e}^{-2x}}}-\frac{i\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}-i\right)}{\sqrt{-\left(\mathrm{e}^{2x}-1\right)^{2}\mathrm{e}^{-2x}}}$	72

input
```
int(tanh(x)/(1-cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
`int/indef0`(sinh(x)/cosh(x)^2/(-sinh(x)^2)^(1/2),sinh(x))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 8.62

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx$$
$$= \log\left(\frac{\cosh\left(x\right)e^{(2x)} + \left(e^{(2x)}-1\right)\sinh\left(x\right) + \sqrt{-(e^{(4x)}-2e^{(2x)}+1)e^{(-2x)}}e^{x} - \cosh\left(x\right)}{e^{(2x)}-1}\right)$$
$$-\log\left(\frac{\cosh\left(x\right)e^{(2x)} + \left(e^{(2x)}-1\right)\sinh\left(x\right) - \sqrt{-(e^{(4x)}-2e^{(2x)}+1)e^{(-2x)}}e^{x} - \cosh\left(x\right)}{e^{(2x)}-1}\right)$$

input
```
integrate(tanh(x)/(1-cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log((cosh(x)*e^(2*x) + (e^(2*x) - 1)*sinh(x) + sqrt(-(e^(4*x) - 2*e^(2*x)
+ 1)*e^(-2*x))*e^x - cosh(x))/(e^(2*x) - 1)) - log((cosh(x)*e^(2*x) + (e^(
2*x) - 1)*sinh(x) - sqrt(-(e^(4*x) - 2*e^(2*x) + 1)*e^(-2*x))*e^x - cosh(x
))/(e^(2*x) - 1))
```

3.80.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{-(\cosh(x)-1)(\cosh(x)+1)}}\,dx$$

input
```
integrate(tanh(x)/(1-cosh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(-(cosh(x) - 1)*(cosh(x) + 1)), x)
```

3.80.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.54

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = -2i\arctan\left(e^{(-x)}\right)$$

input
```
integrate(tanh(x)/(1-cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*I*arctan(e^(-x))
```

3.80.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.92

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = -\frac{\log(e^x+i)}{\operatorname{sgn}(-e^{(3x)}+e^x)} + \frac{\log(e^x-i)}{\operatorname{sgn}(-e^{(3x)}+e^x)}$$

input
```
integrate(tanh(x)/(1-cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(e^x + I)/sgn(-e^(3*x) + e^x) + log(e^x - I)/sgn(-e^(3*x) + e^x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{1-\cosh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{1-\cosh(x)^2}}\,dx$$

input
```
int(tanh(x)/(1 - cosh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)/(1 - cosh(x)^2)^(1/2), x)
```

3.81 $\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx$

3.81.1 Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cosh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} + \frac{\log(\cosh(x))}{a} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cosh(x)\right)}{3a^{5/3}} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cosh(x)+b^{2/3}\cosh^2(x)\right)}{6a^{5/3}} - \frac{\log\left(a+b\cosh^3(x)\right)}{3a} + \frac{\operatorname{sech}^2(x)}{2a}$$

output
```
ln(cosh(x))/a+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*cosh(x))/a^(5/3)-1/6*b^(2/3)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*cosh(x)+b^(2/3)*cosh(x)^2)/a^(5/3)-1/3*ln(a+b*c
osh(x)^3)/a+1/2*sech(x)^2/a-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*cosh
(x))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx$$
$$= \frac{-6x+6\log(\cosh(x))-2\text{RootSum}\left[b+3b\#1^2+8a\#1^3+3b\#1^4+b\#1^6\&,\frac{-bx+b\log\left(e^x-\#1\right)-4ax\#1^3+4a\,\text{lo}}{b+2b\#}\right.}{6a}$$

input
```
Integrate[Tanh[x]^3/(a + b*Cosh[x]^3),x]
```

output
```
(-6*x + 6*Log[Cosh[x]] - 2*RootSum[b + 3*b*#1^2 + 8*a*#1^3 + 3*b*#1^4 + b*
#1^6 & , (-(b*x) + b*Log[E^x - #1] - 4*a*x*#1^3 + 4*a*Log[E^x - #1]*#1^3 -
 3*b*x*#1^4 + 3*b*Log[E^x - #1]*#1^4)/(b + 2*b*#1^2 + 4*a*#1^3 + b*#1^4) &
 ] + 3*Sech[x]^2)/(6*a)
```

3.81.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 3709, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i}{\tan\left(\frac{\pi}{2}+ix\right)^3\left(a+b\sin\left(\frac{\pi}{2}+ix\right)^3\right)}dx$$
$$\Big\downarrow \; 26$$
$$-i\int \frac{1}{\left(b\sin\left(ix+\frac{\pi}{2}\right)^3+a\right)\tan\left(ix+\frac{\pi}{2}\right)^3}dx$$
$$\Big\downarrow \; 3709$$

$$-\int \frac{\left(1-\cosh ^2(x)\right) \operatorname{sech}^3(x)}{b \cosh ^3(x)+a}d\cosh (x)$$

$$\Big\downarrow \textbf{2373}$$

$$-\int \left(\frac{\operatorname{sech}^3(x)}{a}-\frac{\operatorname{sech}(x)}{a}+\frac{b \left(\cosh ^2(x)-1\right)}{a \left(b \cosh ^3(x)+a\right)}\right)d\cosh (x)$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{b^{2/3} \arctan \left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} \cosh (x)}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{5/3}}-\frac{b^{2/3} \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} \cosh (x)+b^{2/3} \cosh ^2(x)\right)}{6 a^{5/3}}+\frac{b^{2/3} \log \left(\sqrt[3]{a}+\sqrt[3]{b} \cosh (x)\right)}{3 a^{5/3}}-\frac{\log \left(a+b \cosh ^3(x)\right)}{3 a}+\frac{\operatorname{sech}^2(x)}{2 a}+\frac{\log (\cosh (x))}{a}$$

input
```
Int[Tanh[x]^3/(a + b*Cosh[x]^3),x]
```

output
```
-((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Cosh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(5/3))) + Log[Cosh[x]]/a + (b^(2/3)*Log[a^(1/3) + b^(1/3)*Cosh[x]])/(
3*a^(5/3)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Cosh[x] + b^(2/3)*Cosh
[x]^2])/(6*a^(5/3)) - Log[a + b*Cosh[x]^3]/(3*a) + Sech[x]^2/(2*a)
```

3.81.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.81.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.61

method	result
risch	$\frac{2\,\mathrm{e}^{2x}}{(1+\mathrm{e}^{2x})^2 a}+\frac{\ln(1+\mathrm{e}^{2x})}{a}+\left(\sum_{\_R=\mathrm{RootOf}(27a^5\_Z^3+27a^4\_Z^2+9a^3\_Z+a^2-b^2)}\_R\ln\left(\mathrm{e}^{2x}+\left(\frac{6a^2\_R}{b}+\frac{2a}{b}\right)\mathrm{e}^{x}+\right.\right.$
default	$\frac{\frac{2}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)-\frac{2}{1+\tanh\left(\frac{x}{2}\right)^2}}{a}-\frac{\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^3+(-3a-3b)\_Z^2+(3a-3b)\_Z-a-b\right)}\frac{\left(\_R^2a-\_R^2b-2\right.}{\_R^2a-}}{3a}$

input
```
int(tanh(x)^3/(a+b*cosh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2*exp(2*x)/(1+exp(2*x))^2/a+1/a*ln(1+exp(2*x))+sum(_R*ln(exp(2*x)+(6*a^2/b
*_R+2*a/b)*exp(x)+1),_R=RootOf(27*_Z^3*a^5+27*_Z^2*a^4+9*_Z*a^3+a^2-b^2))
```

3.81.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 1435, normalized size of antiderivative = 9.38

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^
2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x)
+ a)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3
) + 2/a)*log(b*cosh(x)^2 + b*sinh(x)^2 - (a^2*cosh(x) + a^2*sinh(x))*((1/2
)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a) +
 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b) - 24*cosh(x)^2 + (6*cosh(x)^
4 + 24*cosh(x)*sinh(x)^3 + 6*sinh(x)^4 + 12*(3*cosh(x)^2 + 1)*sinh(x)^2 -
(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*
a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*((1/
2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)
- 3*sqrt(1/3)*(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cos
h(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sin
h(x) + a)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^
2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a
^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2) + 12*cosh(x)^2 + 24*(cosh(x
)^3 + cosh(x))*sinh(x) + 6)*log(b*cosh(x)^2 + b*sinh(x)^2 + 1/2*(a^2*cosh(
x) + a^2*sinh(x))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b
^2)/a^5)^(1/3) + 2/a) + 3/2*sqrt(1/3)*(a^2*cosh(x) + a^2*sinh(x))*sqrt(-((
(1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/
a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2...
```

3.81.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx = \int \frac{\tanh^3{(x)}}{a+b\cosh^3{(x)}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*cosh(x)**3),x)
```

output
```
Integral(tanh(x)**3/(a + b*cosh(x)**3), x)
```

3.81.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx = \int \frac{\tanh(x)^3}{b\cosh(x)^3+a}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)^3),x, algorithm="maxima")
```

output
```
2*b*(x/(a*b) - integrate((b*e^(5*x) + 3*b*e^(3*x) + 8*a*e^(2*x) + 3*b*e^x)
*e^x/(b*e^(6*x) + 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*x) + b), x)/(a*b))
+ 6*b*integrate(e^(4*x)/(b*e^(6*x) + 3*b*e^(4*x) + 8*a*e^(3*x) + 3*b*e^(2*
x) + b), x)/a - 2*(x*e^(4*x) + (2*x - 1)*e^(2*x) + x)/(a*e^(4*x) + 2*a*e^(
2*x) + a) + log(e^(2*x) + 1)/a + 8*integrate(e^(3*x)/(b*e^(6*x) + 3*b*e^(4
*x) + 8*a*e^(3*x) + 3*b*e^(2*x) + b), x)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx = &-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+e^{(-x)}+e^{x}\right|\right)}{3\,a^2} \\ &+\frac{\log\left(e^{(-x)}+e^{x}\right)}{a}-\frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)^3+8\,a\right|\right)}{3\,a} \\ &+\frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+e^{(-x)}+e^{x}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2} \\ &+\frac{(-ab^2)^{\frac{1}{3}}\log\left(\left(e^{(-x)}+e^{x}\right)^2+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\left(e^{(-x)}+e^{x}\right)+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2} \\ &-\frac{3\left(e^{(-x)}+e^{x}\right)^2-4}{2\,a\left(e^{(-x)}+e^{x}\right)^2}\end{aligned}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)^3),x, algorithm="giac")
```

output
```
-1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) + e^(-x) + e^x))/a^2 + log(e^(
-x) + e^x)/a - 1/3*log(abs(b*(e^(-x) + e^x)^3 + 8*a))/a + 1/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + e^(-x) + e^x)/(-a/b)^(1/3))/
a^2 + 1/6*(-a*b^2)^(1/3)*log((e^(-x) + e^x)^2 + 2*(-a/b)^(1/3)*(e^(-x) + e
^x) + 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) + e^x)^2 - 4)/(a*(e^(-x) + e^x)
^2)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 1173, normalized size of antiderivative = 7.67

$$\int \frac{\tanh^3(x)}{a+b\cosh^3(x)}\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^3/(a + b*cosh(x)^3),x)
```

output
```
2/(a + a*exp(2*x)) - 2/(a + 2*a*exp(2*x) + a*exp(4*x)) + symsum(log(-(5033
1648*a^6*exp(2*x) - 786432*b^6*exp(2*x) + 452984832*root(27*a^5*z^3 + 27*a
^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^7 + 50331648*a^6 - 786432*b^6 + 1358
954496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^2*a^8 + 1
358954496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^9
+ 50593792*a^2*b^4 - 102498304*a^4*b^2 + 1358954496*root(27*a^5*z^3 + 27*a
^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^2*a^8*exp(2*x) + 1358954496*root(27*a^
5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^3*a^9*exp(2*x) + 50593792*
a^2*b^4*exp(2*x) - 102498304*a^4*b^2*exp(2*x) + 7602176*root(27*a^5*z^3 +
27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^3*b^4 - 465305600*root(27*a^5*z^
3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^5*b^2 + 524288*a*b^5*exp(x)
+ 24379392*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)^2*a^4
*b^4 - 1383333888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k
)^2*a^6*b^2 + 18874368*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2,
 z, k)^3*a^5*b^4 - 1370750976*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2
 - b^2, z, k)^3*a^7*b^2 + 452984832*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z
 + a^2 - b^2, z, k)*a^7*exp(2*x) - 5242880*a^3*b^3*exp(x) - 524288*root(27
*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^2*b^5*exp(x) - 891289
6*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^4*b^3*exp(x)
 + 7602176*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*a^...
```

3.82 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx$

3.82.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-2/3*arctanh((a+b*cosh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]^3],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.82.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 26, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(\frac{\pi}{2}+ix\right)\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^3}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^3+a}\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \frac{\text{sech}(x)}{\sqrt{a+b\cosh^3(x)}}d\cosh(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\text{sech}(x)}{\sqrt{b\cosh^3(x)+a}}d\cosh^3(x)$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{\cosh^6(x)}{b}-\frac{a}{b}}d\sqrt{b\cosh^3(x)+a}}{3b}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Cosh[x]^3],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

3.82.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.82. $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx$

3.82.4 Maple [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)^3}}dx$$

input
```
int(tanh(x)/(a+b*cosh(x)^3)^(1/2),x)
```

output
```
int(tanh(x)/(a+b*cosh(x)^3)^(1/2),x)
```

3.82.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   fail
ed of mode Union(SparseUnivariatePolynomial(Expression(Integer)),failed) c
annot be coerced to mode SparseUnivariatePolynomial(Expression(Integer))
```

3.82.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)**3)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*cosh(x)**3), x)
```

3.82.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx=\int \frac{\tanh{(x)}}{\sqrt{b\cosh{(x)}^3+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x)^3 + a), x)
```

3.82.8 Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx=\int \frac{\tanh{(x)}}{\sqrt{b\cosh{(x)}^3+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x)^3 + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^3(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{b\cosh{(x)}^3+a}}\,dx$$

input
```
int(tanh(x)/(a + b*cosh(x)^3)^(1/2),x)
```

output
```
int(tanh(x)/(a + b*cosh(x)^3)^(1/2), x)
```

3.83 $\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx$

3.83.1 Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = -\frac{2}{3}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\cosh^3(x)}$$

output
```
-2/3*arctanh((a+b*cosh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*cosh(x)^3)^(1
/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = -\frac{2}{3}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\cosh^3(x)}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]^3]*Tanh[x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Cosh
[x]^3])/3
```

3.83.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\cosh^3(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^3}}{\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^3+a}}{\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \text{sech}(x)\sqrt{a+b\cosh^3(x)}d\cosh(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \sqrt{b\cosh^3(x)+a}\text{sech}(x)d\cosh^3(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\int \frac{\text{sech}(x)}{\sqrt{b\cosh^3(x)+a}}d\cosh^3(x)+2\sqrt{a+b\cosh^3(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2a\int \frac{1}{\frac{\cosh^6(x)}{b}-\frac{a}{b}}d\sqrt{b\cosh^3(x)+a}}{b}+2\sqrt{a+b\cosh^3(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(2\sqrt{a+b\cosh^3(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh^3(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*Cosh[x]^3]*Tanh[x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^3]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]^
3])/3
```

3.83.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.83.4 Maple [F]

$$\int \sqrt{a+b\cosh\left(x\right)^{3}}\,\tanh\left(x\right)dx$$

input
```
int((a+b*cosh(x)^3)^(1/2)*tanh(x),x)
```

output
```
int((a+b*cosh(x)^3)^(1/2)*tanh(x),x)
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(33) = 66$.

Time = 0.79 (sec) , antiderivative size = 1648, normalized size of antiderivative = 36.62

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(x)^3)^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*(cosh(x) + sinh(x))*log(-(b^2*cosh(x)^12 + 12*b^2*cosh(x)*si
nh(x)^11 + b^2*sinh(x)^12 + 6*b^2*cosh(x)^10 + 64*a*b*cosh(x)^9 + 6*(11*b^
2*cosh(x)^2 + b^2)*sinh(x)^10 + 15*b^2*cosh(x)^8 + 4*(55*b^2*cosh(x)^3 + 1
5*b^2*cosh(x) + 16*a*b)*sinh(x)^9 + 192*a*b*cosh(x)^7 + 3*(165*b^2*cosh(x)
^4 + 90*b^2*cosh(x)^2 + 192*a*b*cosh(x) + 5*b^2)*sinh(x)^8 + 24*(33*b^2*co
sh(x)^5 + 30*b^2*cosh(x)^3 + 96*a*b*cosh(x)^2 + 5*b^2*cosh(x) + 8*a*b)*sin
h(x)^7 + 192*a*b*cosh(x)^5 + 4*(128*a^2 + 5*b^2)*cosh(x)^6 + 4*(231*b^2*co
sh(x)^6 + 315*b^2*cosh(x)^4 + 1344*a*b*cosh(x)^3 + 105*b^2*cosh(x)^2 + 336
*a*b*cosh(x) + 128*a^2 + 5*b^2)*sinh(x)^6 + 15*b^2*cosh(x)^4 + 24*(33*b^2*
cosh(x)^7 + 63*b^2*cosh(x)^5 + 336*a*b*cosh(x)^4 + 35*b^2*cosh(x)^3 + 168*
a*b*cosh(x)^2 + 8*a*b + (128*a^2 + 5*b^2)*cosh(x))*sinh(x)^5 + 64*a*b*cosh
(x)^3 + 3*(165*b^2*cosh(x)^8 + 420*b^2*cosh(x)^6 + 2688*a*b*cosh(x)^5 + 35
0*b^2*cosh(x)^4 + 2240*a*b*cosh(x)^3 + 320*a*b*cosh(x) + 20*(128*a^2 + 5*b
^2)*cosh(x)^2 + 5*b^2)*sinh(x)^4 + 6*b^2*cosh(x)^2 + 4*(55*b^2*cosh(x)^9 +
 180*b^2*cosh(x)^7 + 1344*a*b*cosh(x)^6 + 210*b^2*cosh(x)^5 + 1680*a*b*cos
h(x)^4 + 480*a*b*cosh(x)^2 + 20*(128*a^2 + 5*b^2)*cosh(x)^3 + 15*b^2*cosh(
x) + 16*a*b)*sinh(x)^3 + 6*(11*b^2*cosh(x)^10 + 45*b^2*cosh(x)^8 + 384*a*b
*cosh(x)^7 + 70*b^2*cosh(x)^6 + 672*a*b*cosh(x)^5 + 320*a*b*cosh(x)^3 + 10
*(128*a^2 + 5*b^2)*cosh(x)^4 + 15*b^2*cosh(x)^2 + 32*a*b*cosh(x) + b^2)*si
nh(x)^2 + b^2 - 16*(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 +...
```

3.83.6 Sympy [F]

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = \int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx$$

input
```
integrate((a+b*cosh(x)**3)**(1/2)*tanh(x),x)
```

output
```
Integral(sqrt(a + b*cosh(x)**3)*tanh(x), x)
```

3.83.7 Maxima [F]

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = \int \sqrt{b\cosh{(x)}^3+a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x)^3)^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x)^3 + a)*tanh(x), x)
```

3.83.8 Giac [F]

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = \int \sqrt{b\cosh{(x)}^3+a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x)^3)^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x)^3 + a)*tanh(x), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh^3(x)}\tanh(x)\,dx = \int \tanh(x)\,\sqrt{b\cosh{(x)}^3+a}\,dx$$

input
```
int(tanh(x)*(a + b*cosh(x)^3)^(1/2),x)
```

output
```
int(tanh(x)*(a + b*cosh(x)^3)^(1/2), x)
```

3.84 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx$

3.84.1 Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]^n],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.84.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 26, 3709, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(\frac{\pi}{2}+ix\right)\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^n}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^n+a}\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3709}$$

$$\int \frac{\text{sech}(x)}{\sqrt{a+b\cosh^n(x)}}d\cosh(x)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{\text{sech}(x)}{\sqrt{b\cosh^n(x)+a}}d\cosh^n(x)}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{\cosh^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\cosh^n(x)+a}}{bn}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Cosh[x]^n],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.84.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)^{n}}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)^{n}}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24

input
```
int(tanh(x)/(a+b*cosh(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.90

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx$$
$$= \left[\frac{\log\left(\frac{b\cosh(n\log(\cosh(x)))+b\sinh(n\log(\cosh(x)))-2\sqrt{b\cosh(n\log(\cosh(x)))+b\sinh(n\log(\cosh(x)))+a}\sqrt{a}+2a}{\cosh(n\log(\cosh(x)))+\sinh(n\log(\cosh(x)))}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\right.}{}\right.$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*cosh(n*log(cosh(x))) + b*sinh(n*log(cosh(x))) - 2*sqrt(b*cosh(n*lo
g(cosh(x))) + b*sinh(n*log(cosh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(cosh(
x))) + sinh(n*log(cosh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(b*cosh(n
*log(cosh(x))) + b*sinh(n*log(cosh(x))) + a)*sqrt(-a)/a)/(a*n)]
```

3.84.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)**n)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*cosh(x)**n), x)
```

3.84.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\cosh(x)^n + a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x)^n + a), x)
```

3.84.8 Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\cosh(x)^n + a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x)^n + a), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh^n(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)^n}}\,dx$$

input
```
int(tanh(x)/(a + b*cosh(x)^n)^(1/2),x)
```

output
```
int(tanh(x)/(a + b*cosh(x)^n)^(1/2), x)
```

3.85 $\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx$

3.85.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx=-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\cosh^n(x)}}{n}$$

output
```
-2*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*cosh(x)^n)^(1/2
)/n
```

3.85.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx=\frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh^n(x)}}{n}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]^n]*Tanh[x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]^
n])/n
```

3.85.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 3709, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\cosh^n(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)^n}}{\tan\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sqrt{b\sin\left(ix+\frac{\pi}{2}\right)^n+a}}{\tan\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3709$$

$$\int \operatorname{sech}(x)\sqrt{a+b\cosh^n(x)}d\cosh(x)$$

$$\Big\downarrow \; 798$$

$$\frac{\int \sqrt{b\cosh^n(x)+a}\operatorname{sech}(x)d\cosh^n(x)}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{a\int \frac{\operatorname{sech}(x)}{\sqrt{b\cosh^n(x)+a}}d\cosh^n(x)+2\sqrt{a+b\cosh^n(x)}}{n}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{\cosh^{2n}(x)}{b}-\frac{a}{b}}d\sqrt{b\cosh^n(x)+a}}{b}+2\sqrt{a+b\cosh^n(x)}}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{a+b\cosh^n(x)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh^n(x)}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[a + b*Cosh[x]^n]*Tanh[x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]^
n])/n
```

3.85.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3709
```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff^(m + 1)/f   Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m +
1)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] &&
ILtQ[(m - 1)/2, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\cosh(x)^n}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)^n}}{\sqrt{a}}\right)}{n}$	38
default	$\frac{2\sqrt{a+b\cosh(x)^n}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)^n}}{\sqrt{a}}\right)}{n}$	38

input
```
int((a+b*cosh(x)^n)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*cosh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*cosh(x)^n)^(1/2)/a^(1/
2)))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.32

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx$$
$$=\left[\frac{\sqrt{a}\log\left(\frac{b\cosh(n\log(\cosh(x)))+b\sinh(n\log(\cosh(x)))-2\sqrt{b\cosh(n\log(\cosh(x)))+b\sinh(n\log(\cosh(x)))+a}\sqrt{a}+2a}{\cosh(n\log(\cosh(x)))+\sinh(n\log(\cosh(x)))}\right)+2\sqrt{b\cosh(n}{n}\right.$$

input
```
integrate((a+b*cosh(x)^n)^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*cosh(n*log(cosh(x))) + b*sinh(n*log(cosh(x))) - 2*sqrt(b*
cosh(n*log(cosh(x))) + b*sinh(n*log(cosh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*
log(cosh(x))) + sinh(n*log(cosh(x))))) + 2*sqrt(b*cosh(n*log(cosh(x))) + b
*sinh(n*log(cosh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*cosh(n*log(cosh(x
))) + b*sinh(n*log(cosh(x))) + a)*sqrt(-a)/a) + sqrt(b*cosh(n*log(cosh(x))
) + b*sinh(n*log(cosh(x))) + a))/n]
```

3.85.6 Sympy [F]

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx = \int \sqrt{a+b\cosh^n{(x)}}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x)**n)**(1/2)*tanh(x),x)
```

output
```
Integral(sqrt(a + b*cosh(x)**n)*tanh(x), x)
```

3.85.7 Maxima [F]

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx = \int \sqrt{b\cosh{(x)}^n+a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x)^n)^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x)^n + a)*tanh(x), x)
```

3.85.8 Giac [F]

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx = \int \sqrt{b\cosh{(x)}^n+a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x)^n)^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x)^n + a)*tanh(x), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh^n(x)}\tanh(x)\,dx = \int \tanh(x)\,\sqrt{a+b\cosh(x)^n}\,dx$$

input
```
int(tanh(x)*(a + b*cosh(x)^n)^(1/2),x)
```

output
```
int(tanh(x)*(a + b*cosh(x)^n)^(1/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```